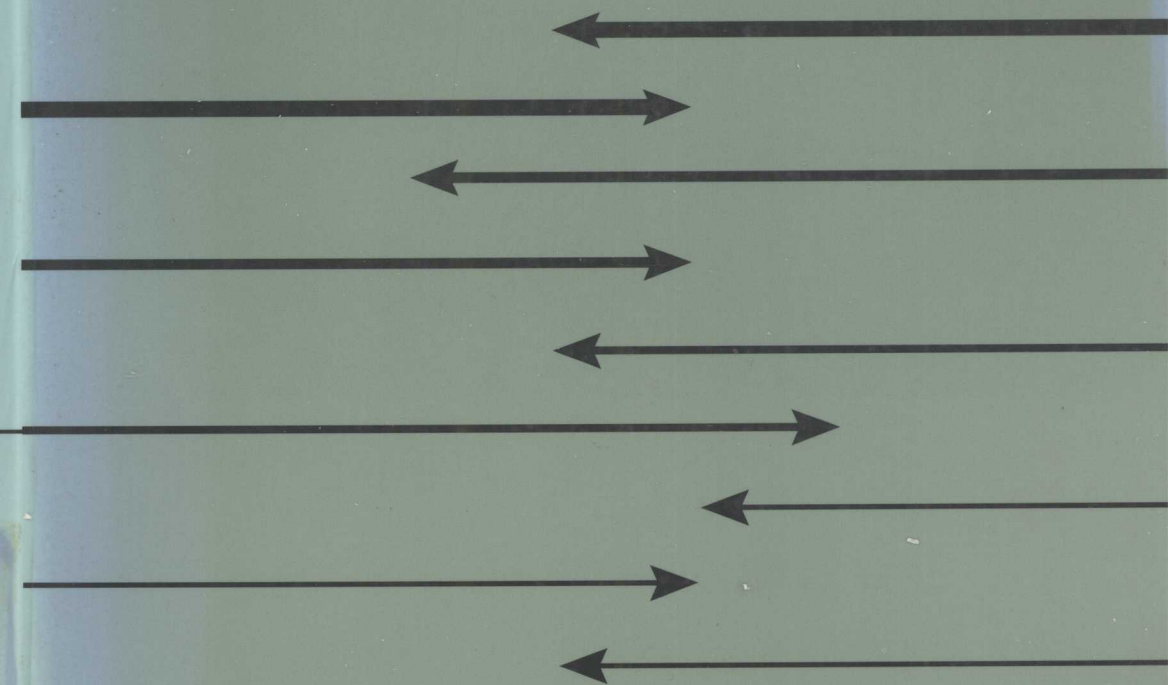


Trade Policy in Developing Countries

EDWARD F. BUFFIE



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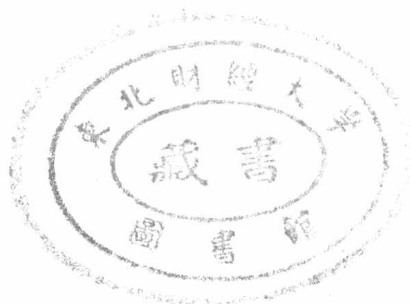


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Trade Policy in Developing Countries

Trade Policy in Developing Countries is an analysis aimed at academics, graduate students, and professional, policy-oriented economists. It is the first work in the field to examine trade policy in an integrated theoretical framework based on optimizing dynamic models that pay careful attention to the structural features of developing country economies. Following a thorough critique of the debate on inward- vs. outward-oriented trade regimes, Buffie explores the main issues of concern to less developed countries in the areas of optimal commercial policy, trade liberalization, and direct foreign investment. In addition to many new and important results, the book contains systematic reviews of the empirical evidence and three expository chapters that show the reader how to use the technical machinery of economic theory to construct and manipulate multisector dynamic general equilibrium models.

Edward F. Buffie is Professor of Economics at Indiana University. He previously taught at the University of Pennsylvania and Vanderbilt University. Professor Buffie has written extensively on trade and macroeconomic policies in less developed countries, publishing his research in diverse scholarly journals such as *International Economic Review*, *Journal of Economic Dynamics and Control*, *Economica*, *Journal of International Economics*, *Journal of Monetary Economics*, *Oxford Economic Papers*, *Journal of Development Economics*, *European Economic Review*, and *Journal of Public Economics*. He was an associate editor of the *Journal of Development Economics* from 1990 to 1995 and has served as a consultant to The World Bank, the Inter-American Development Bank, and the United States Agency for International Development.



