



# **APPLIED DIGITAL SIGNAL PROCESSING**

**Dimitris G. Manolakis  
Vinay K. Ingle**

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# Applied Digital Signal Processing

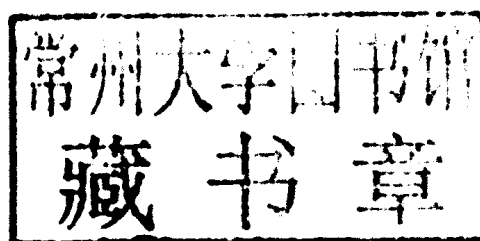
THEORY AND PRACTICE

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## Applied Digital Signal Processing

Master the basic concepts and methodologies of digital signal processing with this systematic introduction, without the need for an extensive mathematical background. The authors lead the reader through the fundamental mathematical principles underlying the operation of key signal processing techniques, providing simple arguments and cases rather than detailed general proofs. Coverage of practical implementation, discussion of the limitations of particular methods, and plentiful MATLAB illustrations allow readers to better connect theory and practice. A focus on algorithms that are of theoretical importance or useful in real-world applications ensures that students cover material relevant to engineering practice, and equips students and practitioners alike with the basic principles necessary to apply DSP techniques to a variety of applications. Chapters include worked examples, problems, and computer experiments, helping students to absorb the material they have just read. Lecture slides for all figures and solutions to the numerous problems are available to instructors.

**Dimitris G. Manolakis** is currently a Member of Technical Staff at MIT Lincoln Laboratory in Lexington, Massachusetts. Prior to this he was a Principal Member of Research Staff at Riverside Research Institute. Since receiving his Ph.D. in Electrical Engineering from the University of Athens in 1981, he has taught at various institutions including Northeastern University, Boston College, and Worcester Polytechnic Institute, and co-authored two textbooks on signal processing. His research experience and interests include the areas of digital signal processing, adaptive filtering, array processing, pattern recognition, remote sensing, and radar systems.

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To my wife and best friend Anna  
and in memory of Eugenia, Gregory, and Elias

DGM

To my loving wife Usha and daughters  
Natasha and Trupti for their endless support.

VKI

## PREFACE

During the last three decades Digital Signal Processing (DSP) has evolved into a core area of study in electrical and computer engineering. Today, DSP provides the methodology and algorithms for the solution of a continuously growing number of practical problems in scientific, engineering, and multimedia applications.

Despite the existence of a number of excellent textbooks focusing either on the theory of DSP or on the application of DSP algorithms using interactive software packages, we feel there is a strong need for a book bridging the two approaches by combining the best of both worlds. This was our motivation for writing this book, that is, to help students and practicing engineers understand the fundamental mathematical principles underlying the operation of a DSP method, appreciate its practical limitations, and grasp, with sufficient details, its practical implementation.

### Objectives

The principal objective of this book is to provide a systematic introduction to the basic concepts and methodologies for digital signal processing, based whenever possible on fundamental principles. A secondary objective is to develop a foundation that can be used by students, researchers, and practicing engineers as the basis for further study and research in this field. To achieve these objectives, we have focused on material that is fundamental and where the scope of application is not limited to the solution of specialized problems, that is, material that has a broad scope of application. Our aim is to help the student develop sufficient intuition as to how a DSP technique works, be able to apply the technique, and be capable of interpreting the results of the application. We believe this approach will also help students to become intelligent users of DSP techniques and good critics of DSP techniques performed by others.

### Pedagogical philosophy

Our experience in teaching undergraduate and graduate courses in digital signal processing has reaffirmed the belief that the ideal blend of simplified mathematical analysis and computer-based reasoning and simulations enhances both the teaching and the learning of digital signal processing. To achieve these objectives, we have used mathematics to support underlying intuition rather than as a substitute for it, and we have emphasized practicality without turning the book into a simplistic “cookbook.” The purpose of MATLAB<sup>®</sup> code integrated with the text is to illustrate the implementation of core signal processing algorithms; therefore, we use standard language commands and functions that have remained relatively stable during the most recent releases. We also believe that in-depth

understanding and full appreciation of DSP is not possible without familiarity with the fundamentals of continuous-time signals and systems. To help the reader grasp the full potential of DSP theory and its application to practical problems, which primarily involve continuous-time signals, we have integrated relevant continuous-time background into the text. This material can be quickly reviewed or skipped by readers already exposed to the theory of continuous-time signals and systems. Another advantage of this approach is that some concepts are easier to explain and analyze in continuous-time than in discrete-time or vice versa.

