PRINCIPLES OF CONTINUA

with Applications

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This book is dedicated to the men of science
whose works form the very basis of that which follows,
and without which this meager effort would truly be impossible;
and to the memory of my father (1895–1966).

Preface

In general, the purpose of this presentation is to demonstrate to the advanced undergraduate engineering student that a major portion of his technical knowledge came from a few basic ideas and that the concepts of his specialized concentration or area of interest were extracted therefrom and expressed in terms of mathematical models for particular applications.

The specific objective is threefold:

1. To present a compendium which blends together the fundamental concepts of the engineering disciplines. The concept of the "continuum" is the most general model for such a treatment; therefore it is the basis upon which the formulations are developed.
2. To present the mathematical models in such a manner that they will be understandable to the advanced undergraduate engineering student.
3. To illustrate the extraction of specific ideas from the general formulation, by the presentation of some "standard" problems in various areas of engineering science.

Although no pretense is made that this book constitutes a treatise on mechanics of continua per se, an attempt has been made to examine the basic principles and some of their applications.

There are three parts, including a current bibliography of the subject areas treated. The several references will be of considerable value for a detailed and rigorous study of any or all of the results presented.

In Part One, the fundamental concepts and the mathematical techniques employed in the formation are reviewed. It is assumed that the reader has been introduced to the use of symbolic notations, that is, vectors and their basic operations. Gibbs's vector notation is employed to replace and extend conventional scalar notation, and indicial notation (summation convention) is used to extend Gibbs's vector notation.

Part Two contains a detailed treatment of the fundamental laws governing the behavior of physical phenomena—the conservation laws. These laws are constructed from a general equation. It is shown that the equations expressing the conservation of mass, balance of momenta, and conservation of energy are special cases of a "general conservation equation." Additional laws are developed for treatment of specific phenomena. The general procedure for combining these "special" laws with the fundamental laws is discussed.

Part Three is devoted to the analysis of some standard problems of elementary elasticity, plasticity, and viscoelasticity, fluid dynamics, thermodynamics, and electromagnetism from the standpoint of the governing equations. Problems involving interaction of these fields are treated to emphasize the scope of application of the governing equations. This part should also indicate the interesting complexity of the analysis of the interaction of various continua. The reader is continually reminded of the relationship between the mathematical models employed in each of the solutions and those of the fundamental system. Most importantly, neglected or revised terms in the formulations are pointed out to give an appreciation for the approach to the solution of physical problems.

Unlike other treatises on continuum mechanics, this book attempts to integrate some of the elementary principles of electromagnetic theory in the text. This is an experiment. The question, of course, is how well this experiment has succeeded.

For more detailed and advanced treatments of the general subject, the reader is referred to the excellent works of Prager (1961), Eringen (1962), and Sedov (1965). Related discussions of the fundamental equations are given by Boley and Weiner (1960), Long (1961), Aris (1962), Fitts (1962), Borg (1963), Cambel (1963), Nowacki (1963), and Bird, Stewart, and Lightfoot (1960), and by other sources which are indicated within the text. All of these publications are listed in the Bibliography under the appropriate headings.

Like every author, I am deeply indebted for help that cannot be adequately acknowledged. During the preparation of this work, several of my colleagues, former and present, assisted me by reading various parts of the preliminary forms of the manuscript. For their valuable suggestions I extend my sincere thanks to Professors R. Erdlac, W. M. Rohrer, Jr., and C. C. Yates, Drs. R. S. Dougall, G. E. Geiger, and G. Fauconneau, Messrs. L. A. Ricardo and D. J. Walukas, and Miss Evelyn Simon of the University of Pittsburgh; Dr. M. L. Walker, Jr., of Howard University; and Professor H. Quijano of the University of Puerto Rico (Mayaguez). There was further assistance from Mr. D. S. Kim, who checked a number of the equations.

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Finally, I express my thanks to Mr. A. R. Beckett, Editor of John Wiley & Sons, for his encouragement throughout this effort, and to the Editorial, Art, and Production departments of the company for contributing their special skills to this book.

