

SALAS AND HILLE'S
CALCULUS

**ONE AND SEVERAL
VARIABLES**

SEVENTH EDITION

REVISED BY
GARRET J. ETGEN



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In fond remembrance of
EINAR HILLE

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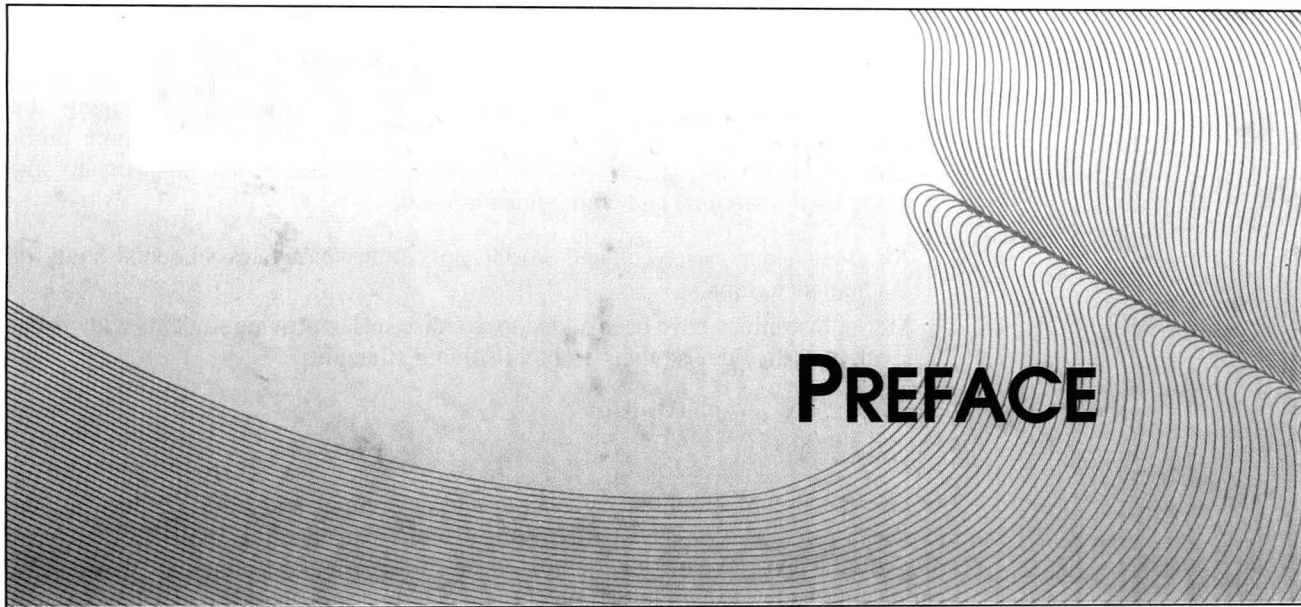
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PREFACE

Above all, this is a text on mathematics. The subject is calculus, and the emphasis is on the three basic concepts: limit, derivative, and integral.

This text is designed for a standard introductory single and multivariable calculus sequence. Our fundamental goal in preparing the Seventh Edition has been to preserve and enhance the notable strengths that characterized previous editions, including:

- An emphasis on the mathematical exposition—an accurate, understandable treatment of the topics.
- A clear, concise approach. Basic ideas and important points are not obscured by excess verbiage.
- An appropriate level of rigor. Mathematical statements are careful and precise, and all important theorems are proved. This formality is presented in a way that is completely accessible to the beginning calculus student.
- A balance of theory and applications, illustrated by many examples and exercises.

At the same time, we recognize that with the rapid advances in computer technology and the current scrutiny of mathematics education at all levels, the teaching of calculus is undergoing a serious examination. Thus, an equally important and parallel goal of the Seventh Edition has been to incorporate modern technology and current trends without sacrificing the acknowledged strengths of the text.

FEATURES OF THE SEVENTH EDITION

Problem-Solving Skills and Real-World Applications

Over 2000 new problems have been added to the Seventh Edition.

