IEEE PES Winter Meeting Papers 1973 Vol.2

CONTENTS

Volume 1

C 73 044-5 Turn-to-Turn Fault Protection with Directional Negative Phase Sequence Relays

C 73 051-0 Study of Maintenance Procedures Used on High Voltage Circuit Breakers

C 73 052-8 Application of Epoxy Resin Bushings to HV Equipment

C 73 053-6 System Application and Operation of the Electronically Controlled Sectionalizer and Its Accessories

C 73 054-4 Development of a Test Circuit for Testing High-Voltage Circuit Breakers According to New IEC and ANSI Standards, Direct Testing

C 73 055-1 The Design and Testing of a New Interrupter for High Voltage Switching

C 73 060-1 Switchgear Applications of a Magnetically-Induced Arc Instability

C 73 061-9 System and Circuit Breaker Combine to Increase Low Natural Frequency Failure Possibility

C 73 065-0 A Novel Power Source for Navigational Lighting on High Voltage Radio Transmitter Towers

C 73 067-6 Copper-Aluminum Joints in the Current Coil of the General Electric I-70 Watthour Meter

C 73 068-4 Meter Interrogation by Reflective Transponder

C 73 069-2 Automatic Meter Reading via Cable

C 73 070-0 Design and Adaptation of Automatic Meter Reading Equipment to Electric Meters

C 73 071-8 Defining a Communications System for Automatic Data Collection from Utility Meters

C 73 072-6 Data Terminal and Other Common Equipment for Automatic Meter Reading

C 73 073-4 Staged Fault and Laboratory Tests on a Cable Pair Protection System Designed to Serve Power Stations

C 73 077-5 Load Flow Studies by Newton-Raphson Method: Convergence Characteristics

C 73 078-3 Equivalent Circuits for Multi-Winding Transformers: A General Procedure

C 73 079-1 Parameter Analysis in Transient Stability of Power Systems Including Saliency Effect

C 73 081-7 Optimal, Sub-Optimal and Practical Excitation Control Schemes for Stability

C 73 082-5 Z-Diakoptics in Sensitivity Studies

C 73 083-3 A Note on Static Equivalent Circuits of Power Systems

C 73 084-1 Analytic Study Aids for Power System Analysis

C 73 087-4 An Application of State Estimation in Modeling Power System Loads

C 73 088-2 A Dynamic Estimator for Tracking the Load-Model of a Power System

C 73 090-8 Variable Integration Step-Transient Analysis: Algorithm

C 73 093-2 Dynamic Modelling of an Arbitrary Number of Interconnected Power Generating Units in State Space Form

C 73 096-5 Unit Commitment Reliability in a Hydro-Termal System

C 73 097-3 Spinning Reserve Allocation in a Complex Power System

C 73 098-1 Direct Economic Dispatch

C 73 099-9 Suboptimal Regulation of Nonlinear Load Frequency Control Systems

C 73 101-3 On the Use Advanced Control Theory in the Design of Automatic Generation Controllers

C 73 102-1 A problem Oriented Algorithm to Load-Flow Computation in Distribution Networks

C 73 103-9 On Convergence of Newton's Method for Load Flow Problems

C 73 104-7 Representative Bibliography on Load-Flow Analysis and Related Topics*

C 73 105-4 A New Graphical Concept for Power Systems: The Graph Γ_S

C 73 106-2 On-Line System Model Error Correction

C 73 107-0 Security Evaluation in Power Systems Using Pattern Recognition

C 73 112-0 A Quadratic Programming Algorithm for Optimal Generation Planning

Volume 2

C 73 119-5 New Programming Approach to the Newton-Raphson Load-Flow

C 73 123-7 Contraction of Plastic Jacket of Power Cable Due to Cyclic Thermal Movement

C 73 124-5 Fault Tests on Embedded Copper wire and Copper Tape Shielded Single Conductor Cables

C 73 129-4 Linear Induction Motor Experiments in Comparision with Mesh/Matrix Analysis

C 73 135-1 Subsynchronous Resonance and Torsional Stresses in Turbine-Generator Shafts

C 73 136-9 Synchronous-Machine Armature-Reaction Harmonics Relative to the Rotor and the Main Field

C 73 138-5 A Superconducting Synchronous Transformer

C 73 140-1 Proposed New American National Standard for Combusion Gas Turbine Driven Synchronous Generators ANSI C50.14

