

Urban  
Evolution  
Studies in the  
Mathematical  
Ecology of  
Cities

Dimitrios S. Dendrinos  
with Henry Mullally

# URBAN EVOLUTION

## Studies in the Mathematical Ecology of Cities

DIMITRIOS S. DENDRINOS  
with HENRY MULLALLY

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To Iris and Spiros Dendrinis

## PREFACE

The main point of this book is that, while the underlying processes in urban dynamics may be very complicated, the macroscopic state of the urban system is simple, easily described and understood. Urban settings are shown to have internal clocks manifested by regular cycles developed within particular environments. A set of non-linear dynamic models based on the Volterra-Lotka<sup>†</sup> formalism, appropriately adapted, are shown to account for certain basic qualitative features of urban systems. This formalism is particularly useful for analysing aspects of dynamic stability. Although the methodology of mathematical ecology is selectively used in this book, no substantive equivalences with general ecology are made. Using empirical evidence, the book develops the thesis of a dynamic interconnectance among the components of the urban dynamic structure, so that non-linearities, multiplicity of equilibria, and bifurcations in urban dynamics are highlighted.

A major theme in the book is that model complexity often implies dynamical instability, a result known in mathematical ecology through the work of Robert May. This complexity vs. stability issue is central in modelling and understanding urban evolution. Recorded inter-urban growth patterns, extensively discussed in this book, exhibit stability which is attributed to highly selective interconnectance among interacting cities. Whereas, the cause for observed intra-urban instability must be found in an extensive random interdependence among various land uses and zones within cities. Both, inter-urban stability and intra-urban instability are demonstrated in a relative growth framework. A number of insights can be drawn from such an approach, and they are extensively analysed in the book.

Certain epistemological topics in urban evolution are also outlined, as they emerge by approaching urban dynamics from the perspective of mathematical ecology. Urban determinism and stochasticity, selection and optimization, fast and slow urban adaptation are concepts defined and reviewed in a Darwinian framework.

Although direct analogues with general ecology are avoided, the common method forces certain fundamental equivalences in the way

<sup>†</sup> In the ecological literature the models are referred to as 'Lotka-Volterra'. In the field of mathematics, Hirsch and Smale (1974), for example, they are referred to as 'Volterra-Lotka'. The latter is adopted here.

problems are stated. As a result of intense theoretical efforts during the past sixty or so years, reinforced by an even longer empirical tradition, general ecology has accumulated a considerable body of principles and methods. Consequently, the preponderant flow of ideas at this stage is from general ecology to urban ecology. As the latter grows, however, the traffic may become more balanced, with questions and solutions in the urban sphere stimulating research in general ecology, much as the work of Darwin was influenced by the writing of Malthus. Ecologists' theoretical efforts can gain significantly from an examination of the economic literature, and in particular that of Samuelson (1948), Hotelling (1929), etc., and the geographic literature, Christaller (1933) and Lösch (1937) among others.

Great care must be exercised in applying concepts or methods developed in one field to problems in a different area. One must ask whether a question pertinent to ecological theory is relevant to urban ecology; and whether other questions not useful to general ecology are pertinent for urban ecology. However, the second task seems more formidable than the first. Almost all theoretical insights that can be obtained using the basic tool of mathematical ecology, namely the Volterra-Lotka formalism, have in all likelihood, already been discovered, in spite of few surprises, e.g. Gilpin (1979). Instead of re-inventing the wheel, the next fruitful source for innovative urban work must come from empirical studies, the source of possibly new phenomena to be identified.

From an urban perspective it is quite surprising that the great advances in the theory of mathematical ecology have gone mostly unnoticed. Here this void is partly filled. It is not accomplished by a wholesale transfer of concepts from ecology to urban dynamics, but rather by a judicious selection of certain basic methodological notions which, when coupled with empirical evidence, provide the main features of an urban theory. One is indeed astonished at the parallel development of the two fields with so many methodological commonalities, and yet so little interaction. Whether this is an example where stability in the evolution of a field requires isolation from other areas of investigation, as increasing connectivity with other disciplines possibly creates instability, remains to be seen.

One, by reading this book, might detect a bias toward description of recorded phenomena, rather than explanation. If so, a comment is in order. Social science events, due to their underlying complexity, whatever that may imply, have multiple suggested explanations. Interest or

opinions among researchers on these possible explanations is in general unevenly distributed, but not highly skewed towards one of them. Thus, the value of explanation may not be as high as that of describing an event, particularly if that description is novel.

The book is addressed to social scientists in general, and those in particular with a background in geography, urban and regional science, and urban demography. It could also be of interest to ecologists. Economists, mathematicians, and systems analysts interested in applications of non-linear dynamics may also find certain elements of the book relevant to their disciplines. The book can be used by advanced level undergraduates or entry level graduate students with some background in differential equations. Although some mathematical analysis is included in proving most of the statements made, the mathematical exposition of the main text has been kept to a minimum with most of the key proofs supplied as appendices. The book's main purpose is to serve as a research source, since in many instances it only posits problems without going very deeply into them, leaving the interested reader to develop them further.

D.S.D., H.M.

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None of the above, however, is responsible for any remaining errors, shortcomings, or the views held in the book for which we retain full responsibility.

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## INTRODUCTION

As an introduction to the book, some broad statements on how one can approach the construction of a theory of urban evolution are made first, and the reasons for the particular path followed are then laid out. An outline of strategic and tactical models sets the stage for the work to be presented. Then, the structure of the book is provided. Finally, certain key terms and their definitions are supplied.