## Instructional aids

We have put in a considerable amount of effort to produce instructional aids that enhance both the teaching and learning of DSP. These aids, which constitute an integral part of the textbook, include:

- **Figures** The graphical illustrations in each figure are designed to provide a mental picture of how each method works or to demonstrate the performance of a specific DSP method.
- **Examples** A large number of examples are provided, many generated by MATLAB<sup>®</sup> to reflect realistic cases, which illustrate important concepts and guide the reader to easily implement various methods.
- **MATLAB<sup>®</sup> functions and scripts** To help the reader apply the various algorithms and models to real-world problems, we provide MATLAB<sup>®</sup> functions for all major algorithms along with examples illustrating their use.
- **Learning summaries** At the end of each chapter, these provide a review of the basic yet important concepts discussed in that chapter in the form of a bullet point list.
- **Review questions** Conceptual questions are provided at the end of each chapter to reinforce the theory, clarify important concepts, and help relate theory to applications.
- **Terms and concepts** Important phrases and notions introduced in the chapter are again explained in a concise manner for a quick overview.
- **Problems** A large number of problems, ranging from simple applications of theory and computations to more advanced analysis and design tasks, have been developed for each chapter. These problems are organized in up to four sections. The first set of problems termed as Tutorial Problems contains problems whose solutions are available on the website. The next section, Basic Problems, belongs to problems with answers available on the website. The third section, Assessment Problems, contains problems based on topics discussed in the chapter. Finally, the last section, Review Problems, introduces applications, review, or extension problems.
- **Book website** This website will contain additional in-depth material, signal datasets, MATLAB<sup>®</sup> functions, power-point slides with all figures in the book, etc., for those who want to delve intensely into topics. This site will be constantly updated. It will also provide tutorials that support readers who need a review of background material.
- **Solutions manual** This manual, which contains solutions for all problems in the text, is available to instructors from the publisher.

## Audience and prerequisites

The book is primarily aimed as a textbook for upper-level undergraduate and for first-year graduate students in electrical and computer engineering. However, researchers, engineers, and industry practitioners can use the book to learn how to analyze or process data for scientific or engineering applications. The mathematical complexity has been kept at a level suitable for seniors and first-year graduate students in almost any technical discipline. More specifically, the reader should have a background in calculus, complex numbers and variables, and the basics of linear algebra (vectors, matrices, and their manipulation).

## Course configurations

The material covered in this text is intended for teaching to upper-level undergraduate or first-year graduate students. However, it can be used flexibly for the preparation of a number of courses. The first six chapters can be used in a junior level signals and systems course with emphasis on discrete-time. The first 11 chapters can be used in a typical one-semester undergraduate or graduate DSP course in which the first six chapters are reviewed and the remaining five chapters are emphasized. Finally, an advanced graduate level course on modern signal processing can be taught by combining some appropriate material from the first 11 chapters and emphasizing the last four chapters. The pedagogical coverage of the material also lends itself to a well-rounded graduate level course in DSP by choosing selected topics from all chapters.

## Feedback

Experience has taught us that errors – typos or just plain mistakes – are an inescapable byproduct of any textbook writing endeavor. We apologize in advance for any errors you may find and we urge you to bring them or additional feedback to our attention at [vingle@ece.neu.edu](mailto:vingle@ece.neu.edu)

## Acknowledgments

We wish to express our sincere appreciation to the many individuals who have helped us with their constructive comments and suggestions. Special thanks go to Sidi Niu for the preparation of the *Solutions Manual*. Phil Meyler persuaded us to choose Cambridge University Press as our publisher, and we have been happy with that decision. We are grateful to Phil for his enthusiasm and his influence in shaping the scope and the objectives of our book. The fine team at CUP, including Catherine Flack, Chris Miller, and Richard Smith, has made the publication of this book an exciting and pleasant experience. Finally, we express our deepest thanks to our wives, Anna and Usha, for their saintly understanding and patience.

Dimitris G. Manolakis  
Vinay K. Ingle



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# 1

## Introduction

Signal processing is a discipline concerned with the acquisition, representation, manipulation, and transformation of signals required in a wide range of practical applications. In this chapter, we introduce the concepts of signals, systems, and signal processing. We first discuss different classes of signals, based on their mathematical and physical representations. Then, we focus on continuous-time and discrete-time signals and the systems required for their processing: continuous-time systems, discrete-time systems, and interface systems between these classes of signal. We continue with a discussion of analog signal processing, digital signal processing, and a brief outline of the book.

