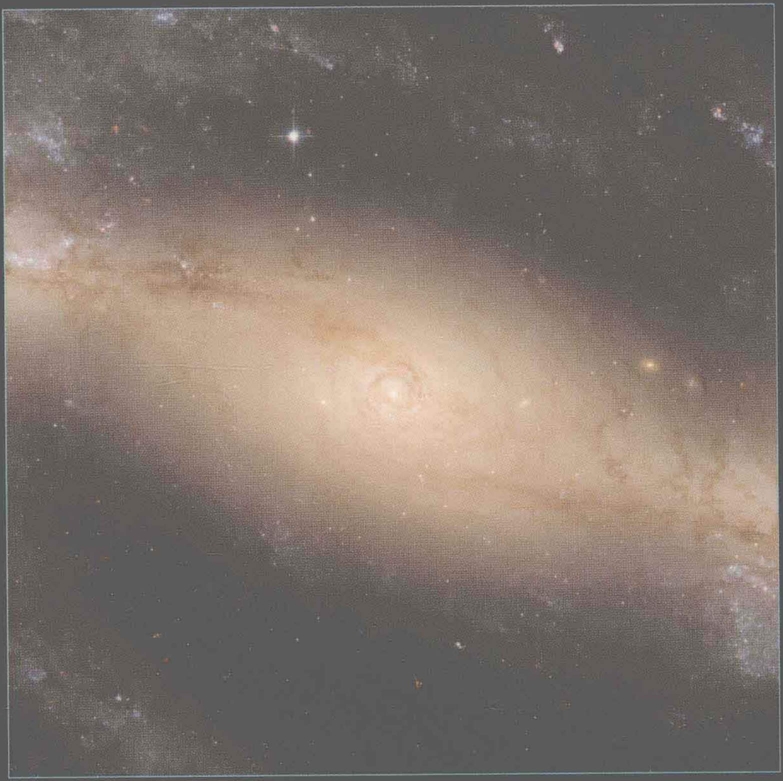
Astrophysics in a Nutshell



Dan Maoz

Astrophysics in a Nutshell

Dan Maoz

PRINCETON UNIVERSITY PRESS · PRINCETON AND OXFORD

Published by Princeton University Press, 41 William Street,
Princeton, New Jersey 08540

In the United Kingdom: Princeton University Press,
3 Market Place, Woodstock, Oxfordshire OX20 1SY

Library of Congress Cataloging-in-Publication Data

Maoz, Dan.
Astrophysics in a nutshell / Dan Maoz.
p. cm.
Includes bibliographical references and index.
ISBN-13: 978-0-691-12584-8 (acid-free paper)
ISBN-10: 0-691-12584-8 (acid-free paper)
1. Astrophysics. I. Title.

QB461.M32 2007
523.01—dc22 2006050588

British Library Cataloging-in-Publication Data is available

This book has been composed in Scala LF and Scala Sans

Printed on acid-free paper. ∞

press.princeton.edu

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

To Orit, Lia and Yonatan—
the three bright stars in my sky; and to my parents

Preface

This textbook is based on the one-semester course "Introduction to Astrophysics," taken by third-year physics students at Tel-Aviv University, which I taught several times in the years 2000–2005. My objective in writing this book is to provide an introductory astronomy text that is suited for university students majoring in physical science fields (physics, astronomy, chemistry, engineering, etc.), rather than for a wider audience, for which many astronomy textbooks already exist. I have tried to cover a large and representative fraction of the main elements of modern astrophysics, including some topics at the forefront of current research. At the same time, I have made an effort to keep this book concise.

I covered this material in approximately forty 45-minute lectures. The text assumes a level of math and physics expected from intermediate-to-advanced undergraduate science majors, namely, familiarity with calculus and differential equations, classical and quantum mechanics, special relativity, waves, statistical mechanics, and thermodynamics. However, I have made an effort to avoid long mathematical derivations, or complicated physical arguments that might mask simple realities. Thus, throughout the text, I use devices such as scaling arguments and order-of-magnitude estimates to arrive at the important basic results. Where relevant, I then state the results of more thorough calculations that involve, e.g., taking into account secondary processes that I have ignored, or full solutions of integrals, or of differential equations.