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Introduction

Indeed, the association between higher growth rates and an export-promotion strategy had already been established before the present project began, although additional evidence has since confirmed the results. . . . It seemed neither necessary nor desirable to cover that ground again. (Krueger, 1983, p. 6)

That a liberal is preferable to a restrictive trade regime is now generally accepted, and a substantial body of empirical research carried out over the last 20 years supports this conclusion. (Michael, Papageorgiou, and Choksi, 1991, p. 1)

The question of the wisdom of an outward-oriented (export-promoting) strategy may be considered to have been settled. (Bhagwati, 1987, p. 257)

[O]ne must resist succumbing to the oversimplifications and generalizations that have too frequently plagued the debates in the sphere of trade strategy . . . what seems to emerge from this survey is a need for a fresh review of fairly major proportions . . . of experience and knowledge of the interaction between trade and other policies and their joint effects upon industrialization and development. Such a review would be particularly valuable if it avoided prejudgements about the relative efficacy of specific trade and other policies in general; and instead explored the specific circumstances in which particular policies, instruments, and policy mixes were less or more effective. (Helleiner, 1990, pp. 880, 894)

The mainstream view is that . . . policy should be directed toward eliminating barriers to trade. There is also an increasing body of literature supporting the opposite point of view. . . . The best summary so far is that the debate is inconclusive: an a priori case for either an open or closed trade policy can never be fully proved . . . this Scotch verdict also applies to the empirical evidence on the relationship between openness and growth. (Shapiro and Taylor, 1990, p. 870)

Neat certainties have a very limited truth. (Robertson Davies, *The Merry Heart*, p. 281)

The last ten years have seen profound changes in the conduct of trade policy in the Third World. After following highly protectionist policies

from the end of WWII until 1990, many less developed countries (LDCs) have eliminated quotas and sharply reduced tariffs. Effective rates of protection above 200%, which were common in earlier decades, are now comparatively rare. But while protection is less extreme, it is by no means dead. Trade policy still retains a strong import-substituting bias in much of Sub-Saharan Africa and South Asia. Even in regions where liberalization has progressed much further, one does not have to search very long to find countries that subject imported consumer goods to tariffs of 30% plus a variety of hidden trade taxes.¹

None of this has escaped the attention of neoclassical economists who believe in the sanctity of free trade. The prevalence of import-substituting industrialization has provoked no less than five major studies aimed at convincing policy makers their countries would fare better under a more outward, export-oriented trade strategy.² These studies have demonstrated successfully that there is no sound economic justification for tariffs and quotas that allow domestic prices to be two or three times higher than prices in world markets. But despite a general consensus that some reduction in trade barriers is desirable, the debate on trade policy remains contentious, at times acrimonious. Critics of the World Bank charge that its ill-conceived programs of trade liberalization are inflicting de-industrialization in Sub-Saharan Africa,³ structuralist economists continue to argue that moderate protection may be beneficial, and the most recent large-scale study of LDC trade policy casts doubt on the objectivity and robustness of the conclusions trumpeted in earlier studies: "... to suggest that there is a universal trade policy prescription that will generate improved economic performance for all is to ignore too much recent experience" (Helleiner, 1994, p. 32).⁴ Clearly, not

¹ In the Dominican Republic, the average tariff on consumer imports was 28% in the mid-nineties. Foreign exchange commissions, consular legalization fees, charges for "services rendered by the port authorities," and selective consumption taxes (that fall only on imports), however, push the effective tariff up to 97%. Various hidden trade taxes are also significant in Brazil, Colombia, and El Salvador. See *Trade Policy Review* (World Trade Organization) for the details.

² See Little, Scitovsky, and Scott (1970), Balassa and Associates (1971), Krueger (1978) and Bhagwati (1978), Krueger et al. (1982), and Michaely, Papageorgiou, and Choksi (1991).

³ See, for example, Stein (1992), Cornia van der Hoeven and Mkandawire (1992), and Stewart (1994).

⁴ The World Institute for Development Economics Research (WIDER) commissioned a new study to provide a "balanced and independent review" of LDC experiences with trade policy and industrialization in the seventies and eighties (Helleiner, 1994).

everyone concedes that proponents of an export-oriented strategy have a monopoly on the truth, or even a commanding market share. The debate is not closed.