- In order to develop students' problem-solving skills, we have significantly increased the number of problems at all levels. A large number of challenging and

routine problems are now available in all exercise sets. Many additional medium-level problems are included to assist students in developing the understanding necessary to attack the challenging problems. In some problems, students are called upon to interpret and justify their answers to improve their analytical and communication skills.

- An even wider variety of real-world applications motivates students' study of mathematical topics.
- More illustrations have been added to exercise sets to provide students with visual support as they devise their problem-solving strategies.

Technology

Because the use of graphing calculators and/or computer algebra systems has increased in calculus courses, we have considerably expanded the application of technology in the text. We do not attempt to teach any particular technology and so use a generic approach. Technology problems are clearly designated with an icon (▶) and may be skipped by instructors who prefer that their students not use calculators or computers.

- New technology-based examples appear within the chapter discussions of the material. These support the numerous exercises requiring the use of a graphics calculator or other graphing software located in the end-of-section problems sets.
- “Projects and Explorations Using Technology,” a set of problems that requires a combination of approaches involving both analytical and technology skills, ends each chapter. As their title suggests, these problems are also suitable for use by students working in groups. A few of the problems introduce concepts to be developed later in the text, while others explore realistic applications of topics that have already been studied.

Increased Emphasis on Visualization

We recognize the importance of visualization in developing students' understanding of mathematical concepts. For that reason:

- All the artwork from the previous edition has been redrawn for increased clarity and understanding.
- Over 130 new figures have been added.
- Representations in three dimensions are now in full color for increased geometric understanding and include many new computer-generated figures of curves and surfaces in space.

Early Introduction of Differential Equations

A subsection on differential equations (separable equations) has been added to the Exponential Growth and Decay section of Chapter 7, allowing exercises on differential equations to be used throughout the rest of the book.

CONTENT AND ORGANIZATION CHANGES IN THE SEVENTH EDITION

In response to the evolutionary state of the current calculus curriculum, many changes have been made in organization and content to meet the needs of today's students and instructors.

Precalculus Review (Chapter 1)

- This material, which provides a brief yet comprehensive review of the precalculus topics basic to the study of calculus, has been rewritten and expanded.
- The real number system and the real line are now discussed; the section covering methods for solving inequalities in one variable has been reorganized; and the treatment of analytic geometry—the Cartesian coordinate system, straight lines, and the conic sections—has been expanded.
- The treatment of the function concept, the elementary functions, and graphing (including technology) has been reorganized and expanded.
- The treatment of one-to-one functions and inverses has been moved to Chapter 7, where it serves to connect logarithmic and exponential functions.
- The brief section on proofs now includes mathematical induction with examples and exercises.

Limits and Continuity (Chapter 2)

- To improve students' understanding of these critical topics, a few of the discussions have been expanded and some figures have been added.
- The section on limits now includes a technology approach, illustrated by examples and new exercises.
- The derivative in various forms is introduced in the exercises.

Differentiation and Applications of the Derivative (Chapters 3 and 4)

- The interpretations of the derivative as a rate of change are treated in one section rather than two, and the coverage includes examples from economics.
- Differentiation of inverse functions has been moved to Chapter 7.
- The derivative of rational powers is now approached through implicit differentiation.
- The treatment of applications of the derivative has been reorganized slightly to emphasize the two objectives of Chapter 4: optimization and curve sketching.

Integration and Applications of the Integral (Chapters 5 and 6)

- The interpretation of the definite integral as “area” has been expanded; a subsection on “signed area” has been added.
- The treatment of u -substitutions has been streamlined; for example, trigonometric integrals are now incorporated in the change of variables section rather than as a separate section.
- The introduction of Riemann sums serves to introduce and motivate the approach taken in the applications chapter.
- The treatments of the various applications of the definite integral have been expanded.

The Transcendental Functions (Chapter 7)

- Inverse functions and the calculus of inverse functions are treated here rather than in separate sections earlier in the text.
- Integration by parts has been moved to Chapter 8, Techniques of Integration, and simple harmonic motion has been moved to Chapter 18, Differential Equations.
- The treatment of applications of exponential and logarithmic functions now includes a subsection on differential equations (separable equations).
- The treatment of the inverse trig functions now includes the inverse secant function.

