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and R Ohayon (Editors)

Computational methods for fluid-structure interaction

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Preface

The first part of this book contains the papers from "Journées Numériques de Besançon 92", the workshop on fluid-structure interaction that was held in Les Moussières, France, on 24-25 September 1992. The conference program covered some computational methods which were developed in order to obtain an approximation of the solution of industrial problems.

Besides these physical situations, another domain where the modelling of phenomena of coupling between a structure and a fluid poses difficulties is biomedical engineering. Some aspects of these difficulties were presented during a second workshop, "Numerical methods for the interaction fluid-structure in Biomechanics" which was held in Métabief, France, on 13-14 October 1992. Several topics presented during this second workshop are summarized in the second part of this book.

It is clearly evident from all these papers that computational methods and numerical simulations are emerging as a major discipline for the solution of such problems. The presentations included descriptions of physical problems, new algorithms, and new methods to take into account the interface. With such a trend towards computer modelling, we think that these meetings have provided a platform for further developments.

Many people contributed to the organization of these two conferences. We thank Pr. C. Oddou for taking on part of the organization of the second meeting. We would like to take the opportunity to thank the invited speakers and authors for the excellent presentations and discussion periods which made these meetings such a success. Finally we would like to offer our thanks to the Conseil Régional de Franche-Comté, the Conseil Général du Doubs, local administrations and Centre National de la Recherche Scientifique for their efforts in supporting the meeting and sponsoring the events.

J.M. CROLET
R. OHAYON

Contents

J PLANCHARD AND B THOMAS

On the added mass, damping and stiffness matrices for an elastic structure placed in a potential cross-flow

1. Introduction

In the study of vibrations of an elastic tube bundle (of mass m and stiffness k) placed in a cross-flow, Chen, from physical considerations [1], poses the tube movement equations a priori in the form:

$$(m+M)\frac{d^2\vec{s}}{dt^2}+C\frac{d\vec{s}}{dt}+(k+K)\vec{s}=0,$$

in which $\vec{s}$ is the displacement vector of the group of tubes; M, C, K are respectively the added mass, stiffness and damping matrices, due to the presence of the fluid (for a general study of fluid-structure interaction and its applications, we refer to Gibert [2], Ohayon [3]). The added mass matrix is that flowing from the potential theory for a perfect still fluid, but the big problem is to calculate the matrices C and K. Paidoussis' and his collaborators' studies have led to a definition of these matrices (see [4] and [5]) which can be obtained experimentally by means of an identification method (Granger [6]). In the case of large displacements, reference [7] had revealed the added damping term although not for K, due to the nonlinearities of the problem.

This paper resumes the definitions of added mass, added damping and added rigidity, taking into account the deformations in the geometry of the fluid region engendered by the tube displacements. This reveals additional terms, depending on time, in the added mass and damping matrices.

2. Dynamical equations

2.1. A single tube between two parallel walls denoted Γ_{lat}, delimiting a channel, is firstly considered to simplify the discussion.

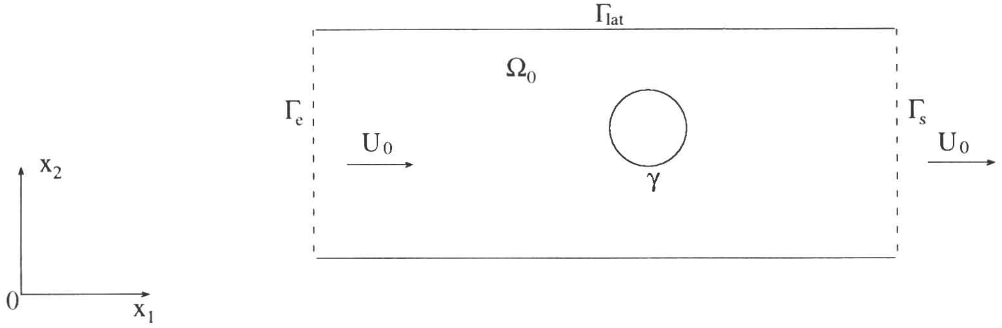

Figure 1

A perfect incompressible fluid of density ρ_0 arrives at the inlet Γ_e of the channel and exits via Γ_s. Let U_0 be the inlet and outlet velocity. Fluid velocity would be U_0 inside the channel should there be no tubes.