C 73 142-7 Stability and Transient Response Analysis of a Reluctance Machine

C 73 143-5 Voltage Control of a Reluctance Generator

*C 73 144-3 Theoretical Analysis of the Stability and Transient Response of Self-Controlled Synchronous Machines

C 73 146-8 Inverter Control in Multiterminal HVDC Power Systems

C 73 148-4 Voltage Regulation Characteristics of AC-DC Interconnecting Point and Its Suppression Method--Application of Automatic Reactive Power Control on Converter Station

C 73 149-2 Development of HVDC Network Analyzer

C 73 150-0 Digital Computer Calculations of Switching Surge Wave Fronts on EHV Transmission Lines

C 73 153-4 The Consideration of Voltage Distribution in HVDC Thyristor Valve

C 73 155-9 A Wind Tunnel Study of Important Parameters for the Existence of Subconductor Vibration

C 73 156-7 Experimental Research on Subspan Galloping

C 73 158-3 Streamer Onset in Air in Nonuniform Fields

C 73 159-1 Digital Computer Calculation of Streamer Threshold of Bundle Conductors*

C 73 160-9 Calculation of Negative Corona Onset Voltages

C 73 161-7 Further Studies on Power Transmission by Multi-Conductor Bundles and on Field and Gradient Distribution Around Them

C 73 163-3 Analysis of the Effects of Wind on DC Transmission Line Corona Losses

C 73 164-1 Spectral Characteristics of Acoustic Noise From Metallic Protrusions and Water Droplets in High Electric Fields

C 73 165-8 Current-Driven, Corona-Terminated Water Jets as Sources of Charged Droplets and Audible Noise

C 73 168-2 Effects of ELF Interference on Power Systems: Analysis, Field Data, Mitigation Techniques

C 73 169-0 Alternating Current Distribution in Parallel Conductors of Arbitrary Cross-Sectional Shape Part I.Analysis and Skin-Effect Current Distribution

C 73 170-8 Alternating Current Distribution in Parallel Conductors of Arbitrary Croos-Sectional Shape Part II. Combined Skin-Effect and Proximity Effect Current Distribution

C 73 172-4 Analysis of Two Layer Aluminum Conductor Ateel Reinforced*

C 73 177-3 Statistical Study of Parallel Gaps of Unequal Charactristics under Switching Surges

C 73 180-7 Application of the Green's Function to Tower Design Analysis

C 73 181-5 Transmission Line Representation of Dynamic Tower and Static Wire Surge Impedance with Applciations

C 73 183-1 Calculation of Transient Recovery Voltage of Extensive Networks

C 73 184-9 Switching Surges on the Four Corners — Eldorado 525 kV Series Compensated Line

C 73 186-4 Including Frequency Dependent Parameters in Transmission Line Simulation Studies

C 73 187-2 Digital Simulation and Theoretical Determination of Surge Waveshapes in DC or AC Power Lines

C 73 188-0 All-Film Power Factor Correction Capacitor

C 73 190-6 Determination of Average Loss in Distribution Lines

C 73 192-2 A Variable Speed Twin Stator Cage Induction Motor

C 73 194-8 Application of D-Q Transformation Technique to Pole-Changing Motors

C 73 195-5 Polyphase Motor with Dual Winding Rotor

C 75 196-3 Stability and Transient Response Analysis of an Induction Machine

C 73 199-7 Static Torque Characteristics of an Induction Motor with a Variable Frequency Supply

C 73 200-3 Multiplexing -- Possible Answer to Wiring Congestion

C 73 202-9 Inplant Remote Multiplexing for Nuclear Power Plants

C 73 203-7 Multiplexed Rod Drive Control System for a General Electric BWR

C 73 205-2 Computer Generated Electrical Elementary Wiring Diagrams

C 73 208-6 Performance Prediction of Annulus Convective Cooling for Electrical Bus-Bar Enclosure System

C 73 212-8 Experimental Study of Central Station MHD Power Generation*

C 73 213-6 Batteries and the Economics of Load Leveling in Large Power Systems

C 73 214-4 Power Generation with the Dry-Cooled Nuclear Gas Turbine Plant

C 73 216-9 A Comparative Study Decompositions in Optimizing Hydro-Thermal Power Scheduling

C 73 219-3 Improving Supplemental Excitation Control Design Using Accurate Model

C 73 221-9 Dynamic System Stability Part II Design Realistic Compensating Network for Excitation Control Using Digital Computer