L. A. Scipio

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Notation

ROMAN

A = area, vector quantity
a = speed of sound propagation, radius, coefficient
B = magnetic induction, vector quantity, thrust flow function
b = coefficient
c_p = heat capacity at constant pressure per unit mass
c_v = heat capacity at constant volume per unit mass
$\mathscr{C}$ = creep function
c = velocity of wave propagation, constant
D = electric displacement, vector quantity
E = internal energy per unit mass, electric field intensity, Young's modulus
$\mathscr{E}$ = internal energy, electromotive force
e = rate of deformation
F = force, function
$\mathscr{F}$ = function, force
G = shear modulus, gravitational constant
$\mathscr{G}$ = gravitational field intensity, function
g = gravitational acceleration
$H = E + p/\rho$ = enthalpy, magnetic flux, magnetic field
h = heat energy per unit mass to some internal source, height
I = moment of inertia
$\mathscr{I}$ = invariant of stress
i = current
i, j, k = unit vectors
J = mechanical equivalent of heat, current density
K = yield shear stress, specific inductive capacity, coefficient

$\mathscr{K}$ = kinetic energy, constant
k = c_p/c_v = ratio of specific heat
k = bulk modulus
L = length, losses, self-inductance of circuit
l, m, n = direction cosines
M = Mach number, mass, moment, magnetic dipole moment, magnetization
$\mathscr{M}$ = momentum, tensor quantity
m = mass of a particle, mass of system
N = number of moles, magnetic torque
$\mathscr{N}$ = magnetic flux
n = unit normal vector
P = power, polarization, tensor quantity
p = pressure intensity, hydrostatic pressure
Q = heat energy, discharge
q = heat flux
R = circuit resistance, gas constant, radius, radius of curvature
$\mathscr{R}$ = relaxation function
r = radius, radius vector
S = surface area, cross-sectional area, entropy, energy source
T = tensor quantity, absolute temperature
t = time, unit tangent vector
U = velocity
u = velocity component
V = volume
v = velocity component
W = energy, work
w = energy, surface charge density, velocity component
x = displacement
x, y, z = rectangular coordinates
y = displacement
z = displacement

Greek

α = direction cosine, coefficient of linear thermal expansion, coefficient
β = coefficient
γ = $(3\mu_E + 2\lambda_E)\alpha$
Δ = volume, displacement, differential quantity
δ = displacement
ϵ = deformation, electric constant

η = open tensor, coefficient of viscosity
Θ = temperature
θ = angle of rotation, angle, curvature
κ = thermal conductivity
$\mathrm{K} = \dfrac{\kappa}{\rho c_p}$ = thermal diffusivity
Λ = material constant
λ = dilatational viscosity coefficient, Lamé constant, scalar factor
μ = coefficient of viscosity, electrochemical potential, Lamé constant, permeability, magnetic moment of dipole
ν = Poisson's ratio, magnetic constant
ξ = displacement
ρ = mass density, radius, volume charge density
$\sum$ = summation, material constant
σ = stress, electrical conductivity
τ = relaxation time
Φ = Rayleigh dissipation function, potential
ϕ = vector function, tensor function
ψ = scalar point function, tensor function, impulse function
Ω = angular velocity, function
ω = rotation, angular velocity

German

$\mathfrak{F}$ = specific quantity per unit mass
$\mathfrak{I}$ = internal source charge per unit volume
$\mathfrak{S}$ = extensive property
$\mathfrak{Z}$ = flux

Russian

А = function
Б = function
В = function
Г = function
Д = function
Л = function

Mathematical Operators and Symbols

$\dfrac{D}{Dt}$ = substantial derivative
$\exp x = e^x$ = exponential function of x

$\ln x$ = logarithm of x to the base e
∇ = del or nabla operator
e_{ijk} = permutation symbol
δ_{ij} = Kronecker delta
$\oint_C$ = integral around boundary C
$\oint_S$ = integral over surface S
[0] = order of
$(\dot{\ }) = \dfrac{\partial}{\partial t}$
P, Q = linear operators
$\overleftrightarrow{(\text{bf})}$ = tensor quantity
(bf) = vector quantity
$\begin{pmatrix}\text{vector} & \text{vector}\\ \text{quantity} \cdot & \text{quantity}\end{pmatrix} \equiv$ dot product
$\begin{pmatrix}\text{vector} & \text{vector}\\ \text{quantity} \wedge & \text{quantity}\end{pmatrix} \equiv$ cross product

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