Undergraduates are often taken aback by their first encounter with this order-of-magnitude approach. Of course, full and accurate calculations are as indispensable in astrophysics as in any other branch of physics (e.g., an omitted factor of π may not be important for understanding the underlying physics of some phenomenon, but it can be very important for comparing a theoretical calculation to the results of an experiment). However, most physicists (regardless of subdiscipline), when faced with a new problem, will first carry out a rough, "back-of-the-envelope" analysis that can lead to some basic intuition about the processes and the numbers involved. Thus, the approach we will follow here is actually valuable and widely used, and the student is well advised to attempt to become proficient at it. With this objective in mind, some derivations and some topics are left as problems at the end of each chapter (usually including a generous amount of guidance), and solving most or all of the problems is highly recommended in order to get the most out of this book. I have not provided full solutions to the problems, to counter the temptation to peek. Instead, at the end of some problems I have provided short answers that permit the reader to check the correctness of the solution, although not in cases where the answer would give away the solution too easily (physical science students are notoriously competent at "reverse engineering" a solution—not necessarily correct—to an answer!).

There is much that does *not* appear in this book. I have excluded almost all descriptions of the historical developments of the various topics, and have, in general, presented them directly as they are understood today. There is almost no attribution of results to the many scientists whose often-heroic work has led to this understanding, a choice that certainly does injustice to many individuals, past and living. Furthermore, not all topics in astrophysics are equally amenable to the type of exposition this book follows, and I naturally have my personal biases about what is most interesting and important. As a result, the coverage of the different subjects is intentionally uneven: some are explored to considerable depth, while others are presented only descriptively, given brief mention, or completely omitted. Similarly, in some cases I develop from "first principles" the physics required to describe a problem, but in other cases I begin by simply stating the physical result, either because I expect the reader is already familiar enough with it, or because developing it would take too long. I believe that all these choices are essential to keep the book concise, focused, and within the scope of a one-term course. No doubt, many people will disagree with the particular choices I have made, but hopefully will agree that all that has been omitted here can be covered later by more advanced courses (and the reader should be aware that proper attribution of results *is* the strict rule in the research literature).

Astronomers use some strange units, in some cases for no reason other than tradition. I generally use cgs units, but also make frequent use of some other units that are common in astronomy, e.g., angstroms, kilometers, parsecs, light-years, years, solar masses, and solar luminosities. However, I have completely avoided using or mentioning "magnitudes," the peculiar logarithmic units used by astronomers to quantify flux. Although magnitudes are widely used in the field, they are not required for explaining anything in this book, and might only cloud the real issues. Again, students continuing to more advanced courses and to research can easily deal with magnitudes at that stage.

A note on equality symbols and their relatives. I use an "$=$" sign, in addition to cases of strict mathematical equality, for numerical results that are accurate to better than ten percent. Indeed, throughout the text I use constants and unit transformations with only two significant digits (they are also listed in this form in "Constants and Units," in the hope that the student will memorize the most commonly used among them after a while), except in a few places where more digits are essential. An "$\approx$" sign in a mathematical relation (i.e., when mathematical symbols, rather than numbers, appear on both sides) means some approximation has been made, and in a numerical relation it means an accuracy somewhat worse than ten percent. A "$\propto$" sign means strict proportionality between the two sides. A "$\sim$" is used in two senses. In a mathematical relation it means an approximate functional dependence. For example, if $y = ax^{2.2}$, I may write $y \sim x^2$. In numerical relations, I use "$\sim$" to indicate order-of-magnitude accuracy.