C 73 222-7 Choice of the Excitation System Parameters for Maximum Possible Capacitive Power Loading of Dual-Excited Synchronous Generators

C 73 223-5 Limitations of Increasing the Capacitive Power Loading of Regulated Dual-Excited Synchronous Condensers

C 73 225-0 Electricity Generation and District Heating by means of Combined Generation of Heat and Electric Power — Limits and Effects--

C 73 226-8 An Analytical Study of Turbine Generator Radial Ventilation

C 73 227-6 Block and Double-Block Connections for HVDC Power Station Infeed

C 73 231-8 Pole-Changing Single-Winding Repulsin-Start Induction Motor

NEW PROGRAMMING APPROACH TO THE NEWTON-RAPHSON LOAD-FLOW

N. Nabona
Imperial College of Science and Technology
London, Great Britain

L.L. Freris
Imperial College of Science and Technology
London, Great Britain

ABSTRACT

There are several differences between the conventional Newton-Raphson load-flow programs using sparsity techniques and that described in this paper, the principal one being the reuse of the location vectors of the sparse clustered Jacobian after the first triangularisation-factorisation.

In the first iteration the location vectors are modified to take into account the new created elements during triangularisation. In the subsequent iterations a different and faster subroutine is used as no logical instructions are necessary for the location of the new created elements. It is shown that when this technique is applied to large systems, computing time improvements approaching 50% in comparison to conventional programs are obtained.

INTRODUCTION

During the last few years the Newton-Raphson (N.R.) technique has emerged as the most efficient method for the solution of the load-flow problem in power systems. This came about through the exploitation of the sparsity of the matrices involved and through ordered elimination. The resulting saving in storage and computing time transformed the technique from what was considered initially as a non-starter to an extremely fast and powerful load-flow method.

The N.R. algorithm itself is a simple enough technique but the advances in programming efficiency to which the method owes its success are complex and meagrely documented. The existing literature on the exploitation of sparsity gives scanty details on programming and organisation[1,2,3]. However, through the information in this literature an overall picture can be put together of the features that these N.R. programs have in common:

1) use of polar co-ordinates.
2) use of one slack bus - usually node No. 1.
3) provision of storage for the Jacobian modified with reference to a pre-established slack bus and to a fixed pre-established set of voltage controlled nodes.
4) calculation of the injected currents and power mismatches, formation, triangularisation and factorisation of the Jacobian all carried out simultaneously node by node.
5) use of the same triangularisation subroutine throughout the program.

At Imperial College an N.R. based program (referred to henceforth as I.C.N.R.) was developed as a part of a system optimisation program. The object of this paper is to describe the I.C.N.R. and show that considerable savings in computing time are achieved through the adoption of the following features:

CONFERENCE PAPER

C 73 119-5. A paper recommended by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE PES Winter Meeting, New York, N.Y., January 28-February 2, 1973. Manuscript submitted September 8, 1972; made available for printing November 29, 1972.

Price: Members $1.35
Nonmembers $1.80
At Meeting: $1.00

1) use of rectangular co-ordinates.
2) facility to employ as many slack nodes as one wishes and distributed in any desired manner.
3) storage and location vectors of the Jacobian can be adapted to any set of slack nodes and voltage controlled nodes during the execution of the program.
4) the calculation of the injected currents and power mismatches, formation, triangularisation and factorisation of the Jacobian are carried out in three separate stages.
5) a normal triangularisation subroutine is used only in the first iteration with a different one in the subsequent iterations.

It is shown later that feature 5 and to a lesser degree feature 1, result in a considerable reduction of computing time. In the N.R. programming technique proposed in reference 2 the problem is solved in the last-but-one iteration. However, this is not detected until the end of the last iteration. This redundant extra iteration is avoided through feature 4 which is thus instrumental in the reduction of solution time. Features 1, 2 and 3 enable the program to change the type of node variables during the execution of the program with the minimum of programming effort and computation time, a facility necessary for economic optimisation techniques that employ change of type of control variables.

Another factor influencing the speed of an N.R. algorithm is the ordering of the nodes. It has been shown that the number of newly created elements and, therefore, the number of operations can be minimised if optimal ordering is implemented[4]. The programming aspects considered in this paper are, however, independent of the ordering scheme adopted and will reduce the computation time whether the ordering is optimal or otherwise.