### Study objectives

After studying this chapter you should be able to:

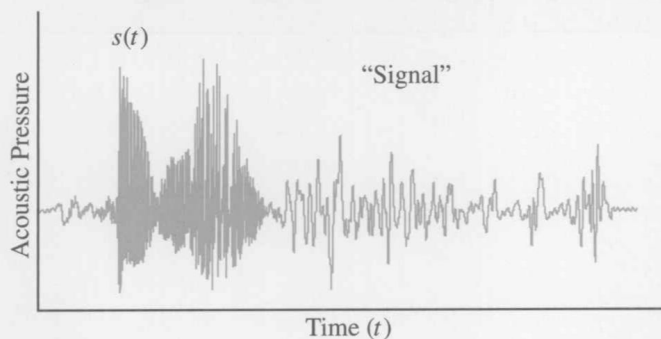
- Understand the concept of signal and explain the differences between continuous-time, discrete-time, and digital signals.
- Explain how the physical representation of signals influences their mathematical representation and vice versa.
- Explain the concepts of continuous-time and discrete-time systems and justify the need for interface systems between the analog and digital worlds.
- Recognize the differences between analog and digital signal processing and explain the key advantages of digital over analog processing.

For our purposes a *signal* is defined as any physical quantity that varies as a function of time, space, or any other variable or variables. Signals convey information in their patterns of variation. The manipulation of this information involves the acquisition, storage, transmission, and transformation of signals.

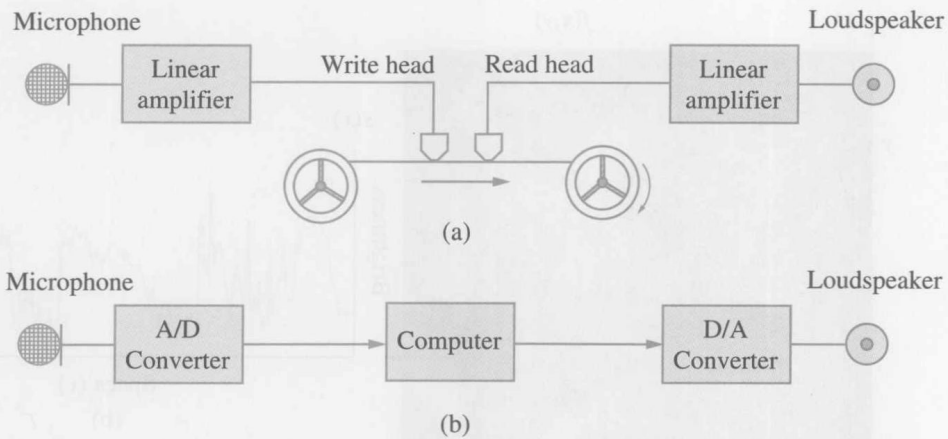
There are many signals that could be used as examples in this section. However, we shall restrict our attention to a few signals that can be used to illustrate several important concepts and they will be useful in later chapters. The speech signal, shown as a *time waveform* in Figure 1.1, represents the variations of acoustic pressure converted into an electric signal by a microphone. We note that different sounds correspond to different patterns of temporal pressure variation.

To better understand the nature of and differences between analog and digital signal processing, we shall use an analog system which is near extinction and probably unknown to many readers. This is the magnetic tape system, used for recording and playback of sounds such as speech or music, shown in Figure 1.2(a). The recording process and playback process, which is the inverse of the recording process, involve the following steps:

- Sound waves are picked up by a microphone and converted to a small analog voltage called the audio signal.
- The audio signal, which varies continuously to “mimic” the volume and frequency of the sound waves, is amplified and then converted to a magnetic field by the recording head.
- As the magnetic tape moves under the head, the intensity of the magnetic field is recorded (“stored”) on the tape.
- As the magnetic tape moves under the read head, the magnetic field on the tape is converted to an electrical signal, which is applied to a linear amplifier.
- The output of the amplifier goes to the speaker, which changes the amplified audio signal back to sound waves. The volume of the reproduced sound waves is controlled by the amplifier.



**Figure 1.1** Example of a recording of speech. The time waveform shows the variation of acoustic pressure as a function  $s(t)$  of time for the word “signal.”



**Figure 1.2** Block diagrams of (a) an analog audio recording system using magnetic tape and (b) a digital recording system using a personal computer.

Consider next the system in Figure 1.2(b), which is part of any personal computer. Sound recording and playback with this system involve the following steps:

- The sound waves are converted to an electrical audio signal by the microphone. The audio signal is amplified to a usable level and is applied to an analog-to-digital converter.
- The amplified audio signal is converted into a series of numbers by the analog-to-digital converter.
- The numbers representing the audio signal can be stored or manipulated by software to enhance quality, reduce storage space, or add special effects.
- The digital data are converted into an analog electrical signal; this signal is then amplified and sent to the speaker to produce sound waves.