### *Towards constructing a theory of urban evolution*

A theory of urban evolution and its method can be formulated from a number of very distinct perspectives. One might start from a very microscopic level, that of the behaviour of a single individual, and climb up through a series of aggregations to a very macroscopic level. By stating a dynamic theory of micro-behaviour one then might be able to arrive at a dynamic theory of macro- (urban, regional) evolution. Even were such a monumental task feasible, it would not be an efficient undertaking. This book is premised on the belief that in spite of their micro-level complexity basic insights into urban evolution could be obtained by making relatively few, strategically placed, macro-level observations of urban growth and form. It suffices that these observations be made at particular time-periods, and be generic ones, in the sense that they contain if not all at least most of the important qualitative features of urban evolution. Thus, an alternative to the microscopic approach is adopted here. A partial and local view is taken on the subject from a macro-level although reference to micro-level behaviour is made. Specifically, the aggregate dynamic behaviour of cities from the United States is examined as it is manifested in the collective dynamic behaviour of their population.

Over centuries of metropolitan evolution a recurring feature is that of urban cycles. Although a host of factors obviously affect the dynamics of any particular city at any particular time-period (may be too numerous to fully account in a model), urban population and income seem to be most important. Looked at in isolation, population and income oscillate. Considered in combination, urban population and income behave according to a simple and regular dynamic: a damped cyclical motion. Data covering approximately one hundred years

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indicate that urban oscillations occur at the inter-urban level, obeying certain regularities. Within this time-scale, the cities of the US are characterized by properties of dynamic stability. More drastic events occur if one considers obviously much longer time-horizons: cities suddenly appear, grow, and then possibly become extinct. Some cities' growth is consistently associated with others' decline. Naturally, these aggregate urban dynamic patterns of various time-scales must not be haphazard or random. Some simple mathematical models of Volterra-Lotka type seem to adequately describe such events.

At the intra-urban level, some isolated but key phenomena from US intra-metropolitan recent history (the past forty-year period) seem to indicate dynamic instability. Empirical evidence regarding the qualitative aspects of the widely discussed phenomena of suburbanization, slum formation, gentrification, etc., seems to point to models capable of producing bifurcating behaviour and switches from stable to unstable equilibria. Bifurcations occur when certain qualitative properties of dynamic equilibria change suddenly over time, when smooth changes in key parameters take place, and certain thresholds are crossed. A host of inter- and intra-urban events can be captured through such bifurcations.

The mathematical ecology-based models presented here address *dynamic non-linear interdependencies, stability and multiple equilibria*, conditions argued to be necessary components of urban systems theory and modelling. These advances are carried out in a manner which *allows for an analytical treatment*. In these models phenomena of particular interest, including bifurcation in behaviour, can be captured analytically and be verified empirically. This is where the novelty of this work lies.

As will be seen later, some isolated earlier attempts do exist in the geographic literature where non-linear dynamic models have been proposed. Very few are empirically verified. An extensive literature also exists in the urban planning and regional science fields on large-scale urban models. With very few exceptions, these models are static and equilibrium bound and they do not possess analytical solutions. They are also not problem oriented, i.e. they do not address particular urban phenomena or events. A few dynamic models also exist in the urban economic literature, as will be indicated later in the text. They, however, are not empirically tested, and not capable of capturing *bifurcating behaviour*. Obviously this book is not a forum for a comprehensive review of this particular issue. A comparative study of models found in the various disciplines mentioned with the ecological construct will have to wait. The task here is to lay the foundations for the

development of an urban mathematical ecology theory and leave comparisons and/or integration to future studies.

In summary, the main objectives of the book are to show that in spite of their underlying complexity metropolitan dynamics at a macro-scale seem to be consistent with certain simple but powerful population models of the Volterra-Lotka type. These models can be used to obtain basic insights into the topic of urban evolution, to classify a rather broad selection of urban phenomena, and to demonstrate that bifurcation theory provides a means to extend the urban ecological field. By combining these fields of study one can bring the topic of urban evolution closer to the mainstream of the general theory of evolution and possibly contribute to the present epistemological debate regarding smooth and abrupt evolutionary change, determinism and stochasticity, necessity and chance in urban and other social or biological or natural events.

#### *A note on models*

Over the past thirty years or so, the relevance of mathematical modelling has been debated in a variety of fields. Empirical and theoretical ecologists, applied and theoretical economists, urban policy-makers and urban theorists, to mention but a few, have extensively debated the issue. Still, the relevance of model-building remains a source for confusion and disagreement. Criticism is fuelled when the dichotomy between micro- (disaggregate) and macro- (aggregated) models is made, mostly centring on the relevance of macro-models. It is even further heated when the use of small versus large dimensionality models is discussed.

Clearly, the two dimensions of the discussion are different. One can have a low dimensionality disaggregated model, or a large dimensionality aggregated one. The use of macro-models, according to this book, lies in the way that one interprets such models. In Part IV of this volume, the epistemological insights gained by such models are provided. This is only one part of the answer. The other, of course, is that knowledge is gained by studying the regular patterns of aggregate behaviour, so that micro-level expectations can be based on macro-system performance. This is further elaborated in Part III.

More critical in the discussion is the low versus high dimensionality argument. It relates to whether a complex system needs a complex (high dimensionality) model to simulate it. In general, low dimensionality models are *strategic* means toward understanding the functioning of