THE STORAGE OF THE SPARSE JACOBIAN

The major programming task in the N.R. method involves the handling, storing and manipulation of the system Jacobian matrix, which is dimensionally large but is sparse. For a large system, the Jacobian is far too big to be stored in matrix form and the time to triangularise it conventionally is far too long. Sparsity techniques have therefore been developed[5] to store only a minimum number of elements and to triangularise performing a minimum of operations.

It is computationally convenient not to operate on the vector of constants during the triangularisation of the Jacobian. Instead, the factors which will eventually operate on and modify the vector of constants are stored in the lower diagonal part of the triangularised Jacobian. The resulting matrix is topologically symmetrical and is referred to as the "Table of Factors".

Take, for example, the problem of storing the matrix of Fig. 1. The non-zero upper-diagonal elements can be efficiently stored in a vector 'ELEM' together with the location vectors 'CLN' and 'FPR' as shown in Table I.

Vector CLN gives the column number of the element and contains as many elements as there are non-zero upper-diagonal elements. Vector FPR indicates the first of the elements in ELEM that belongs to a new row. The

	1	2	3	4	5
1	A	å	å	å	
2	a	A	å		å
3	a	a	A		å
4	a			A	
5		a	a		A

A: diagonal element
å: upper-diagonal elem.
a: lower-diagonal elem.

Figure 1. Example of a small sparse matrix

NUM	ELEM.	CLN	ROW	FPR
1	a(1,2)	2	1	1
2	a(1,3)	3		
3	a(1,4)	4		
4	a(2,3)	3	2	4
5	a(2,5)	5		
6	a(3,5)	5	3	6
7	-	-	4	7
8	-	-	5	8

TABLE I. Compact storage of a sparse matrix

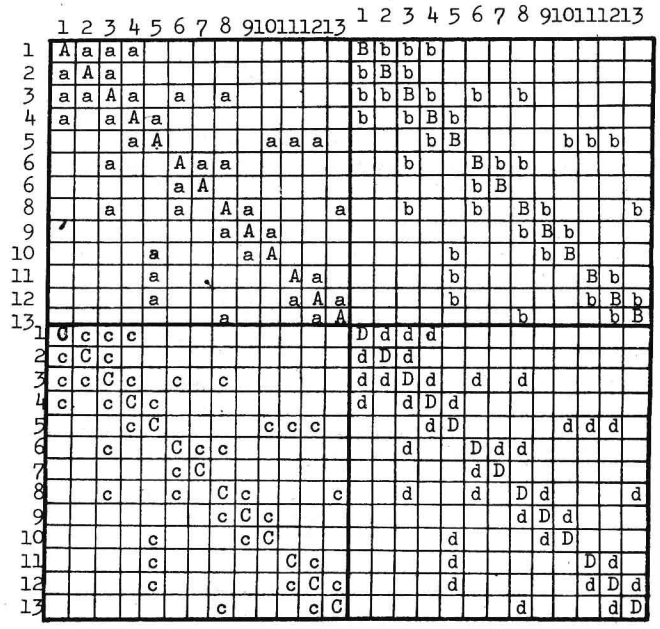
The diagonal elements are stored in a separate vector. If the matrix is topologically symmetrical about the diagonal the same pair of vectors CLN and FPR describe the lower triangular matrix if the row and column labelling is interchanged.

The following four points have to be taken into account when planning the strategy for optimum storage of the Jacobian matrix:

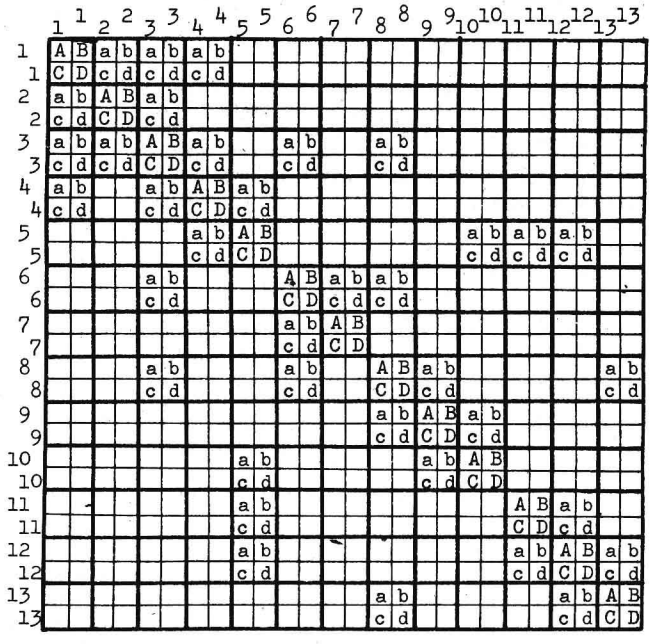
a) as the Jacobian is a topologically symmetrical matrix its table of factors is also symmetrical even if new elements have appeared during the triangularisation-factorisation process. Therefore, any location vectors used to describe the Jacobian before or after triangularisation-factorisation only need to describe the location of the upper-diagonal elements.

