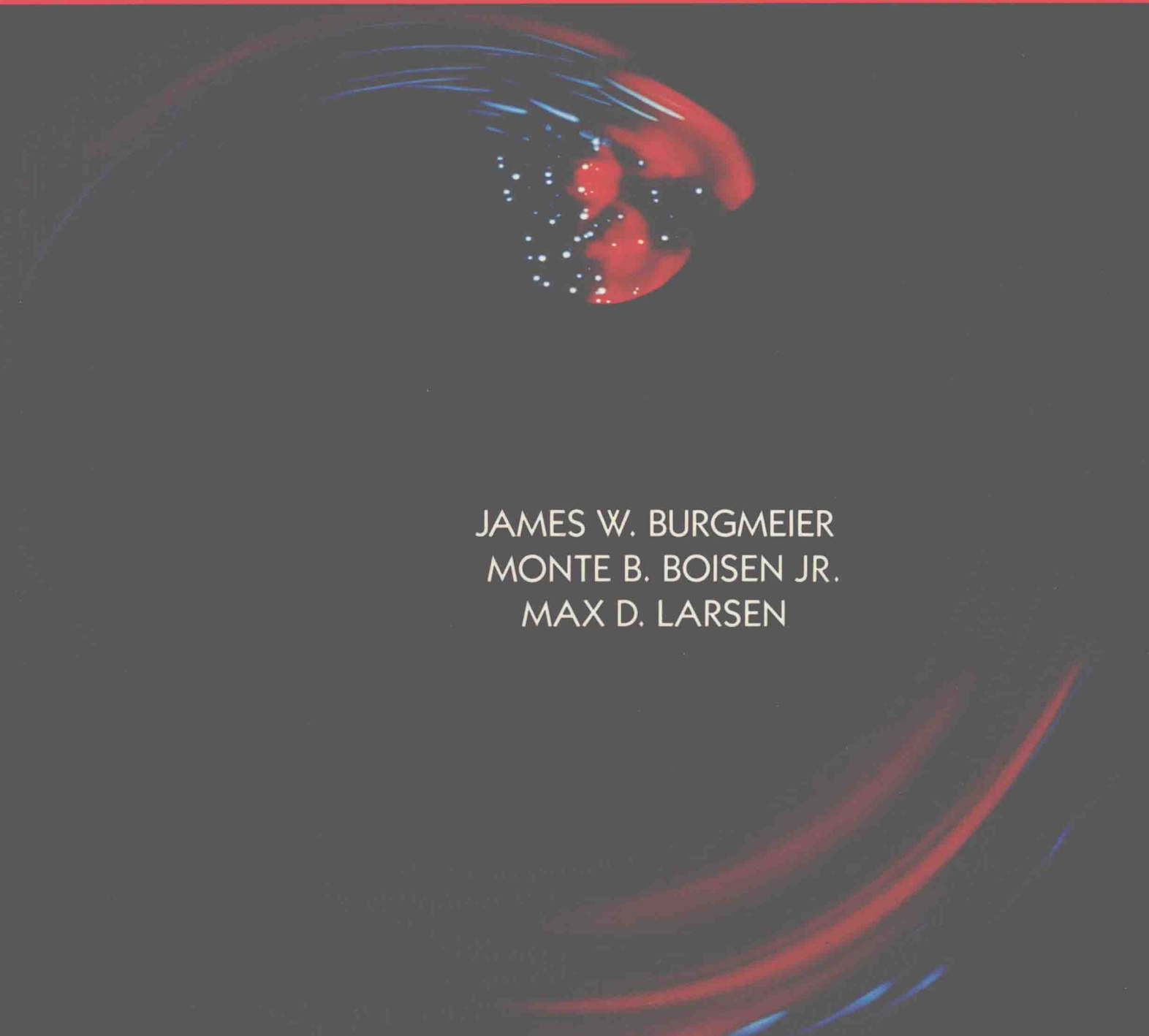


# BRIEF CALCULUS

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# WITH APPLICATIONS

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JAMES W. BURGMEIER  
MONTE B. BOISEN JR.  
MAX D. LARSEN

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# • B • R • I • E • F • • C • A • L • C • U • L • U • S • WITH APPLICATIONS

James W. Burgmeier  
University of Vermont

Monte B. Boisen, Jr.  
Virginia Tech

Max D. Larsen  
The Gallup Organization

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## BRIEF CALCULUS WITH APPLICATIONS

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This text is dedicated to our families, for their patience, support, and love.