Techniques of Integration (Chapter 8)

- Integration by parts has been moved from Chapter 7 to Section 8.2, and the section on partial fractions now follows trig substitutions.
- Almost all the treatments have been expanded, with many new examples, increased coverage of reduction formulas, and a discussion of hyperbolic substitutions.

Conic Sections; Polar Coordinates and Parametric Equations (Chapter 9)

- This new chapter combines material from two chapters in the previous edition. The conic sections are treated in one section; translation of axes and rotation of axes have been moved to Chapter 1 or the Appendices.
- The treatment of arc length has been softened a little: the proof based on the least upper bound axiom has been replaced by an intuitive argument.

Sequences and Series (Chapters 10 and 11)

- The least upper bound axiom now serves as a prelude to sequences, and there is more emphasis on boundedness in the treatment of sequences.
- The treatment of indeterminate forms has been modified: The “other” indeterminate forms—differences, products, exponential forms—are now treated in a separate subsection rather than integrating them with the $0/0$ and ∞/∞ forms.
- The treatment of power series has been expanded slightly; there are some new examples and figures, and the Lagrange form of the remainder is stated explicitly and used to derive bounds on the remainder.

Multivariable Calculus (Chapters 12–17)

- Substantial changes were not necessary in the treatment of these chapters. The major effort in this edition was to upgrade the illustrations and the exercises.
- A large number of computer-generated figures illustrating curves and surfaces in space have been added, and full color has been used where it is most helpful to students’ understanding—in three-dimensional figures.

Differential Equations (Chapter 18)

- The material on differential equations has been thoroughly updated and revised to include numerous examples and applications throughout the chapter.
- A new introductory section familiarizes students with the basic terminology and concepts of differential equations.

FEATURES OF THE BOOK

Concise exposition The concepts of calculus are presented clearly and accurately without hand waving.

Theorems and proofs Highlighted theorems direct students to accurate mathematical statements. Most proofs are included to provide a high level of precision.

2.5 THE PINCHING THEOREM; TRIGONOMETRIC LIMITS

Figure 2.5.1 shows the graphs of three functions f, g, h . Suppose that, as suggested by the figure, for x close to c , f is trapped between g and h . (The values of these functions at c itself are irrelevant.) If, as x tends to c , both $g(x)$ and $h(x)$ tend to the same limit L , then $f(x)$ also tends to L . This idea is made precise in what we call the *pinching theorem*.

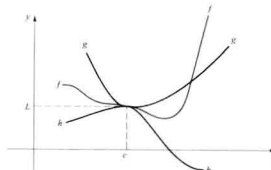


Figure 2.5.1

THEOREM 2.5.1 THE PINCHING THEOREM

Let $p > 0$. Suppose that, for all x such that $0 < |x - c| < p$,

$$h(x) \leq f(x) \leq g(x).$$

If

$$\lim_{x \rightarrow c} h(x) = L \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = L,$$

then

$$\lim_{x \rightarrow c} f(x) = L.$$

PROOF Let $\epsilon > 0$. Let $p > 0$ be such that

$$\text{if } 0 < |x - c| < p, \quad \text{then } h(x) \leq f(x) \leq g(x).$$

Choose $\delta_1 > 0$ such that

$$\text{if } 0 < |x - c| < \delta_1, \quad \text{then } L - \epsilon < h(x) < L + \epsilon.$$

Choose $\delta_2 > 0$ such that

$$0 < |x - c| < \delta_2, \quad \text{then } L - \epsilon < g(x) < L + \epsilon.$$

δ_2 . For x satisfying $0 < |x - c| < \delta$, we have

$$L - \epsilon < h(x) \leq f(x) \leq g(x) < L + \epsilon,$$

$$|f(x) - L| < \epsilon. \quad \square$$

Example 3 Figure 4.7.10 is a computer-generated graph of the function

$$f(x) = \frac{\cos x}{x}.$$

As $x \rightarrow 0^-$, $f(x) \rightarrow -\infty$; as $x \rightarrow 0^+$, $f(x) \rightarrow \infty$. The line $x = 0$ (the y -axis) is a vertical asymptote.