b) let $S_p = P_p + jQ_p$ be the complex power injected into node p and $(e_q + jf_q)$ the complex voltage at bus q. The order of the variables and of the equations which have the Jacobian as matrix of coefficients can be arranged so as to segregate in clusters of four elements the coefficients: $\delta P_p/\delta e_q$, $\delta P_p/\delta f_q$, $\delta Q_p/\delta e_q$, $\delta Q_p/\delta f_q$. In this way the location vectors that describe any of the topologically identical sub-matrices or 'parts' of the Jacobian $(\delta P_i/\delta e_j$ i,j = 1,...,N), $(\delta P_i/\delta f_j$ i,j = 1,...,N), $(\delta Q_i/\delta e_j$ i,j = 1,...,N), $(\delta Q_i/\delta f_j$ i,j = 1,...,N) can also describe the whole Jacobian since each cluster of elements corresponding to the same node can be topologically considered as one element. This is illustrated in Fig. 2 where the Standard and the Clustered Jacobians of the IEEE 14 bus system with bus number one as the slack bus are shown. It is clear that the four parts of the Jacobian are topologically identical and that by changing the order of the equations and of the variables the Clustered Jacobian is obtained. The topology of the Clustered Jacobian, assuming that a cluster represents a term, is identical to the topology of one part of the original Jacobian. When the triangularisation-factorisation is performed on the Clustered Jacobian the newly created elements during this process also appear in clusters of four. The benefit resulting from this arrangement is illustrated in Figure 3 where the newly created elements after triangularisation-factorisation of the Standard and

Standard Jacobian



Clustered Jacobian



a: $\delta P_i/\delta e_j$ b: $\delta P_i/\delta f_j$ c: $\delta Q_i/\delta e_j$ d: $\delta Q_i/\delta f_j$
A: $\delta P_k/\delta e_k$ B: $\delta P_k/\delta f_k$ C: $\delta Q_k/\delta e_k$ D: $\delta Q_k/\delta f_k$

Figure 2. Standard Jacobian and Clustered Jacobian

the Clustered Jacobians of Figure 2 are shown. The difference in the generation of new elements is striking: in this case clustering results in a created element number reduction of almost 2 to 1.

c) the topological structure of any part of the Jacobian and, therefore, of the whole Jacobian is the same as that of the nodal admittance matrix. As a result, the same location vectors can be used to describe in a compact manner both matrices. In a load

flow however, there is at least one slack bus, therefore, at least two of the Jacobian rows and columns have to be removed. The resulting reduced Jacobian requires a new set of location vectors. For every new set of slack buses chosen a modified set of location vectors is required.

d) although the nodal-admittance matrix can be stored in its most compact form, this is not possible with the Jacobian because triangularisation will create new elements for which space should be reserved. Furthermore, the number and location of the new elements will depend on the choice of the slack bus or buses. For example, Figure 4 shows the table of factors corresponding to the matrix of Figure 1 and Table II shows the compactly stored matrices before and after triangularisation. Table III illustrates the effect on the triangularised Jacobian of removing the line and column corresponding to a slack bus; this is done for all possible locations of the slack bus. This table shows that irrespective of the bus or buses chosen, the resulting set of elements after the triangularisation is always a subset of the triangularised full matrix.

LOCATION VECTORS FOR THE REDUCED JACOBIAN

The location vectors of the nodal admittance matrix and the set of slack buses chosen for the load-flow study determine the structure of the location vectors of the reduced Jacobian before triangularisation. Table IV shows the triangularised full and reduced Jacobian with node 3 chosen as the slack bus. A new set of location vectors CLN_3^* and FPR_3^* can be worked out for the storage of the reduced Jacobian. From the table it can be deduced that for any choice of slack bus or buses FPR* will be the same as FPR with the difference that FPR* has no elements corresponding to the row of the slack bus.

	1	2	3	4	5
1	A	å	å	å	
2	a	A	å	nå	å
3	a	a	A	nå	å
4	a	na	na	A	nå
5		a	a	na	A

A: diagonal element.