Pat, Glenn, and Robin

Helen, John, and Jennifer

Lillie, Mike, Paul, and Charlie

# PREFACE

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The purpose of this text is to develop the student's understanding of the basic concepts of calculus through realistic applications from the managerial, social, and life sciences. Numerous examples motivate the theoretical concepts and show how calculus can be applied in these areas. Realistic applications show that the mathematics presented in this course will have an impact on the student's future work. This approach ensures that the student learns the concepts, power, and usefulness of calculus.

The text presents the fundamentals of calculus intuitively and clearly without sacrificing accuracy. The balance of prose explanations and algebraic details found in the solutions to examples is the result of years of classroom experience. Instructors and students alike will find the text clear, concise, accurate, and readable.

## Distinguishing Features

- ***Emphasis on examples:*** The many carefully explained examples illustrate procedures, show applications of the material, and elaborate on mathematical content. In many cases, the examples are somewhat idealized: we have adjusted the numerical constants to avoid tedious and distracting arithmetic. Students can thus focus attention on the calculus concept involved and grasp the mathematics more easily and quickly.
- ***Emphasis on applications:*** We have included a wealth of applications to the managerial, social, and life sciences adapted from the recent literature of these sciences. Each application has been chosen to represent faithfully the discussion from which it came as well as to provide a straightforward illustration of the calculus topic under study. Examples from each of the managerial, social, and life sciences are presented for most of the mathematical topics in this text, so instructors can emphasize one discipline or another according to the interests of the students. These examples require no specialized knowledge. The range of applications provides evidence that the mathematics learned in this course has relevancy and usefulness beyond the classroom.
- ***Exercises:*** Over 2400 exercises are included to give students ample opportunity to test their understanding and apply their

skills to real-world problems. Our experience shows that the student's understanding of calculus is linked directly to the number of problem-types solved successfully; we believe that the quality and variety of our exercises will foster this success. The answers to the odd-numbered exercises are given at the back of the book, and many of these include intermediate results so students can check their work. Answers to the even-numbered exercises are given in the Instructor's Manual.

- ♦ **Figures and photographs:** Wherever possible, we have used figures to motivate new concepts and as an aid in problem-solving. These figures are clearly drawn and carefully captioned and labeled to enhance student comprehension. In addition, the abundance of photographs throughout the text attests to the range of applied problems in this book and the diverse uses of mathematics. Every photo is carefully keyed to an applied example or topic in the text, illustrating the usefulness of calculus in the real world.
- ♦ **Bibliography:** The annotated bibliography lists sources for the applied problems and other works in which calculus is used at the level presented in this text. Readers who would like more information on the applications will find numerous source materials listed here.

## Special Pedagogical Issues

- ♦ **Limits and continuity:** While our discussion of limits is intuitive, it does not present the material in a dishonest fashion. We have made a special effort to use applications to set the stage in Section 2.1 for the need for limits. Section 2.2 then discusses limits and how to calculate the limits of various functions. We have deliberately postponed continuity until Section 2.7 so that students can go immediately to the derivative in Section 2.3. Instructors who wish to omit Section 2.7 may do so without disruption. This organization should hold the student's attention and clarify why these concepts are important.
- ♦ **Derivatives:** Section 2.3 uses the informal treatment of limits given in Sections 2.1 and 2.2 to define the derivative and cover the simple algebraic rules for finding derivatives. The product, quotient, and chain rules are postponed until Section 3.1 in order to present some of the uses of the derivative immediately. Our experience has shown that this approach has more appeal for students.

Sections 2.4 to 2.6 discuss several ways to interpret the derivative. Section 2.4 covers the slope of the tangent line and how it is related to the derivative. Section 2.5 discusses increasing and decreasing functions so students see a direct implementation of the tangent line concept. Section 2.6 uses applications to explore the derivative as a rate of change. These sections discuss various interpretations of the derivative and several of its applications.

- ♦ **Curve sketching:** In Section 4.2 we present a checklist for graphing a function. Sign charts and carefully labeled preliminary sketches accompany the examples; the final graphs are labeled with the important features of the functions. A detailed summary of the data is given under the caption of each figure. The examples and exercises in this section cover various types of functions to give students plenty of practice in graphing functions.
- ♦ **Numerical methods:** Computing technology has evolved enormously in the past several years and an increasing number of disciplines use numerical methods to approximate solutions to mathematical problems and to analyze data. Therefore, we introduce a variety of numerical methods throughout the text to acquaint students with the power and scope of these techniques. We discuss the total differential of several variables (Section 8.5), numerical integration (Sections 6.5 and 7.4), and the method of least squares (Section 8.6).