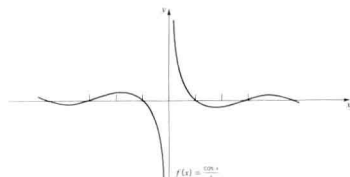


Figure 4.7.10

As $x \rightarrow \pm\infty$,

$$f(x) = \frac{\cos x}{x} \rightarrow 0.$$

This follows from the fact that

$$\left| \frac{\cos x}{x} \right| \leq \frac{1}{|x|} \quad \text{for all } x$$

and $1/|x| \rightarrow 0$ as $x \rightarrow \pm\infty$. Thus, the line $y = 0$ (the x -axis) is a horizontal asymptote. Note that f is an odd function [$f(-x) = -f(x)$] so its graph is symmetric with respect to the origin. \square

Example 4 Find the vertical and horizontal asymptotes, if any, of the function

$$g(x) = \frac{x+1-\sqrt{x}}{x^2-2x+1} = \frac{x+1-\sqrt{x}}{(x-1)^2}.$$

SOLUTION The domain of g is $0 \leq x < \infty$, $x \neq 1$. As $x \rightarrow 1$, $g(x) \rightarrow \infty$. Thus, the line $x = 1$ is a vertical asymptote. The behavior of g as $x \rightarrow \infty$ can be made more apparent by writing

$$g(x) = \frac{x+1-\sqrt{x}}{x^2-2x+1} = \frac{x\left(1+\frac{1}{x}-\frac{1}{\sqrt{x}}\right)}{x^2\left(1-\frac{2}{x}+\frac{1}{x^2}\right)} = \frac{1+\frac{1}{x}-\frac{1}{\sqrt{x}}}{x\left(1-\frac{2}{x}+\frac{1}{x^2}\right)}.$$

Now, it is easy to see that $g(x) \rightarrow 0$ as $x \rightarrow \infty$. The line $y = 0$ (the x -axis) is a horizontal asymptote. \square

New examples To facilitate students' understanding, many examples have been revised and new examples have been added.

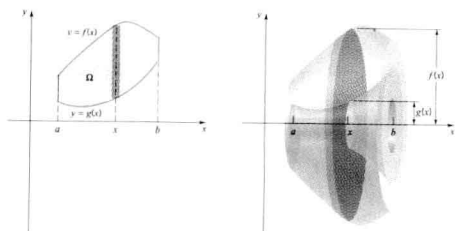


Figure 6.2.12

PROOF The cross section with coordinate x is a washer of outer radius $f(x)$, inner radius $g(x)$, and area

$$A(x) = \pi[f(x)]^2 - \pi[g(x)]^2 = \pi([f(x)]^2 - [g(x)]^2).$$

We can get the volume of the solid by integrating this function from a to b . \square

Suppose now that the boundaries are functions of y rather than x (see Figure 6.2.13). By revolving Ω about the y -axis, we obtain a solid. It is clear from (6.2.4) that in this case

(6.2.6)
$$V = \int_c^d \pi[F(y)]^2 - [G(y)]^2 dy. \quad (\text{washer method about } y\text{-axis})$$

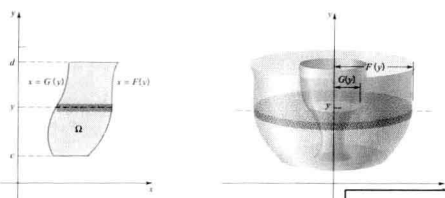


Figure 6.2.13

Improved visualization A completely new and expanded illustration program, including three-dimensional illustrations in full color, provides a better visual representation of concepts.

Real-world applications Students see how the concepts and methods of calculus connect with important problems in science and engineering.

Example 2 A metal plate in the form of a trapezoid is affixed to a vertical dam as in Figure 6.6.5. The dimensions shown are given in meters; the weight density of water in the metric system is approximately 9800 newtons per cubic meter. Find the force on the plate.