This book has benefitted immeasurably from the input of the following colleagues, to whom I am grateful for providing content, comments, insights, ideas, and many corrections: T. Alexander, R. Barkana, M. Bartelmann, J.-P. Beaulieu, D. Bennett, D. Bram, D. Champion, M. Dominik, H. Falcke, A. Gal-Yam, A. Ghez, O. Gnat, A. Gould, B. Griswold, Y. Hoffman, S. Jha, M. Kamionkowski, S. Kaspi, V. Kaspi, A. Laor, A. Levinson, J. R. Lu, J. Maos, T. Mazeh, J. Peacock, D. Poznanski, P. Saha, D. Spergel, A. Sternberg, R. Thompson, R. Webbink, L. R. Williams, and S. Zucker. The remaining errors are, of course, all my own. Orit Bergman patiently produced most of the figures—one more of the many things she has granted me, and for which I am forever thankful.

D.M.
Tel-Aviv, 2006

Constants and Units (to two significant digits)

Gravitational constant	$G = 6.7 \times 10^{-8}$ erg cm g^{-2}
Speed of light	$c = 3.0 \times 10^{10}$ cm s^{-1}
Planck's constant	$h = 6.6 \times 10^{-27}$ erg s $\hbar = h/2\pi = 1.05 \times 10^{-27}$ erg s
Boltzmann's constant	$k = 1.4 \times 10^{-16}$ erg K^{-1} $= 8.6 \times 10^{-5}$ eV K^{-1}
Stefan-Boltzmann constant	$\sigma = 5.7 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
Radiation constant	$a = 4\sigma/c = 7.6 \times 10^{-15}$ erg $cm^{-3}K^{-4}$
Proton mass	$m_p = 1.7 \times 10^{-24}$ g
Electron mass	$m_e = 9.1 \times 10^{-28}$ g
Electron charge	$e = 4.8 \times 10^{-10}$ esu
Electron volt	1 eV $= 1.6 \times 10^{-12}$ erg
Thomson cross section	$\sigma_T = 6.7 \times 10^{-25}$ cm^2
Wien's law	$\lambda_{\text{max}} = 2900$ Å 10^4 K$/T$ $h\nu_{\text{max}} = 2.4$ eV $T/10^4$ K
Angstrom	1 Å $= 10^{-8}$ cm
Solar mass	$M_\odot = 2.0 \times 10^{33}$ g
Solar luminosity	$L_\odot = 3.8 \times 10^{33}$ erg s^{-1}
Solar radius	$r_\odot = 7.0 \times 10^{10}$ cm

Solar distance	$d_\odot = 1\ \mathrm{AU} = 1.5 \times 10^{13}$ cm
Jupiter mass	$M_J = 1.9 \times 10^{30}$ g
Jupiter radius	$r_J = 7.1 \times 10^{9}$ cm
Jupiter distance	$d_J = 5\ \mathrm{AU} = 7.5 \times 10^{13}$ cm
Earth mass	$M_\oplus = 6.0 \times 10^{27}$ g
Earth radius	$r_\oplus = 6.4 \times 10^{8}$ cm
Moon mass	$M_{\mathrm{moon}} = 7.4 \times 10^{25}$ g
Moon radius	$r_{\mathrm{moon}} = 1.7 \times 10^{8}$ cm
Moon distance	$d_{\mathrm{moon}} = 3.8 \times 10^{10}$ cm
Astronomical unit	$1\ \mathrm{AU} = 1.5 \times 10^{13}$ cm
Parsec	$1\ \mathrm{pc} = 3.1 \times 10^{18}\ \mathrm{cm} = 3.3$ ly
Year	$1\ \mathrm{yr} = 3.15 \times 10^{7}$ s