This book analyzes the main issues of concern to LDCs in the areas of commercial policy, trade liberalization, and direct foreign investment in an integrated theoretical framework. Is there really a need for this given all the work that has been done on the pure theory of international trade and all that has been written about trade policy in developing countries? I believe there is. Students of development economics will find much that is useful and relevant in existing trade theory. They may also, however, doubt the value of time spent studying the factor-price equalization theorem or grow weary after a while of laboring through models more appropriate for Canada than Bangladesh or Costa Rica. The trade and development literature, on the other hand, is conspicuously lacking in formal theory and rigorous analysis. This criticism applies with special force to the heart of the literature: the major studies of LDC trade policy are long on advocacy and assertion but distressingly short on clean analytical and empirical results.

In what follows the reader will encounter many optimizing dynamic general equilibrium models. The emphasis on dynamics is unusual in a book devoted to trade policy. It is essential, however, to a theory-based analysis of trade policy in developing countries. Policy makers in LDCs wish to know *inter alia* how large the short-run costs of liberalization are relative to its long-run benefits, what underemployment and underinvestment imply for the optimal export subsidy and the optimal tariffs on consumer goods, intermediate inputs and capital goods, the extent to which lack of credibility may undermine reforms that would otherwise work well, whether foreign investment displaces or is complementary to domestic investment, and how different trade taxes affect real wages and the distribution of income in the short vs. long run. The analysis of these and other issues requires an intertemporal framework that pays careful attention to sectoral interactions and the structural features of production. Consequently, most of the analysis in the book is based on dynamic general equilibrium models of varying complexity. I am trying to sell not only a particular set of conclusions but also a general approach to the analysis of policy issues.

Calibration techniques are an important part of the approach. The analysis of optimizing dynamic models is inherently demanding because current economic actions both depend on and influence the economy's future path. In two- or three-sector dynamic models this can produce complicated interactions and dense, tortuous algebra. It often proves useful, therefore, to supplement theory with numerical solutions that

cover a wide range of potentially relevant cases. Sometimes the numerical solutions will argue strongly in favor of a particular outcome. At the very least, they can help narrow the zone of disagreement; that is, if the model is judged acceptable, then all interested parties can agree that policy x is desirable when parameter z lies between 0 and .75.

1.1 Overview of the Book

The main body of the book consists of five chapters on different aspects of trade policy and three expository chapters on duality theory and solution techniques in continuous-time dynamic models. The expository chapters do not pretend to be comprehensive. My objective is rather to show the reader how to use the technical machinery of economic theory to build and solve interesting general equilibrium models. In my view, we do a bad job of this in economics. For some reason, many of the tools widely used in the application of theory have not yet found their way into graduate textbooks. The texts on dynamics are especially deficient in this regard. The solution for the perfect foresight path in dynamic general equilibrium models typically requires solutions from a pseudostatic variant of the model that relates the paths of endogenous variables to the paths of the variables that form the dynamic system. Strangely, most texts do not discuss this or provide a clear statement of the condition for saddlepoint stability in higher dimensional models, and none explains how to solve for the transition path when a policy or shock is temporary instead of permanent. Nor is it easy to learn the relevant techniques from the journals. The journals are full of cryptic statements of the type "Using standard solution techniques, we obtain . . ." or "It is straightforward to show that . . ." But for the uninitiated (e.g., graduate students), nothing is "straightforward" and solution techniques are not "standard."

The expository chapters are introduced when needed. Since duality theory is used throughout the text, it is covered in Chapter 2. The material on dynamics appears later, in Chapters 4 and 7. The rationale for this organizational structure is that it is probably most efficient to study the solution techniques just before the chapters which utilize the techniques intensively. Nothing, however, prevents those who have a robust appetite for mathematics from reading Chapters 4 and 7 immediately after Chapter 2 – the three "Tools and Tricks of the Trade" chapters are a self-contained unit.

I begin the analysis of trade policy proper in Chapter 3 with a critique of the debate on the merits of inward- vs. outward-oriented trade regimes. The central message of the critique is that there is a consider-

able imbalance in the literature between what is asserted and what is actually known. If defenders of protection are too quick to dismiss the policy prescriptions of neoclassical economics, it is equally true that proponents of export promotion have repeatedly made claims far stronger than either theory or empirical evidence can support. It can be said with some assurance that extreme protectionist policies are economically harmful. But the case for free trade is not airtight, and there is no general theoretical presumption that the market failures common in LDCs favor an export-oriented rather than an import-substituting trade strategy. At present, far more is known about the consequences of bad trade policy than about the makeup of optimal trade policy.