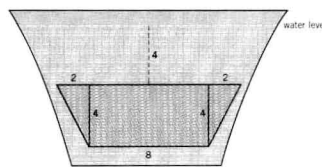


Figure 6.6.5

SOLUTION First we find the width of the plate x meters below the water level. By similar triangles (see Figure 6.6.6)

$$t = \frac{1}{4}(8 - x) \quad \text{so that} \quad w(x) = 8 + 2t = 16 - x.$$

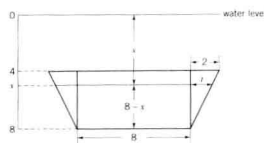



Figure 6.6.6

The force against the plate is

$$\begin{aligned} \int_4^8 9800x(16 - x) dx &= 9800 \int_4^8 (16x - x^2) dx \\ &= 9800 \left[8x^2 - \frac{1}{3}x^3 \right]_4^8 \approx 2,300,000 \text{ newtons. } \square \end{aligned}$$

New exercises The exercise sets have been revised and over 2000 new problems added, resulting in an improved balance between drill problems and more challenging exercises involving either theory or applications.

Technology problems Problems marked by the icon  encourage students to use technology as a tool to enhance understanding and problem-solving skills.

- 26. $f(x) = x + \cos 2x$, $0 < x < \pi$.
- 27. $f(x) = \sin^2 x - \sqrt{3} \sin x$, $0 < x < \pi$.
- 28. $f(x) = \sin^2 x$, $0 < x < 2\pi$.
- 29. $f(x) = \sin x \cos x - 3 \sin x + 2x$, $0 < x < 2\pi$.
- 30. $f(x) = 2 \sin^3 x - 3 \sin x$, $0 < x < \pi$.
- 31. Prove Theorem 4.3.4 by applying Theorem 4.2.3.

32. Prove the validity of the second-derivative test in the case that $f''(c) < 0$.

33. Find the critical numbers and the local extreme values of the polynomial

$$P(x) = x^4 - 8x^3 + 22x^2 - 24x + 4.$$

Then show that the equation $P(x) = 0$ has exactly two real roots, both positive.

34. A function f has derivative f' given by

$$f'(x) = x^3(x-1)^2(x+1)(x-2).$$

At what numbers x , if any, does f have a local maximum? A local minimum?

35. A polynomial function $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ has critical numbers at $x = -1, 1, 2$, and 3 , and corresponding values $p(-1) = 6$, $p(1) = 1$, $p(2) = 3$, and $p(3) = 1$. Sketch a possible graph for p if: (a) n is odd. (b) n is even.

36. The quadratic function $f(x) = Ax^2 + Bx + C$ has a local minimum at $x = 2$ and passes through the points $(-1, 3)$ and $(3, -1)$. Find A, B , and C .

37. Determine a and b such that the function $f(x) = ax(x^2 + b^2)$ has a local minimum at $x = -2$ and $f'(0) = 1$.

38. Let $f(x) = x^p(1-x)^q$, where $p \geq 2$ and $q \geq 2$ are integers.

- (a) Show that the critical numbers of f are $x = 0$, $p/(p+q)$, and 1 .
- (b) Show that if p is even, then f has a local minimum at 0 .
- (c) Show that if q is even, then f has a local minimum at 1 .
- (d) Show that f has a local maximum at $p/(p+q)$ for all p and q .

39. Let

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

In Exercise 67, Section 3.1, we saw that f is differentiable at 0 and that $f'(0) = 0$. Show that f has neither a local maximum nor a local minimum at 0 .

40. Suppose that $C(x)$, $R(x)$, and $P(x)$ are the cost, revenue, and profit functions corresponding to the production and sale of x units. Then, since $P = R - C$, it follows that P is differentiable. Prove that if it is possible

to maximize the profit by producing and selling x_0 items, then $C'(x_0) = R'(x_0)$. That is, the marginal cost equals the marginal revenue when the profit is maximized.

41. Let $y = f(x)$ be differentiable and suppose that the graph of f does not pass through the origin. Then the distance D from the origin to a point $P(x, f(x))$ on the graph is given by

$$D = \sqrt{x^2 + [f(x)]^2}.$$

Show that if D has a local extreme value at c , then the line through $(0, 0)$ and $(c, f(c))$ is perpendicular to the tangent line to the graph of f at c .