A strophysics in a Nutshell

Contents

1 Introduction

Astrophysics is the branch of physics that studies, loosely speaking, phenomena on large scales—the Sun, the planets, stars, galaxies, and the Universe as a whole. But this definition is clearly incomplete; much of astronomy[1] also deals, e.g., with phenomena at the atomic and nuclear levels. We could attempt to define astrophysics as the physics of distant objects and phenomena, but astrophysics also includes the formation of the Earth, and the effects of astronomical events on the emergence and evolution of life on Earth. This semantic difficulty perhaps simply reflects the huge variety of physical phenomena encompassed by astrophysics. Indeed, as we will see, practically all the subjects encountered in a standard undergraduate physical science curriculum—classical mechanics, electromagnetism, thermodynamics, quantum mechanics, statistical mechanics, relativity, and chemistry, to name just some—play a prominent role in astronomical phenomena. Seeing all of them in action is one of the exciting aspects of studying astrophyics.

Like other branches of physics, astronomy involves an interplay between experiment and theory. Theoretical astrophysics is carried out with the same tools and approaches used by other theoretical branches of physics. Experimental astrophysics, however, is somewhat different from other experimental disciplines, in the sense that astronomers cannot carry out controlled experiments,[2] but can only perform **observations** of the various phenomena provided by nature. With this in mind, there is little difference, in practice, between the design and the execution of an experiment in some field of physics, on the one hand, and the design and the execution of an astronomical observation, on the other. There is certainly no particular distinction between the methods of data analysis in either case. But, since everything we discuss in this book will ultimately be based on observations, let us begin with a brief overview of how observations are used to make astrophysical measurements.

[1] We will use the words "astrophysics" and "astronomy" interchangeably, as they mean the same thing nowadays. For example, the four leading journals in which astrophysics research is published are named *The Astrophysical Journal, The Astronomical Journal, Astronomy and Astrophysics,* and *Monthly Notices of the Royal Astronomical Society*, but their subject content is the same.

[2] An exception is the field of laboratory astrophysics, in which some particular properties of astronomical conditions are simulated in the lab.

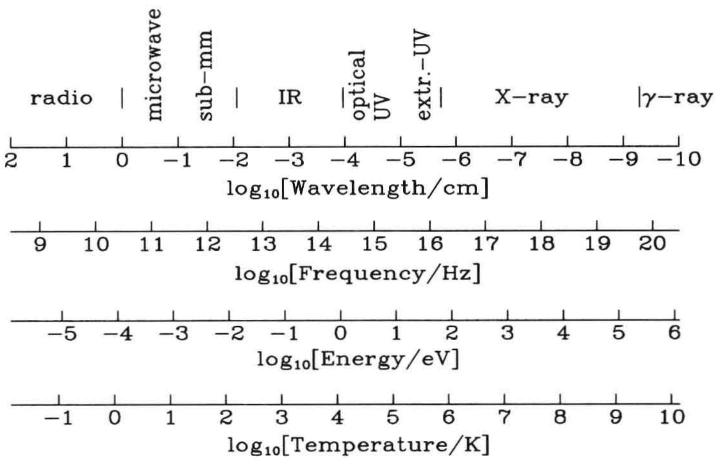


Figure 1.1 The various spectral regions of electromagnetic radiation, their common astronomical nomenclature, and their approximate borders in terms of wavelength, frequency, energy, and temperature. Temperature is here associated with photon energy E via the relation $E = kT$, where k is Boltzmann's constant.

1.1 Observational Techniques

With several exceptions, astronomical phenomena are almost always observed by detecting and measuring electromagnetic (EM) radiation from distant sources. (The exceptions are in the fields of cosmic ray astronomy, neutrino astronomy, and gravitational wave astronomy.) Figure 1.1 shows the various, roughly defined, regions of the EM spectrum. To record and characterize EM radiation, one needs, at least, a camera that will focus the approximately plane EM waves arriving from a distant source and a detector at the focal plane of the camera, which will record the signal. A "telescope" is just another name for a camera that is specialized for viewing distant objects. The most basic such camera–detector combination is the human eye, which consists (among other things) of a lens (the camera) that focuses images on the retina (the detector). Light-sensitive cells on the retina then translate the light intensity of the images into nerve signals that are transmitted to the brain. Figure 1.2 sketches the optical principles of the eye and of two telescope configurations.