## Features for the Student

- ♦ **Algebra review:** We have assumed that students taking this course will have had two years of high-school algebra or the equivalent. However, Appendix A includes a review of basic algebra. The numerous examples and exercises enable students to refresh their knowledge of algebra and to pinpoint those areas in which a deficiency may exist. They also give the instructor an option to cover as much review as needed.
- ♦ **Algebraic steps:** We know that students get discouraged if too many algebraic details are omitted. Therefore, in the exposition and in the examples, algebraic steps are clearly marked and explained. Many answers in the back of the book also include steps to the solution so students can check intermediate results. By providing intermediate steps we encourage students to persist in their problem-solving efforts.
- ♦ **Table format:** Except for some of the examples, the material in the text has been classroom-tested over a period of several years. Many pedagogical aspects of the text stem from this testing, including our special table format for many procedures. This format takes advantage of the fact that a compact array of information is more easily assimilated than a long list. These table formats have proved to be successful for learning and implementing many procedures, including integration by parts, the trapezoidal rule, Simpson's rule, the derivative tests for one and two independent variables, and the method of least squares.
- ♦ **Plausibility arguments:** The primary pedagogical tool we use is examples. Therefore, we have included only those proofs and theoretical discussions that contribute to an understanding of calculus and its applications. Even in these cases we often use

informal arguments since we believe students will find them more meaningful than formal mathematical proofs.

### For Each Chapter

- ♦ **Chapter introduction:** Most of the chapter introductions discuss applications of the material; these applications and many others are then explored and developed in the examples and exercises of that chapter.
- ♦ **Summary:** Each chapter concludes with a concise summary highlighting the chapter's important concepts, definitions, formulas, and procedures. Students will find this summary an excellent study and review tool.
- ♦ **Review exercises:** In addition, at the end of each chapter, there is an extensive set of review exercises to test student comprehension and problem-solving skills. In keeping with our emphasis on applications, almost every review exercise set includes applied problems as well as numerous exercises to test skills and understanding.

### Accuracy

To ensure accuracy, the exercises were solved independently by James W. Burgmeier and Wendy Fenwick. The examples have been solved and checked by several people, including Garret Etgen, Linda Holden, and Vincent P. Schielack. We are grateful for their keen eyes and attention to detail. If, however, you do find an error, please advise our publisher immediately so that it can be corrected in subsequent printings.