å: element of the triangularised matrix.

a: factor for the forward substitution.

na: newly created element.

Figure 4. Example of triangularised-factorised matrix

Before Triangularisation

NUM	ELEMT.	CLN	FPR
1	a(1,2)	2	1
2	a(1,3)	3	
3	a(1,4)	4	
4	a(2,3)	3	4
5	a(2,5)	5	
6	a(3,5)	5	6
7	-	-	7
8	-	-	8

After Triangularisation

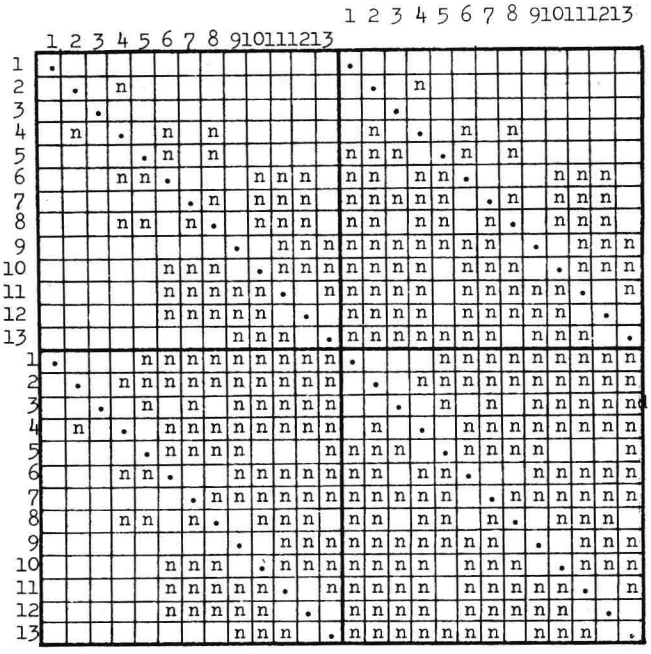
NUM	ELEMT.	CLN	FPR
1	a(1,2)	2	1
2	a(1,3)	3	
3	a(1,4)	4	
4	a(2,3)	3	4
5	na(2,4)	4	
6	a(2,5)	5	
7	na(3,4)	4	7
8	a(3,5)	5	
9	na(4,5)	5	9
10	-	-	10

(the underlined numbers in vector CLN correspond to the newly created elements)

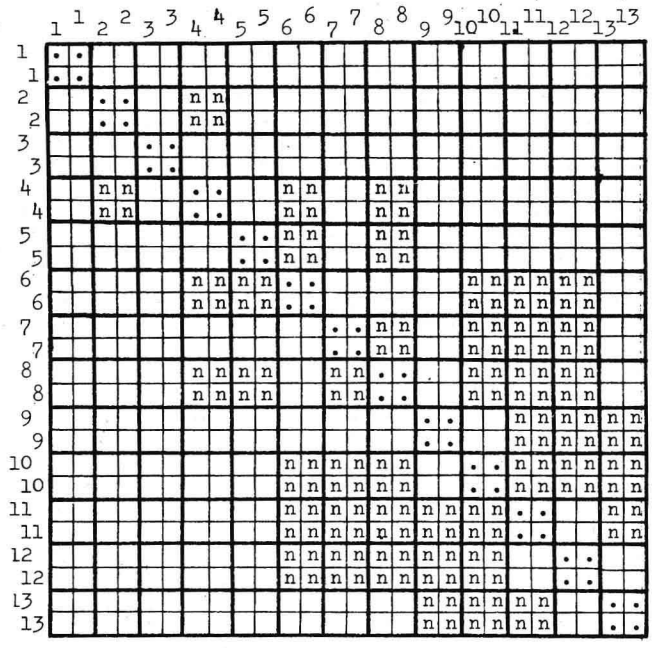
TABLE II. Compactly Stored Matrix Before and After Triangularisation.

At the beginning of the load-flow the elements about to be created are not known because the reduced Jacobian has not yet been triangularised, (these new elements are underlined in the CLN vector of Table IV). During the first triangularisation the new elements corresponding to the particular reduced Jacobian will be created. Determining new elements and fitting these

Standard



Clustered



n: newly created element during the triangularisation-factorisation

Figure 3. Created Elements for the Standard and Clustered Jacobian.

in the location vectors entails a considerable amount of logical computation in the first triangularisation. As shown in Tables V and VI this computation can be appreciably reduced in the subsequent triangularisations.