Until the introduction of telescope use to astronomy by Galileo in 1609, observational astronomy was carried out solely using human eyes. However, the eye as an astronomical tool has several disadvantages. The **aperture** of a dark-adapted pupil is <1 cm in diameter, providing limited **light-gathering area** and limited **angular resolution**. The light-gathering capability of a camera is set by the area of its aperture (e.g., of the objective lens, or of the primary mirror in a reflecting telescope). The larger the aperture, the more photons, per unit time, can be detected, and hence fainter sources of light can be observed. For example, the largest visible-light telescopes in operation today have 10-meter primary mirrors, i.e., more than a million times the light gathering area of a human eye.

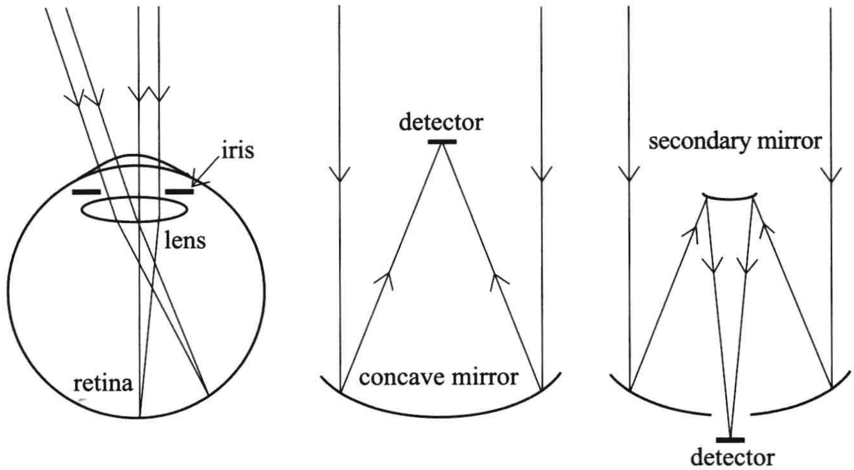


Figure 1.2 Optical sketches of three different examples of camera-detector combinations. *Left:* Human eye, shown with parallel rays from two distant sources, one source on the optical axis of the lens and one at an angle to the optical axis. The lens, which serves as the camera in this case, focuses the light onto the retina (the detector), on which two point images are formed. *Center:* A reflecting telecope with a detector at its *prime focus.* Plotted are parallel rays from a distant source on the optical axis of the telescope. The concave mirror focuses the rays onto the detector at the mirror's focal plane, where a point image is formed. *Right:* Reflecting telescope, but with a secondary, convex, mirror, which folds the beam back down and through a hole in the primary concave mirror, to form an image on the detector at the so-called *Cassegrain focus.*

The angular resolution of a camera or a telescope is the smallest angle on the sky between two sources of light that can be discerned as separate sources with that camera. From wave optics, a plane wave of wavelength λ passing through a circular aperture of diameter D, when focused onto a detector, will produce a diffraction pattern of concentric rings, centered on the position expected from geometrical optics, with a central spot having an angular radius (in radians) of

$$\theta = 1.22\frac{\lambda}{D}. \tag{1.1}$$

Consider, for example, the image of a field of stars obtained through some camera, and having also a bandpass filter that lets through light only within a narrow range of wavelengths. The image will consist of a set of such diffraction patterns, one at the position of each star (see Fig. 1.3). Actually seeing these diffraction patterns requires that blurring of the image not be introduced, either by imperfectly built optics or by other elements, e.g., Earth's atmosphere. The central spots from the diffraction patterns of two adjacent sources on the sky will overlap, and will therefore be hard to distinguish from each other, when their angular separation is less than about λ/D. Similarly, a source of light with an intrinsic angular size smaller than this **diffraction limit** will produce an image that is *unresolved,* i.e., indistinguishable from the image produced by a **point source** of zero angular extent. Thus, in principle, a 10-meter telescope working at the same visual wavelengths as the eye can have an angular resolution that is 1000 times better than that of the eye.