42. Prove that a polynomial of degree n has at most $n - 1$ local extreme values.

43. Let $f(x) = x^4 - 2x^2 - 3x + 2$.

(a) Show that f has exactly one critical number c in the interval $(1, 2)$.

(b) Use the bisection method (see Section 2.6) to approximate c to within $\frac{1}{16}$. Does f have a local maximum, a local minimum, or neither a maximum nor a minimum at c ?

44. Let $f(x) = 2 + 20x + 4x^2 - x^4$.

(a) Show that f has exactly one critical number in the interval $(2, 3)$.

(b) Use the bisection method to approximate c to within $\frac{1}{16}$. Does f have a local maximum, a local minimum, or neither a maximum nor a minimum at c ?

45. Let $f(x) = x^4 - 7x^2 + 2x - 3$.

(a) Show that f has exactly one critical number c in the interval $(2, 3)$.

(b) Use the Newton-Raphson method to approximate c ; calculate x_3 and round your answer to four decimal places. Does f have a local maximum, a local minimum, or neither a maximum nor a minimum at c ?

46. Let $f(x) = x \cos x$.

(a) Show that f has exactly one critical number in the interval $(0, \pi/2)$.

(b) Use the Newton-Raphson method to approximate c ; calculate x_3 and round your answer to four decimal places. Does f have a local maximum, a local minimum, or neither a maximum nor a minimum at c ?

47. Let $f(x) = \sin x + (x^2/2) - 2x$.

(a) Show that f has exactly one critical number in the interval $[2, 3]$.

(b) Use the Newton-Raphson method to approximate c ; calculate x_3 and round your answer to four decimal places. Does f have a local maximum, a local minimum, or neither a maximum nor a minimum at c ?

48. In Exercises 48–51, use a graphing utility to graph the function f on the indicated interval. (a) Use the graph to estimate the critical numbers and the local extreme values; and (b) estimate the intervals on which f increases and the intervals on which f decreases. Round off your estimates to three decimal places.

Chapter Highlights End-of-chapter lists stress important terms, ideas, and theorems.

Projects and Explorations Using Technology Special problem sets encourage deeper investigation of the material and can be used for cooperative learning activities.

PROJECTS AND EXPLORATIONS USING TECHNOLOGY ■ 485

The inverse secant, $y = \sec^{-1} x$, is the inverse of $y = \sec x$, $x \in [0, \frac{1}{2}\pi] \cup (\frac{3}{2}\pi, \pi]$.

graph of $y = \sin^{-1} x$ (p. 462) graph of $y = \tan^{-1} x$ (p. 465)
graph of $y = \sec^{-1} x$ (p. 468)

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \quad (a > 0)$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \quad \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \quad (a \neq 0)$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}} \quad \int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{|x|}{a}\right) + C \quad (a > 0)$$

definition of the remaining inverse trigonometric functions (p. 471)

7.9 The Hyperbolic Sine and Cosine

$$\sinh x = \frac{1}{2}(e^x - e^{-x}), \quad \cosh x = \frac{1}{2}(e^x + e^{-x}),$$

$$\frac{d}{dx}(\sinh x) = \cosh x, \quad \frac{d}{dx}(\cosh x) = \sinh x.$$

graphs (pp. 475–476) basic identities (p. 477)

7.10 The Other Hyperbolic Functions

$$\tanh x = \frac{\sinh x}{\cosh x}, \quad \coth x = \frac{\cosh x}{\sinh x},$$

$$\operatorname{sech} x = \frac{1}{\cosh x}, \quad \operatorname{csch} x = \frac{1}{\sinh x}.$$

derivatives (p. 479) hyperbolic inverses (p. 481)

derivatives of hyperbolic inverses (p. 482)

PROJECTS AND EXPLORATIONS USING TECHNOLOGY

To do these exercises you will need a graphics calculator or a computer with graphing capability. The majority of these problems are open-ended so different approaches may be used to solve them. You should be aware that different approaches can result in slight variations in the answers. Round your numerical answers to at least four decimal places. The rounding method that your calculator or computer uses also may cause variations in answers.

7.1 The functions $f(x) = a \ln x$, where a is a constant, have a number of applications, one of which will be considered in a later exercise.