The presence of the tube modifies the flow; we therefore have:

$$u_0(x) = U_0 + \overline{u}_0(x) \ ,$$

in which $\overline{u}_0(x)$ is the change in the velocity field due to the presence of γ. Let $\overline{\phi}_0(x)$ be the velocity potential associated with $\overline{u}_0(x)$.

We therefore have:

$$\Delta\overline{\phi}_0(x) = 0 \text{ in } \Omega_0 \text{ (fluid domain)} \tag{2.1}$$

with the boundary conditions:

$$\left\{ \begin{array}{l} \dfrac{\partial\overline{\phi}_0}{\partial n} = 0 \text{ on } \Gamma_{\text{lat}} \ \text{ (rigid wall) .} \\ \left.\dfrac{\partial\overline{\phi}_0}{\partial n}\right|_{\Gamma_e} = \left.\dfrac{\partial\overline{\phi}_0}{\partial n}\right|_{\Gamma_s} = 0 \text{ (i.e } \overline{u}_0 = 0 \text{ on } \Gamma_e \text{ and } \Gamma_s\text{) .} \\ \dfrac{\partial\overline{\phi}_0}{\partial n}(x) = -\vec{U}_0.\vec{n} \text{ on } \gamma \ . \end{array} \right. \tag{2.2}$$

The resulting velocity field $u_0(x)$ derives from potential $\phi_0(x) = \overline{\phi}_0(x) + U_0x_1$ (because U_0 is parallel to x_1).

2.2. Relation between pressure p and potential ϕ

It is useful to recall how this relation is determined. Let $u(x,t)$ be a velocity field for a perfect incompressible fluid. The movement equation can be written:

$$\begin{cases} \rho_0 \left[\dfrac{\partial u}{\partial t} + \dfrac{1}{2} \nabla |u|^2 + \text{ curl } u \times u \right] + \nabla p = 0 \ , \\ \\ \text{div } u = 0 \ . \end{cases}$$

If the flow is irrotational, then curl $u = 0$ and setting $u = \nabla\phi$, we have:

$$\nabla \left[\rho_0 \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} |\nabla\phi|^2 \right) + p \right] = 0 \ ,$$

$$p = -\rho_0 \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} |\nabla\phi|^2 \right) + c(t) \ . \tag{2.3}$$

in which $c(t)$ is an integration constant independent of x. Since pressure differences alone are considered below, we pose $c(t) = 0$.

2.3. Equilibrium equations of the system

Under the effect of the assumed steady flow, tube γ moves by $\vec{s}_0$ given by:

$$\vec{s}_0 = \frac{1}{k} \int_\gamma p_0(x) \vec{n} d\gamma \ , \tag{2.4}$$

in which p_0 is the pressure and k is tube rigidity. $p_0(x)$, from (2.3), satisfies:

$$p_0(x) = -\frac{\rho_0}{2} |\nabla \phi_0(x)|^2 \ . \tag{2.5}$$

2.4. Small vibrations about the state equilibrium

Short movements $\vec{s}(t)$ of the tube are considered around its position of equilibrium defined by vector $\vec{s}_0$ given by (2.4) and (2.5).

These small movements produce pressure $p(x,t)$ and velocity potential $\phi(x,t)$ fluctuations around $p_0(x)$ and $\phi_0(x)$. We therefore have:

$$\begin{aligned} p_0(x) + p(x,t) &= -\rho_0 \left[\frac{\partial \phi}{\partial t} + \frac{1}{2} |\nabla(\phi_0 + \phi)|^2 \right] \\ &= -\rho_0 \left[\frac{\partial \phi}{\partial t} + \frac{1}{2} \left(|\nabla\phi_0|^2 + 2\nabla\phi_0 . \nabla\phi + |\nabla\phi|^2 \right) \right] \ . \end{aligned}$$

Neglecting the term $|\nabla\phi|^2$ and in the light of (2.5), we have:

$$p(x,t) = -\rho_0 \left[\frac{\partial\phi}{\partial t} + \nabla\phi_0.\nabla\phi\right] \ . \tag{2.6}$$