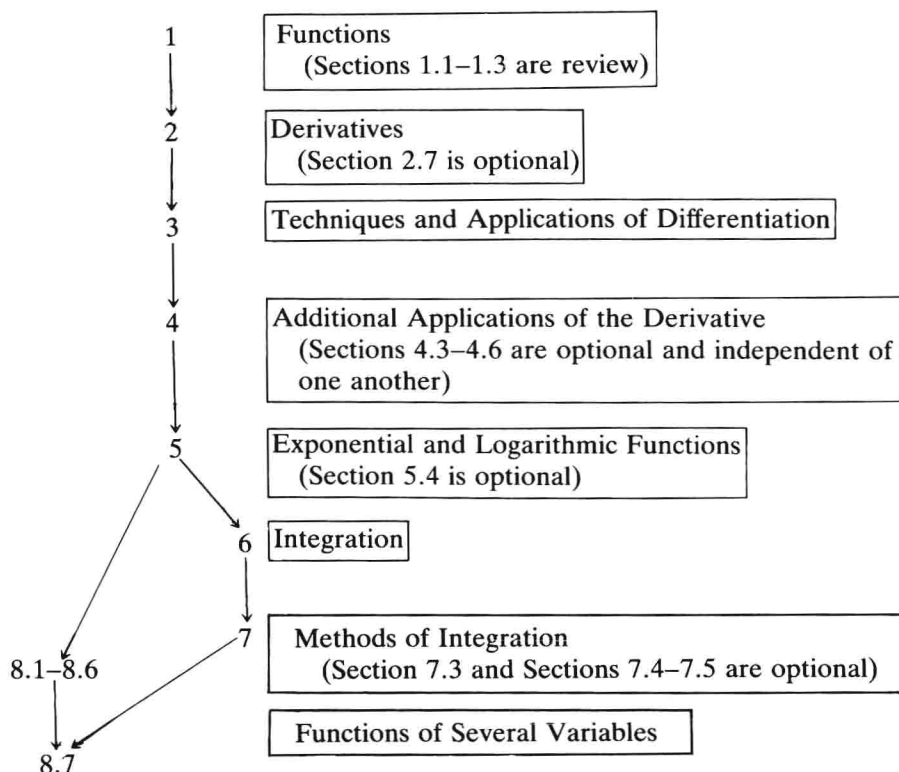
### Flexibility

Since several chapters are independent of one another, there are various possible sequences of chapters, as the accompanying schema illustrates. For a one-semester course for business or managerial students, Sections 8.1 through 8.6 may be substituted for Chapter 6. Students who have a thorough precalculus background could skip Sections 1.1 through 1.3 and briefly study the material in the remainder of Chapter 1. The other optional sections in Chapters 1 through 7 are: 2.7; 4.3 through 4.6; 5.4; 6.5; 7.3; and 7.4 through 7.5.

### Section Length

Generally, each section presents the amount of material that is appropriate for one fifty-minute lecture. Exceptions to this rule, such as Sections 4.2, 5.5, and 6.5, contain many examples and applications, rather than new mathematical material. Time devoted to these sections should depend on the disciplines represented in a particular class.



Chart of Dependent Chapters for *Brief Calculus with Applications*

## Supplements

This text is accompanied by a well-rounded supplements package, including Instructor's Manual, a Student Solutions Manual, Test Bank, Computerized Test Bank and software.

- *Student Solutions Manual*: The Student Solutions Manual was prepared by James W. Burgmeier and Wendy Fenwick and contains detailed solutions to selected odd-numbered exercises.
- *Instructor's Manual*: The Instructor's Manual was prepared by James W. Burgmeier and Wendy Fenwick, who prepared answers to even-numbered exercises, and Jan E. H. Johansson, who prepared sample tests. It includes three forms of a chapter test for each chapter, three forms for two mid-term exams, and three forms of two final exams to insure a plentiful supply of questions for instructors.
- *Test Bank*: The questions found in the sample tests, and additional test questions, are also available in a test bank form which corresponds to a computerized version for IBM-PC and compatibles.

The student supplements package also includes software. For more details, please contact your local McGraw-Hill Sales Representative.

## Reviewers

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*James W. Burgmeier  
Monte B. Boisen, Jr.  
Max D. Larsen*

# INDEX OF APPLICATIONS

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The following list is a selection from the approximately 600 applications in the text.

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## Business and Economics

Advertising in newspapers, 330, 364  
Agricultural equipment sales, 179  
Agricultural production, 332  
Cable-television subscription campaign, 185  
Cash management model, 177, 180  
Class size related to fee, 80  
Cobb–Douglas production function, 354–355  
College fund, 21, 23, 200  
Compound interest, 192–193, 194, 195, 214–216, 223, 224, 225  
Constructing houses, 6  
Consumer surplus, 264–266  
Cost function, 25, 40, 167, 168, 173, 174, 177, 180, 181, 187, 188, 234, 238, 252, 306, 313, 330, 331, 340  
Cost of building aquarium, 336–337  
Crop yield, 80, 144, 306, 307, 320, 324, 330, 332  
Demand function, 10, 27, 28, 29, 43, 112, 164, 174, 186, 188, 264, 265, 266, 323  
Effect of tax on price and profit, 171–172, 173  
Egg production rate on a chicken farm, 257  
Electronics manufacturing, 102  
Equilibrium price, 29  
Farm profits related to choice of livestock, 331  
Federal program to reduce unemployment, 120  
Import fees, 173–174  
Inspection for defective parts, 313  
Interest-bearing account, 192–193, 194, 200–201, 214–216, 223, 224, 225, 226  
Inventory model, 176  
Investment rate of return, 304  
Long-distance telephone costs, 30  
Manufacturing batch size, 174–176, 288, 291  
Manufacturing costs, 3, 10, 25–26, 80–81  
Marginal, cost function, 168, 170, 172, 173, 187, 230, 234, 235, 238, 252, 257, 268, 277