This latter conclusion serves as the motivation for Chapter 5. Guidelines for optimal commercial policy should be based on the results obtained from optimizing dynamic models that occupy the middle ground between toy models that leave out too much and CGE black boxes that include too much and rely on ad hoc behavioral assumptions. I try to make some progress in this direction by developing a model that incorporates export, import-competing, and nontradables production, capital accumulation in all three sectors, imports of intermediate inputs, consumer goods, and capital goods, and a realistic government budget constraint. The allocation of resources at the initial free trade equilibrium is distorted by underemployment and underinvestment, and the task of the social planner is to choose the three import tariffs, the export subsidy, and the value added tax to maximize social welfare subject to the constraint that revenues cover the cost of export subsidies and other government expenditures.

The results provide something for everyone. Advocates of *sensible* import-substituting policies will be happy with the conclusion that an escalated structure of protection (i.e., higher tariffs on consumer goods than on intermediate inputs and capital goods) is more effective in stimulating capital accumulation and reducing underemployment than export promotion. But moderately high levels of protection are optimal only when combined with substantial export subsidies in the primary sector. If primary export subsidies are not feasible because of political or administrative constraints, the optimal effective rate of protection is quite low (10–30%). Moreover, in many cases, the direction of trade changes and the former import-substituting sector becomes a net exporter of manufactured goods. Overall, therefore, the results support a mixed ISEP strategy – either import substitution plus export promotion or import substitution then export promotion.

The finding that most countries would be better off had they opted for a moderate ISEP strategy does not necessarily justify calls for aggres-

sive trade liberalization. When contemplating cuts in protection, the government has to be sure it can handle any adverse effects on employment and the balance of payments, and that the long-run gains from liberalization suffice to compensate for losses suffered during the adjustment process. Chapters 6 and 8 deal with these issues. Chapter 6 focuses mainly on the problem of transitory unemployment. This is not a subject that lends itself to sharply defined conclusions because much depends on the nature of technology, private sector expectations, the speed of adjustment in the labor market, and the other policies that comprise the reform program. Several themes emerge, however, from the analysis: (i) different types of liberalization programs would be expected to have different *qualitative* effects on labor demand and unemployment; (ii) weak credibility exacerbates the problem of transitory unemployment; and (iii) in some cases the losses from transitory unemployment are relatively large and the optimal tariff cut stops well short of the *ex ante* optimal tariff.

The credibility problem shows up in Chapter 6 because fears that liberalization may not last affect labor mobility and the duration of transitory unemployment. Chapter 8 investigates two other aspects of the problem. The first part of the chapter develops Guillermo Calvo's crucial insight that expectations of a policy reversal distort intertemporal choice by creating an incentive to consume more in the near term while imports are temporarily cheap. Employing a mix of theory and numerical methods, I analyze how large the losses from the intertemporal distortion are relative to the gains from trade and the implications of this for the optimal tariff cut. The second part of the chapter seeks explanations for the key stylized fact that many liberalization programs have been abandoned in the face of unexpectedly large balance of payments deficits. One explanation for the policy reversals is simply that the government lacks the foreign exchange reserves to support the liberalization attempt. A second, equally straightforward explanation holds that persistent payments deficits stem from the failure to properly coordinate fiscal policy and trade reform. These explanations may be correct in many cases, but they are not the only possible explanations. The analysis in Chapter 8 shows that the causal links connecting credibility, payments deficits, and fiscal adjustment are subtle and bidirectional when the government's reputation has been damaged by past failures. It may not be easy therefore to judge whether private sector pessimism or incompatible policies are the source of failure: reform programs that are fundamentally sound may fail merely because they are expected to fail.

Chapter 9 takes up the question of what LDCs have to gain from direct foreign investment. I first analyze how foreign investment affects

welfare and the dynamics of domestic capital accumulation and underemployment in the simplest case where foreign firms use the same technology as domestic firms and are not subject to any special regulations. This is followed by analysis of more complicated cases involving technology transfer, joint ventures and minimum export requirements. It turns out that a lot depends on the details of the overall package, including the sector in which foreign firms invest. The prospects for a welfare gain are best when foreign investment generates favorable technological spillovers and the government imposes minimum export and local equity requirements. Even then, however, it is risky to allow foreign firms to compete with domestic firms in the home manufactures market. This type of foreign investment often reduces the *aggregate* capital stock (i.e., it crowds out domestic capital more than one-for-one) and worsens underemployment in the long run. Thus, the results do not support the current trend toward laissez faire policies. In a second-best environment characterized by underemployment and underinvestment, it does not make sense to drop performance requirements and let foreign firms invest in any sector they like.