For small disturbances, the coupled system therefore obeys:

$$\left\{\begin{array}{l} \Delta\phi(x,t) = 0 \text{ in } \Omega_0 \ , \\ \dfrac{\partial\phi}{\partial n} = \vec{n}.\dfrac{d\vec{s}}{dt} \text{ over } \gamma \ , \\ \dfrac{\partial\phi}{\partial n} = 0 \text{ over } \Gamma = \partial\Omega_0\backslash\gamma \ , \\ \left(m\dfrac{d^2}{dt^2} + k\right)\vec{s} = -\rho_0 \displaystyle\int_\gamma \left[\frac{\partial\phi}{\partial t} + \nabla\phi_0.\nabla\phi\right]\vec{n}d\gamma \ . \end{array}\right. \tag{2.7}$$

in which m is the mass of the tube.

2.5. Added mass

Since ϕ linearly depends on $\dfrac{d\vec{s}}{dt}$, we set:

$$\phi(x,t) = \sum_{j=1}^{2} \chi_j(x)\frac{ds_j}{dt} \equiv \chi(x).\frac{d\vec{s}}{dt} \ , \tag{2.8}$$

in which s_j is the j^{th} component of $\vec{s}$ and χ_j satisfies:

$$\left\{\begin{array}{rcl} \Delta\chi_j & = & 0 \text{ in } \Omega_0 \ , \\ \dfrac{\partial\chi_j}{\partial n} & = & 0 \text{ on } \Gamma \ , \\ \dfrac{\partial\chi_j}{\partial n} & = & \cos(\vec{n},x_j) \text{ on } \gamma \ , \ \displaystyle\int_{\Omega_0}\chi_j = 0 \ . \end{array}\right.$$

Matrix H is built with vectors $\displaystyle\int_\gamma \chi_j\vec{n}d\gamma$, $\rho_0 H$ is the added mass matrix (it is symmetrical and defined positive [8]).

2.6. Added damping matrix

The integral $\int_\gamma (u_0.\nabla\phi)\vec{n}\,d\gamma$ $(u_0 = \nabla\phi_0)$ must be expressed in explicit manner.

We have:

$$\nabla\phi = \sum_j \frac{ds_j}{dt}\nabla\chi_j(x) \ .$$

Let C be the matrix built with the two vectors $\int_\gamma (u_0.\nabla\chi_j)\vec{n}\,d\gamma$. Then the 4th equation of (2.7) is written as

$$(m + \rho_0 H)\frac{d^2\vec{s}}{dt} + \rho_0 C\frac{d\vec{s}}{dt} + k\vec{s} = 0 \ . \tag{2.9}$$

$\rho_0 C$ is the added damping matrix due to the presence of flow.

Remarks:

a) C depends linearly on u_0, therefore on inflow U_0. We can thus write:

$$C = U_0 C_0 \ .$$

b) The properties of matrix C should be examined in a little more detail.

c) If the term $|\nabla\phi|^2$ is not neglected, then the term $\frac{1}{2}\int_\gamma |\nabla\phi|^2\vec{n}\,d\gamma$ can be put in the form:

$$\frac{1}{2}\int_\gamma |\nabla\phi|^2\vec{n}\,d\gamma \equiv \left(B\left(\frac{ds}{dt}\right)\right)\frac{d\vec{s}}{dt} \ ,$$

in which $B\left(\frac{ds}{dt}\right)$ is a matrix, linear function of the tube velocity. In this case, there is a damping term which depends on movement.

Case of several tubes

If N parralel tubes are present in the channel, ϕ should verify:

$$\begin{cases} \Delta\phi(x,t) = 0 \text{ in } \Omega_0 \ , \\ \dfrac{\partial\phi}{\partial n} = 0 \text{ on } \Gamma \ , \\ \dfrac{\partial\phi}{\partial n} = \dfrac{d\vec{s}_\ell}{dt}.\vec{n} \text{ on } \gamma_\ell \ , \ \ell = 1 \text{ to } N \ , \\ m\dfrac{d^2\vec{s}_\ell}{dt^2} + k\vec{s}_\ell = -\rho_0\displaystyle\int_{\gamma_\ell}\left[\frac{\partial\phi}{\partial t} + u_0.\nabla\phi\right]\vec{n}d\gamma_\ell \ , \end{cases} \tag{2.10}$$

$\vec{s}_\ell$ is the displacement of the tube γ_ℓ.