- demand, 323
- price, 257
- productivity of capital, 319, 323, 344, 346
- productivity of labor, 319, 323, 344–345, 346
- productivity of money, 336
- profit, 268
- revenue function, 169, 172, 173, 187, 234, 238, 239, 257, 277, 295, 299, 304
- Maximizing, manufacturing profits, 84–85, 144, 331
  - production on a budget, 332, 333–334, 335–336, 340
  - profit, 74, 138, 142, 144, 169–170, 171, 173, 174, 187, 329, 331, 364
  - profits with limited resources, 142
  - rental income, 144
  - revenue, 140, 172, 173, 174, 364
- Military spending versus welfare spending, 143
- Minimizing, batching and storage costs, 174–177, 179, 180, 181, 188
  - inventory ordering costs, 180
  - production costs, 340
- Newsletter design, 143
- Optimal company size, 109, 112
- Optimal order size, 176–177, 179, 180, 188
- Optimal production level, 170, 171, 173
- Penalty for price-fixing, 172–173
- Plumbing costs, 30
- Pollution from manufacturing plant, 323, 340
- Postal rate function, 3, 4, 58, 86
- Poverty level income, 31
- Price elasticity, of demand, 181–183, 184, 185, 186, 188
  - of supply, 186, 188
- Producer surplus, 267
- Production as a function of shift, 331
- Production cost, 25, 30, 40–41, 42, 73, 74, 112, 143, 145, 230, 235, 238, 252, 277
- Production function, 39, 86, 170, 171–172, 252, 268, 295, 298–299, 304, 306–307, 313, 319, 323, 330, 331, 332, 333, 335, 340, 344, 346, 364
- Production-inventory problem, 174, 179
- Production-storage problem, 174, 179, 188
- Productivity of manufacturing plant, 319, 323
- Profit change, from advertising, 84, 85
  - from using fertilizer, 75
- Profit from special edition of book, 144
- Profit function, 42, 44, 74, 84, 85, 86, 138, 142, 144, 148, 169, 170, 171, 173, 187, 268, 328–329, 330, 331, 364
- Publisher's profit from sale of novel, 330
- Publishing, 143, 144, 313, 330
- Rental car costs, 43
- Restaurant inventory, 180
- Restaurant tax, 184–185
- Revenue function, 10, 31, 80, 167, 169, 172, 174, 187, 234, 238, 239, 264, 313, 330

Sales analysis, 225, 238, 239, 240, 247, 248, 257, 266, 286, 300, 323, 356  
 Sales from advertising, 225, 240, 306, 323, 356  
 Salesperson's income, 43  
 Setting advertising level, 166  
 Shipping costs, 364  
 Stock market investment strategy, 138, 144, 187  
 Store's profit, 328–329  
 Supply function, 29, 43, 265, 266  
 Tax effect on price, 171–172, 173  
 Television advertising, 323  
 Venture-capital investment profits, 74, 86  
 Worth of manufacturing company, 102

## Life Sciences

Absorption rate of medicine, 220–221, 224  
 Animal diet cost, 340  
 Animal metabolism rate, 353  
 Athlete's workout, 284, 286  
 Bacteria population growth, 41, 85, 219, 223, 225, 268, 323, 351  
 Bacterial infection, 95, 225  
 Codling moth life cycle, 33, 37, 132  
 Cold medication to minimize symptoms, 330  
 Counting cricket chirps to measure temperature, 85  
 Decay of radioactive medicine, 223  
 Deer population program, 37  
 Drug concentration in bloodstream, 224  
 Earth's population, 223, 224  
 Field mice population growth, 257  
 Flu epidemic, 137–138, 143–144  
 Gerbil colony population, 42  
 Grasshopper population at various temperatures, 10  
 Growth rate of malignant tumor, 86  
 Insect infestation, 46–48, 51, 84  
 Landfill monitoring system, 41  
 Maze experiments, 2, 224  
 Mediterranean fruit fly infestation, 46–48, 51, 84  
 Mouse-to-elephant function (metabolism), 353  
 Park lands in city, 108–109  
 Pesticide/insecticide use, 46–48, 144, 330, 340  
 Photosynthesis rate of trees, 26–27, 30  
 Plant growth rate, 328, 331  
 Pollution index, 31, 320, 323, 343, 345  
 Population, density, 262–263, 266  
     growth, 145, 216–217, 223, 224  
     of United States, 224  
 Reaction rate to drugs, 306, 324