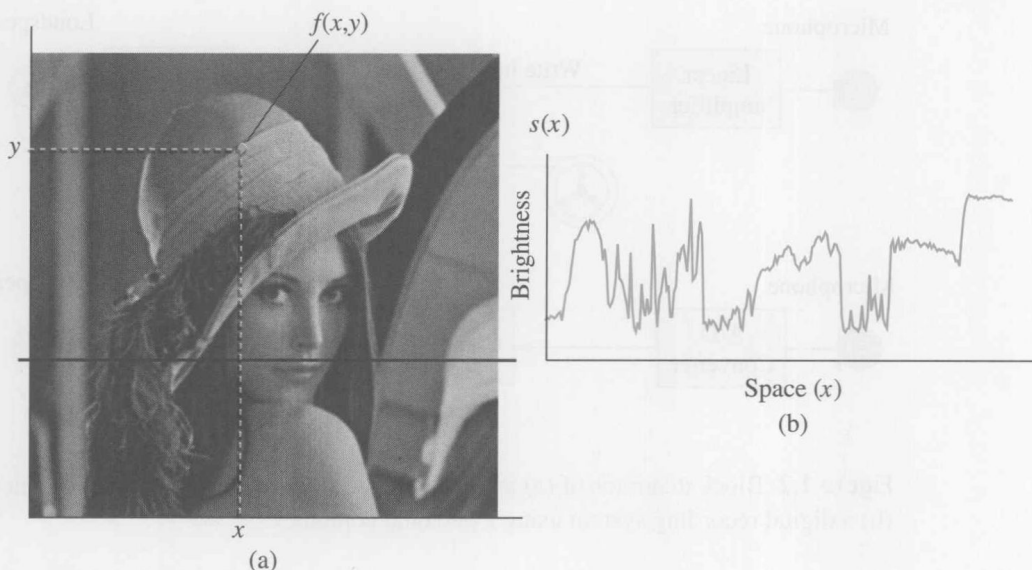
The major limitation in the quality of the analog tape recorder is imposed by the recording medium, that is, the magnetic tape. As the magnetic tape stretches and shrinks or the speed of the motor driving the tape changes, we have distortions caused by variations in the time scale of the audio signal. Also, random changes in the strength of the magnetic field lead to amplitude distortions of the audio signal. The quality of the recording deteriorates with each additional playback or generation of a copy. In contrast, the quality of the digital audio is determined by the accuracy of numbers produced by the analog-to-digital conversion process. Once the audio signal is converted into digital form, it is possible to achieve error-free storage, transmission, and reproduction. An interesting discussion about preserving information using analog or digital media is given by Bollacker (2010). Every personal computer has a sound card, which can be used to implement the system in Figure 1.2(b); we shall make frequent use of this system to illustrate various signal processing techniques.

### 1.1.1

#### Mathematical representation of signals

To simplify the analysis and design of signal processing systems it is almost always necessary to represent signals by mathematical functions of one or more independent variables. For example, the speech signal in Figure 1.1 can be represented mathematically by a function  $s(t)$  that shows the variation of acoustic pressure as a function of time. In contrast,





**Figure 1.3** Example of a monochrome picture. (a) The brightness at each point in space is a scalar function  $f(x, y)$  of the rectangular coordinates  $x$  and  $y$ . (b) The brightness at a horizontal line at  $y = y_0$  is a function  $s(x) = f(x, y = y_0)$  of the horizontal space variable  $x$ , only.

the monochromatic picture in Figure 1.3 is an example of a signal that carries information encoded in the spatial patterns of brightness variation. Therefore, it can be represented by a function  $f(x, y)$  describing the brightness as a function of two spatial variables  $x$  and  $y$ . However, if we take the values of brightness along a horizontal or vertical line, we obtain a signal involving a single independent variable  $x$  or  $y$ , respectively. In this book, we focus our attention on signals with a single independent variable. For convenience, we refer to the dependent variable as *amplitude* and the independent variable as *time*. However, it is relatively straightforward to adjust the notation and the vocabulary to accommodate signals that are functions of other independent variables.

Signals can be classified into different categories depending on the values taken by the amplitude (dependent) and time (independent) variables. Two natural categories, that are the subject of this book, are continuous-time signals and discrete-time signals.

The speech signal in Figure 1.1 is an example of a *continuous-time signal* because its value  $s(t)$  is defined for every value of time  $t$ . In mathematical terms, we say that  $s(t)$  is a function of a continuous independent variable. The amplitude of a continuous-time signal may take any value from a continuous range of real numbers. Continuous-time signals are also known as *analog signals* because their amplitude is “analogous” (that is, proportional) to the physical quantity they represent.

The mean yearly number of dark spots visible on the solar disk (sunspots), as illustrated in Figure 1.4, is an example of a discrete-time signal. *Discrete-time signals* are defined only at discrete times, that is, at a discrete set of values of the independent variable. Most signals of practical interest arise as continuous-time signals. However, the use of digital signal processing technology requires a discrete-time signal representation. This is usually done by *sampling* a continuous-time signal at isolated, equally spaced points in time