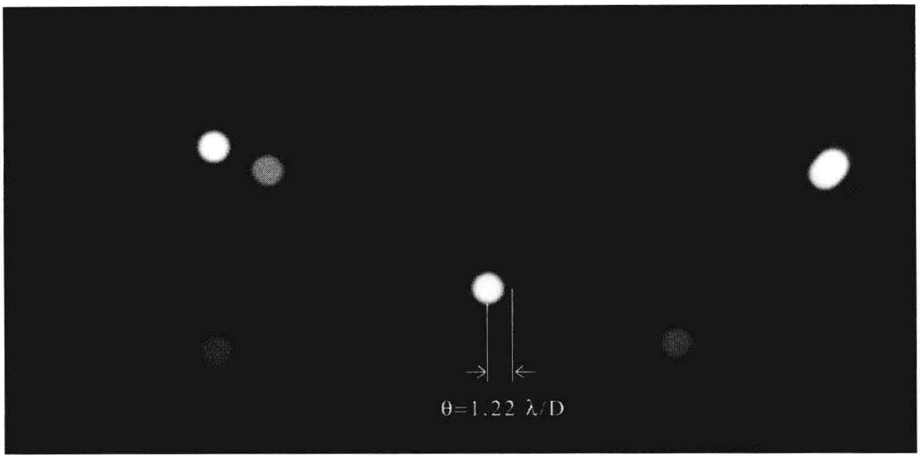


Figure 1.3 Simulated diffraction-limited image of a field of stars, with the characteristic diffraction pattern due to the telescope's finite circular aperture at the position of every star. Pairs of stars separated on the sky by an angle $\theta < \lambda/D$ (e.g., on the right-hand side of the image) are hard to distinguish from single stars. Real conditions are always worse than the diffraction limit, due to, e.g., imperfect optics and atmospheric blurring.

In practice, it is difficult to achieve diffraction-limited performance with ground-based *optical* telescopes, due to the constantly changing, blurring effect of the atmosphere. (The **optical** wavelength range of EM radiation is roughly defined as 0.32–1 μm.) However, observations with angular resolutions at the diffraction limit are routine in radio and infrared astronomy, and much progress in this field has been achieved recently in the optical range as well. Angular resolution is important not only for discerning the fine details of astronomical sources (e.g., seeing the moons and surface features of Jupiter, the constituents of a star-forming region, or subtle details in a galaxy), but also for detecting faint unresolved sources against the background of emission from the Earth's atmosphere, i.e., the "sky." The night sky shines due to scattered light from the stars, from the Moon, if it is up, and from artificial light sources, but also due to fluorescence of atoms and molecules in the atmosphere. The better the angular resolution of a telescope, the smaller the solid angle over which the light from, say, a star, will be spread out, and hence the higher the contrast of that star's image over the statistical fluctuations of the sky background (see Fig. 1.4). A high sky background combined with limited angular resolution are among the reasons why it is difficult to see stars during daytime.

A third limitation of the human eye is its fixed integration time, of about 1/30 second. In astronomical observations, faint signals can be collected on a detector during arbitrarily long exposures (sometimes accumulating to months), permitting the detection of extremely faint sources. Another shortcoming of the human eye is that it is sensitive only to a narrow **visual** range of wavelengths of EM radiation (about 0.4–0.7 μm, i.e., within the optical range defined above), while astronomical information exists in all regions of the EM spectrum, from radio, through infrared, optical, ultraviolet, X-ray, and gamma-ray bands. Finally, a detector other than the eye allows keeping an objective record of the observation,