- (a) Find the values of a for which the graph of f is tangent to the line $y = x$.
- (b) For each real number a , how many solutions will there be of $f(x) = x^2$? What is the value of f' at each solution of $f(x) = x^2$?
- (c) How many solutions are there to $f[f(x)] = x^2$? What is the value of f' at each of these solutions?
- (d) Represent f as a logarithm function in another base.

7.2 Let $A(t)$ denote the area of the rectangle of width 2 that has its lower vertices on the x -axis and its upper vertices on the graph of

$$f(x) = e^{1-x} \cos 2x.$$

See the figure.

SUPPLEMENTS

Student Aids

Answers to Odd-Numbered Exercises Answers to all the odd-numbered exercises are included at the back of the text.

Student Solutions Manual, Prepared by Garret J. Etgen, University of Houston This manual contains worked-out solutions to all the odd-numbered exercises and is available through your bookstore.

Instructor Aids

Instructor's Manual, by Garret J. Etgen and Sylvain Laroche This manual contains solutions to all the problems in the text.

Test Bank, by Sylvain Laroche A wide range of problems and their solutions are keyed to the text material and exercise sets.

Computerized Test Bank Available in both IBM and Macintosh formats, the Computerized Test Bank allows instructors to create, customize, and print a test containing any combination of questions from the test bank. Instructors can also edit the questions or add their own.

Technology Manuals

Discovering Calculus with Derive, by Jerry Johnson, University of Nevada-Reno, and Benny Evans, Oklahoma State University

- Derive instructions and tutorials
- Solved problems
- Practice problems
- Laboratory exercises

Discovering Calculus with Mathematica, by Cecilia A. Knoll, Florida Institute of Technology, Michael D. Shaw, Florida Institute of Technology, Jerry Johnson, and Benny Evans

- Mathematica introduction and commands
- Solved problems
- Exercises
- Laboratory projects

Discovering Calculus with Maple, by Kent Harris, Western Illinois University, and Robert J. Lopez, Rose-Hulman Institute of Technology

- Maple commands
- Example problems and step-by-step solutions
- Exercises

Discovering Calculus with Graphing Calculators, by Joan McCarter, Arizona State University

- Introductions to various calculators currently on the market (this manual is calculator nonspecific)
- Projects

- Additional exercises
- Critical thinking questions

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The revision of a text of this magnitude and stature requires a lot of encouragement and a lot of help. I was fortunate to have an ample supply of both from a variety of sources.

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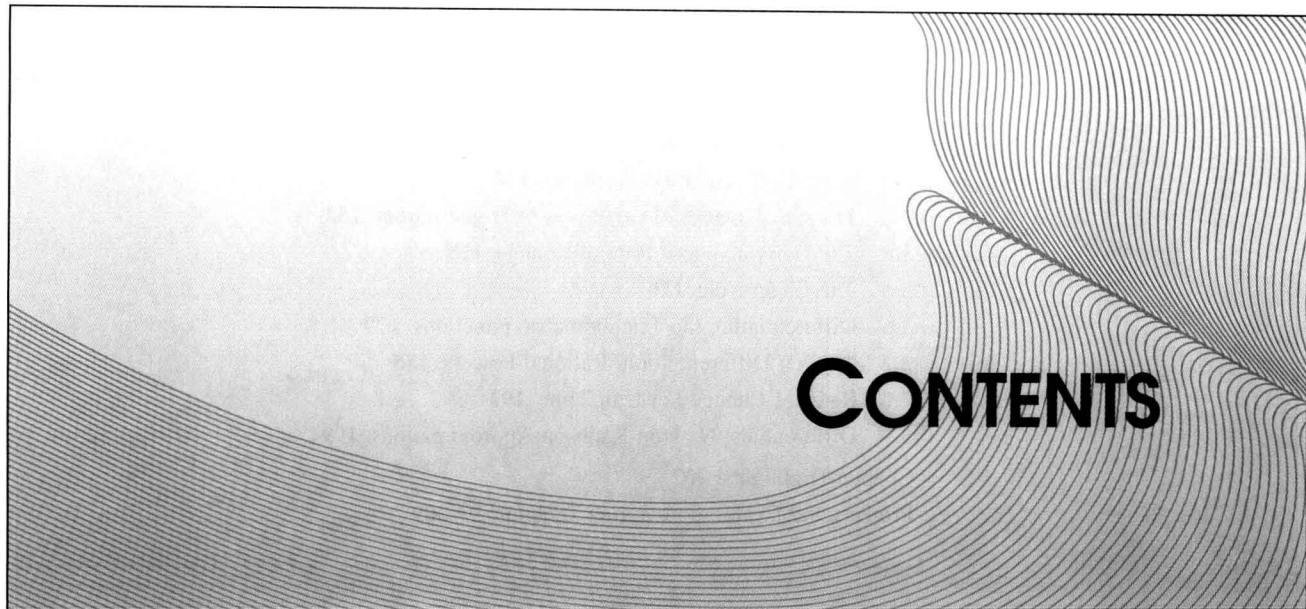
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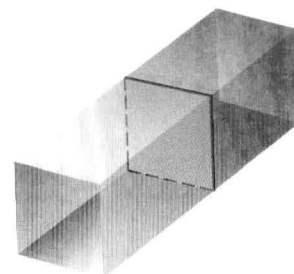