Reineke's equation, 222, 224  
Rodent control, 41, 80  
Runner's rate of respiration, 41  
    training program, 43, 143  
Sparrowhawk flight height, 277  
Standard urea clearance function, 343–344  
Tree, destruction by pests, 291  
    diameter at maturity, 222–223, 224  
Wildlife management approaches, 37  
Wood production, 71, 74, 85, 300

---

## Physical Sciences

Acceleration measurements, 83–84, 115, 239, 268, 277  
Air-monitoring system, 221, 225–226  
Animal holding pen, 364  
Average speed of a car, 38–39, 81–82, 247, 248  
Block of ice sliding down chute, 239  
Capacity of livestock watering tank, 268  
Car's speed before accident, 345  
Chemical reaction rate, 80  
Contamination of hay from Chernobyl explosion, 217–218  
Deceleration measurements, 85, 236, 237, 239  
Distance traveled, 85, 247–248, 253, 286–287, 291  
Diving-pool depth, 85  
Engine efficiency, 144  
Fahrenheit-to-Celsius conversion function, 24–25  
Fencing maximum pasture area, 139, 142, 146  
Flight of arrow, 83–84, 239  
Grain pile shape, 110  
Highway safety, 208  
Human cannonball, 85  
Maximizing capacity, 141  
Maximizing volume, 48–49, 51, 80, 129–130, 136–137  
Measurement conversion, 24–25, 30  
Minimizing surface area, 139, 142, 146  
Missile's altitude recorder, 61  
Oil slick, 112, 145  
Projectile, 236, 239, 240, 248, 253, 268  
Radioactive decay, 217–219, 223, 224  
Rocket trajectory, 239  
Stopping distances for various speeds, 237–238  
Temperature measurements, 10, 24, 30  
Trailer broken loose from car, 236–237  
Velocity, measurements, 104, 253, 257, 277–278, 286, 291, 296  
    of bomb dropped from airplane, 239  
    of bullet, 85  
    of hockey puck, 268

Water testing program, 253  
Weather, 266, 312  
Weather map showing isobars and isotherms, 311–312  
Window design, 143

---

## Social Sciences

Age of excavated bone, 218–219, 223  
Burglaries during police strike, 300  
Census, 93  
Community cooperation index, 357  
Crime rate, 330  
Customer receptivity, 331  
Education level and income level, 350–351, 358  
Election campaign strategies, 113, 117, 120, 143  
Final-exam grade versus study time, 223  
Fundraising campaign, 43  
Job-creation program, 34, 37  
Job satisfaction, 340  
Learning random sequences, 224  
Mobility index, 357  
Participation rank in foster-care center, 357  
Popularity rank in foster-care center, 357  
Psychology experiments, 2, 19–20, 23, 76–77, 80, 135, 143, 224, 277, 331, 339, 340  
Quality-of-living index in rural area, 112  
Reading proficiency test, 112  
Researcher productivity, 257  
Restaurant popularity, 268  
Sociology, 30, 257, 300, 330, 349–350, 357, 358  
    survey cost, 30  
Spatial perception experiment, 19, 23  
Standardized tests and amount of instruction, 6  
Student achievement index, 339  
Student receptivity to new concepts, 76–77, 80  
Students' test scores, 356  
Vacation planning, 340  
Worker efficiency, 357

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Chapter 4: page 147, Barbara Rios/Photo Researchers; page 170, Courtesy Harley-Davidson Co.; page 176, The Photo Works/Photo Researchers  
Chapter 5: page 189, Runk/Schoenberger/Grant Heilman; page 193, Hazel Hankin/Stock, Boston; page 214, Culver Pictures; page 219, Stan Levy/Photo Researchers  
Chapter 6: page 227, Earth Observation Satellite Co.; page 241, Paul Conklin  
Chapter 7: page 271, Barrera/TexaStock; page 278, Michael D. Sullivan, TexaStock; page 288, Topham/The Image Works  
Chapter 8: page 305, Ellis Herwig/Stock, Boston; page 328, Grant Heilman; page 353, Culver Pictures  
Chapter 9: page 365, Jack Spratt/The Image Works; page 395, John J. Krieger/Picture Cube  
Chapter 10: page 405, George Gardner; page 410, Mark Antman/The Image Works; page 415, Wide World Photos; page 443, Leonard Lee Rue III/NAS/Photo Researchers  
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