NUM	full ELEMNT.	1 out ELEMNT.	2 out ELEMNT.	3 out ELEMNT.	4 out ELEMNT.	5 out ELEMNT.
1	a(1,2)	-	-	a(1,2)	a(1,2)	a(1,2)
2	a(1,3)	-	a(1,3)	-	a(1,3)	a(1,3)
3	a(1,4)	-	a(1,4)	a(1,4)	-	a(1,4)
4	a(2,3)	a(2,3)	-	-	a(2,3)	a(2,3)
5	na(2,4)	-	-	na(2,4)	-	na(2,4)
6	a(2,5)	a(2,5)	-	a(2,5)	a(2,5)	-
7	na(3,4)	-	na(3,4)	-	-	na(3,4)
8	a(3,5)	a(3,5)	a(3,5)	-	a(3,5)	-
9	na(4,5)	-	na(4,5)	na(4,5)	-	-
10	-	-	-	-	-	-

TABLE III. Triangularised Full and Reduced Matrices

NUM	ELEMNT.	CLN	FPR	ELEMN.$_3$	CLN$_3$	CLN$_3$*	FPR$_3$	FPR$_3$*
1	a(1,2)	2	1	a(1,2)	2	2	1	1
2	a(1,3)	3		a(1,4)	4	3		
3	a(1,4)	4		0	0	0		
4	a(2,3)	3	4	na(2,4)	4	3	4	4
5	na(2,4)	4		a(2,5)	5	4		
6	a(2,5)	5		0	0	0		
7	na(3,4)	4	7	0	0	0	0	
8	a(3,5)	5		0	0	0		
9	na(4,5)	5	9	na(4,5)	5	4	9	9
10	-	-	-	-	-	-	-	-

TABLE IV. Comparison of Storage for Triangularised Full and Reduced Matrix

RECTANGULAR VERSUS POLAR CO-ORDINATES

In power system networks there are some nodes at which a certain level of voltage must be maintained by appropriate injection of reactive power. If rectangular co-ordinates are used, the reactive power mismatch equation is replaced by a voltage magnitude mismatch equation. The row representing this equation in the Jacobian will have terms only in the position corresponding to the diagonals of the two lower parts of the Standard Jacobian. As a result, the topological location vectors for a normal reactive power mismatch equation can be used provided that the non-diagonal elements of the row are made zero. In this way the topological symmetry of the four parts of the Jacobian is preserved.

If instead, polar co-ordinates were used, the voltage magnitude mismatch equation for a voltage controlled bus would disappear with the resulting break-up of the symmetry of the four parts of the Jacobian. The problem of course can still be successfully solved but at the expense of many logical instructions which are time consuming.

THE I.C.N.R. PROGRAM

Assuming that some optimum or sub-optimum ordering of the buses has been chosen, enough storage is provided in the location vectors so that a Jacobian with every possible row and column can be stored. This involves a few redundant storage places corresponding to the nodes subsequently chosen as slack buses but with the advantage that the program is very flexible and capable of dealing with any number and any position of slack and voltage controlled buses.

The first part of the N.R. subroutine contains the logical instructions to adapt the generalised location vectors of the nodal admittance matrix to the particular location vectors corresponding to the set of slack buses chosen.

The subsequent steps are as follows:

a) the currents and powers injected into each node are calculated using the last estimates of the node voltages. The mismatch at each node is checked as soon as the power is calculated. If the mismatches are less than a predeterminated tolerance the computation is terminated.

b) the Jacobian is built up using the last estimates of the node voltages.

c) the Jacobian is triangularised and left in its factored form. During the first iteration a conventional sparse triangularisation subroutine is used. This subroutine will create some new elements thus filling some zeros in the Jacobian and consequently modifying the location vectors. From the second iteration onwards a different triangularisation subroutine is employed. The available modified location vectors are used and zeros are placed in the positions of the non-triangularised Jacobian where the new created elements are about to appear. This implies that some useless multiplications by zero will be performed but the gain in time due to the reduced logic instructions far outstrips this loss.

d) the voltage increments are found by forward and backward substitution of the triangularised and factorised Jacobian in the vector of power mismatch. The bus voltages are then updated and the process is started again.

RESULTS AND CONCLUSIONS

The I.C.N.R. program was tried very successfully on several systems. The results of load-flow studies on six of these systems ranging from 14 to 118 nodes are presented in Table VI.