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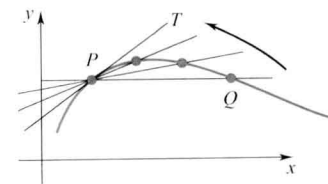


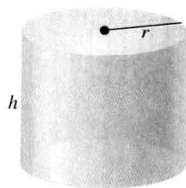
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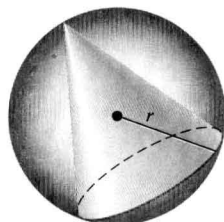




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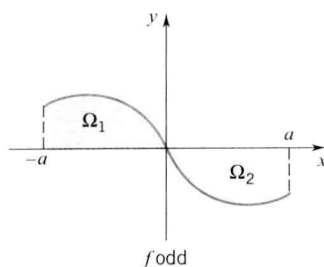
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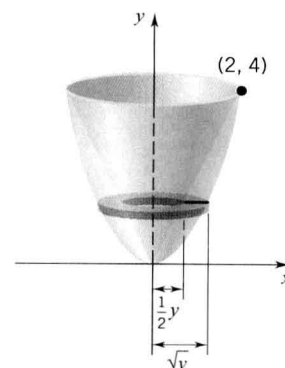
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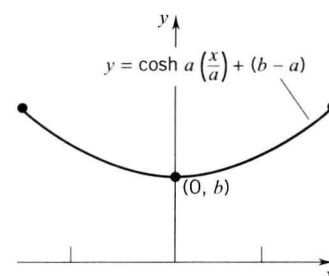
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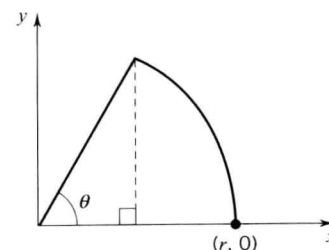
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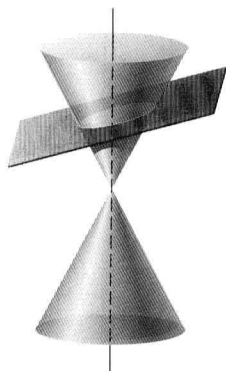
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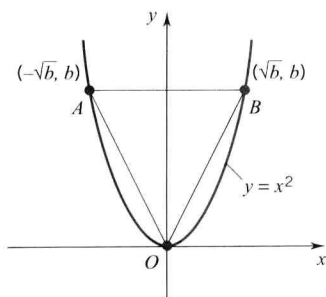
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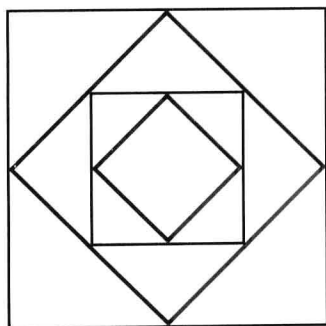
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