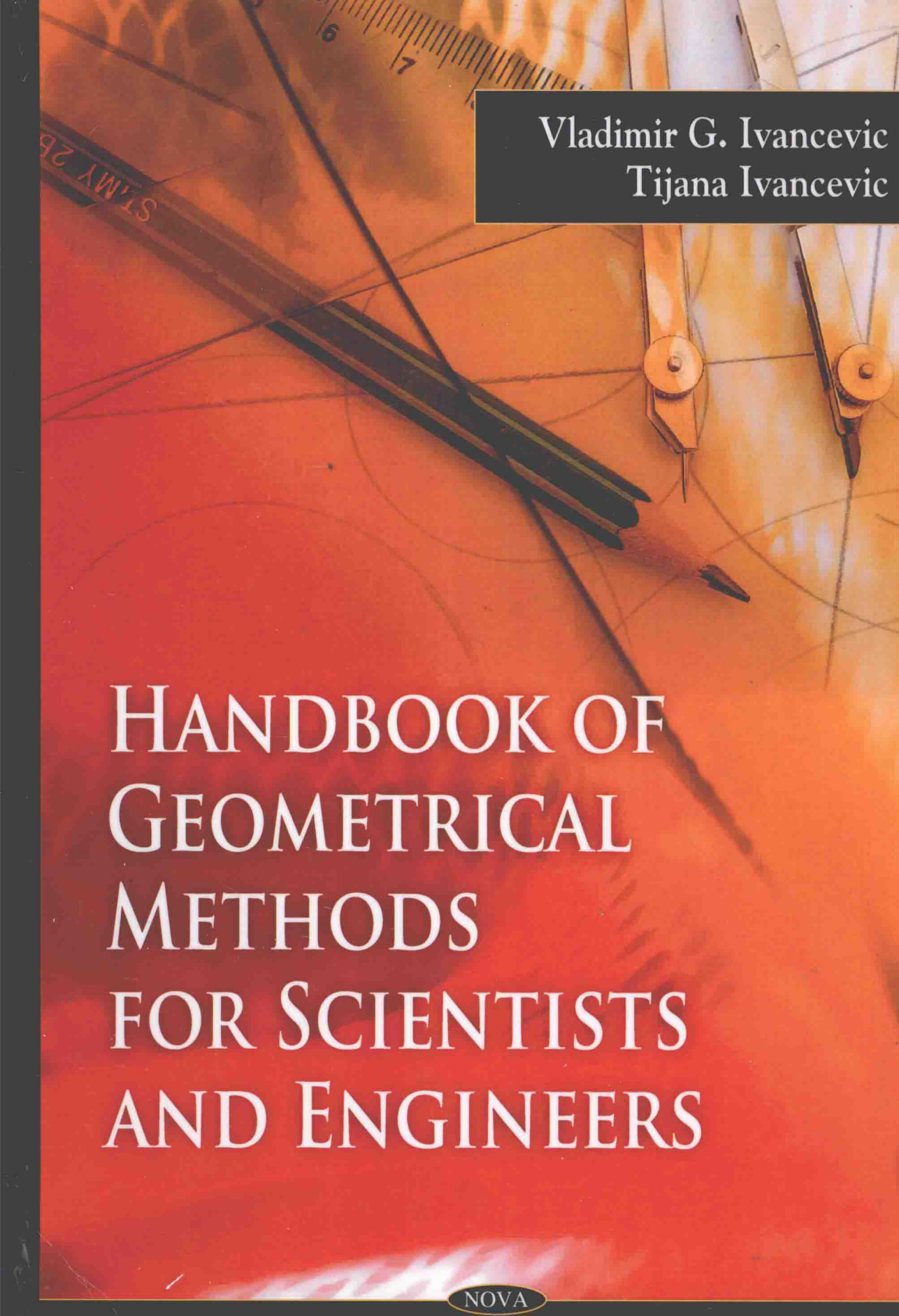


The background of the cover is a detailed illustration of geometric construction. It features a large orange circle, a ruler with markings for 6 and 7, a compass, and a pair of dividers. A pencil is also visible. The text is overlaid on this background.

Vladimir G. Ivancevic
Tijana Ivancevic

HANDBOOK OF GEOMETRICAL METHODS FOR SCIENTISTS AND ENGINEERS

HANDBOOK OF GEOMETRICAL METHODS FOR SCIENTISTS AND ENGINEERS

VLADIMIR G. IVANCEVIC

AND

TIJANA IVANCEVIC



Nova Science Publishers, Inc.

New York

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LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Ivancevic, Vladimir G.

Handbook of geometrical methods for scientists and engineers / Vladimir G. Ivancevic, Tijana Ivancevic.
p. cm.

Includes bibliographical references and index.

ISBN 978-1-60741-769-9 (hardcover)

1. Geometric analysis. 2. Geometrical models. I. Ivancevic, Tijana T. II. Title.

QA300.I855 2009

516--dc22

2009018844

Published by Nova Science Publishers, Inc. ✦ New York

**HANDBOOK OF GEOMETRICAL
METHODS FOR SCIENTISTS
AND ENGINEERS**

Dedicated to Nick, Atma and Kali

Preface

Handbook of Geometrical Methods for Scientists and Engineers is an undergraduate applied mathematics text, compiled as a collection of *concepts* and *formulas* of modern geometrical and topological methods designed for use in science and engineering. These geometrical methods are currently being used for modelling complex systems in theoretical physics, chemistry and biology, nonlinear dynamics and nonlinear control, as well as mathematically-enriched human sciences (medicine, psychology, sociology and economics).

The *Handbook* contains an easy-to-follow essence of geometrical and topological methods for modelling complex dynamical systems, extracted from our five graduate-level monographs (including over 2000 cited references in total):

1. Geometrical Dynamics of Complex Systems: A Unified Modelling Approach to Physics, Control, Biomechanics, Neurodynamics and Psycho-Socio-Economical Dynamics. Springer, 2006;
2. Complex Dynamics: Advanced System Dynamics in Complex Variables, Springer, 2007;
3. Applied Differential Geometry: A Modern Introduction. World Scientific, 2007;
4. Complex Nonlinearity: Chaos, Phase Transitions, Topology Change and Path Integrals, Springer, 2008; and
5. Quantum Leap: From Dirac and Feynman, Across the Universe, to Human Body and Mind. World Scientific, Singapore, 2008.

The only necessary background for efficient understanding and using of the *Handbook* is standard Engineering Mathematics IB (namely, Calculus and Linear Algebra).

Adelaide,
Feb 2009

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Acknowledgments

The authors wish to thank Land Operations Division, Defence Science & Technology Organisation, Australia, for their support in developing the world-leading human motion simulator called *Human Biodynamics Engine* and all related text in this Handbook.

We also express our sincere gratitude to *Nova Science Publishers* and especially to Mr. Frank Columbus, Ms. Stephanie Gonzalez and Ms. Maya Columbus.

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Part I

Geometrical Preliminaries