In Table V the calculation times for the 118 node system without and with a pseudo-optimal ordering are shown. It can be concluded that:

a) most of the iteration time is taken by the triangularisation-factorisation.

b) there is a considerable saving in computing time from the first to the subsequent iterations due to the omission of the logical steps associated with the location of the new created elements.

Table VI gives pertinent information of the six systems solved and the saving in time involved when I.C.N.R. is used instead of conventional N.R.

The solution times for the pseudo-optimally ordered 118 bus system show a gain of 32% of the I.C.N.R. over the N.R. as against a 40% for the 118 bus system without reordering. This was to be expected because the closer a system is to its optimal ordering, the fewer elements are created during triangularisation therefore, there is less improvement in saving the computation to find the new created elements.

Using the improved N.R. programming technique described, improvements on computation time of 20% on small systems to figures approaching 50% on large systems were obtained.

REFERENCES

(1) TINNEY, W.F. and WALKER, J.F.: "Direct solution of sparse network equations by optimally ordered triangular factorization", IEEE Proceedings, Vol.55, pp. 1801-1809, November, 1967.

(2) TINNEY, W.F. and HART, C.E.: "Power flow solution by Newton's method", IEEE Trans. (Power Apparatus and Systems), Vol.86, pp.1449-1460, November, 1967.

(3) TINNEY, W.F.: "Some examples of sparse matrix methods for power system problems", Paper N.A.5, Proceedings of the Power System Computation Conference, Rome, 1969.

(4) STOTT, B. and HOBSON, E.: "Solution of large power-system networks by ordered eliminations: a comparison of ordering schemes", Proceedings of the IEE (London), Vol. 118, pp.125-134, January, 1971.

Times in seconds. Programs written in Fortran IV run on the Imperial College CDC 6400.

After the first iteration a subroutine to triangularise-factorise the Jacobian without finding the new created elements is used. The maximum P,Q mismatch at the end of the load-flow was 2.1×10^{-2} MVA for both cases.

Ordinary 118 bus system: calculation time for the load-flow 10.852 seconds.

Iteration number	Time per iteration	Calculation of new currents and powers	Formation of Jacobian	Triangularisation and factorisation	Forward, back. subs. Updating of voltages
1	6.131	0.209	0.341	5.435	0.146
2	2.310	0.052	0.339	1.775	0.144
3	2.317	0.055	0.340	1.777	0.145
4	0.094	0.094	-	-	-

Reordered 118 bus system: calculation time for the load-flow 3.011 seconds.

Iteration number	Time per iteration	Calculation of new currents and powers	Formation of Jacobian	Triangularisation and factorisation	Forward, back. subs. Updating of voltages
1	1.569	0.215	0.419	0.874	0.061
2	0.688	0.057	0.413	0.157	0.061
3	0.678	0.052	0.406	0.159	0.061
4	0.076	0.076	-	-	-

TABLE V. Computation Times for the IEEE 118 Bus Test System

System bus number	14*	23	30*	43	57*	118*	118**
Workspace required (words)	946	1,298	2,159	2,334	6,374	13,943	6,393
Number of lines/Number of buses	2.14	1.3	1.37	0.98	1.37	1.52	1.52
No. of non-zero, non-diag. Jacobian elements after triang./idem before triang.	2.1	2.07	3.12	2.71	6.06	5.72	1.47
Newton-Raphson iterations to converge	3	3	3	6	3	3	3
Per unit maximum P,Q mismatch (base 100MVA)	5×10^{-6}	8×10^{-5}	10^{-5}	2×10^{-5}	3×10^{-5}	2×10^{-4}	2×10^{-4}
Triang.-factostn. time with normal subrtn.	0.056	0.070	0.248	0.225	1.768	5.435	0.874
Triang.-factostn. time with subroutine not finding the new created elements	0.036	0.032	0.146	0.100	0.865	1.775	0.158
Total solution time using N.R. method	0.261	0.341	0.953	1.789	5.872	18.170	4.443
Total solution time using I.C.N.R. method	0.227	0.227	0.761	1.164	4.066	10.852	3.011
% time saving of I.C.N.R. over N.R.	13.0	18.8	20.0	34.9	30.7	40.2	32.1

All times in seconds. Programs written in Fortran IV run on the Imperial College CDC 6400.
* IEEE test system without reordering
** IEEE test system reordered

TABLE VI. Characteristics of Six Test Systems