Chapters 2–9 contain many models and many results. They also leave a lot of territory unexplored. In some chapters, the analysis ignores important policies; in others, it is restricted to models that are appropriate for region A of the Third World but not regions B, C, and D. The concluding chapter elaborates on this and the closely related subject of promising directions for future research.

Tools and Tricks of the Trade, Part I: Duality Theory

Many policy issues in development economics cannot be addressed in a rigorous manner without building models that allow for considerable structural detail. Depending on the issue, it may be important to distinguish between agriculture and industry, between importable, exportable, and nontraded goods, between employment in high-wage vs. low-wage sectors, between domestically produced and imported capital goods, or between private and parastatal production. Unfortunately, there is a basic problem with the generally laudable strategy of including all relevant structural detail in a model: the more complicated the economic interactions, the messier the analysis and the more difficult it is to derive clean, insightful results. This is why duality theory should be part of the policy-oriented development economist's tool kit. Duality theory provides the model builder with functions based on the solutions to various static optimization problems. The functions summarize in a compact manner how demand and supply depend on preferences, technology, and optimizing behavior on the part of competitive, price-taking firms and consumers. This enables multisector general equilibrium models to be specified and manipulated with comparative ease as it is not necessary to explicitly solve the optimization problems that govern private agents' behavior. When duality theory is used to characterize demand and supply responses, general comparative statics results can be derived directly by exploiting the properties of the relevant maximum or minimum value functions.

2.1 Duality Theory and Supply

I start by discussing the duality functions that describe production and supply. The exposition will be heuristic, with an emphasis on how to apply duality theory for the purpose of constructing and manipulating models. Readers who desire a more in-depth treatment of the subject should consult Blackorby, Primont, and Russel (1978); Diewert (1978); McFadden (1978); Chambers (1988); and Cornes (1992).

2.1.1 The Cost Function

Let $\mathbf{x} = (x_1, \dots, x_n)$ be a vector of inputs, $\mathbf{w} = (w_1, \dots, w_n)$ an associated vector of factor prices, Q output, and $f(\mathbf{x})$ an increasing, continuous, quasi-concave production function. The cost function is the solution to the problem of choosing inputs so as to minimize the cost of producing a given level of output:

$$C(\mathbf{w}, Q) = \underset{\{\mathbf{x}\}}{\text{Min}} \{ \mathbf{w}\mathbf{x} \mid f(\mathbf{x}) \geq Q, \mathbf{x} \geq 0 \}. \quad (1)$$

Since more inputs have to be purchased to produce more output, C is increasing in Q . It is also increasing, homogeneous of degree one, and concave in $\mathbf{w}$. Linear homogeneity follows from the observation that replacing $\mathbf{w}$ in (1) by $\lambda \mathbf{w}$ does not alter the optimal input choices. The concavity property falls out directly from a comparison of the true cost function and the cost function for Leontief technology. In the case of Leontief technology, C is a linear function of factor prices because the inputs x_i are fixed. Hence, if any substitution is possible, costs increase less than linearly when factor prices rise. This argument underlies the convexity/concavity properties of other duality functions – the general implication of substitution is that minimum value functions are concave in prices and maximum value functions are convex.

The structure of the optimization problem in (1) suggests a dual relationship between the cost function and the production function that appears in the constraint set. This conjecture is correct when technology is convex [i.e., $f(\mathbf{x})$ is quasi-concave] as the underlying production function can be recovered from the cost function. The procedure for doing so is straightforward. A continuous quasi-concave production function has convex isoquants. Every input vector on an isoquant is thus an optimal choice for some set of factor prices. By holding output constant and varying $\mathbf{w}$, the entire isoquant can be reconstructed from the cost function.

There is one slightly tricky aspect to the dual nature of cost and production functions. While the two functions embody the same information about technology, they need not bear a family resemblance. Functions that *do* possess this property are said to be self-dual. For example, the Cobb–Douglas production function

$$Q = x_1^b x_2^{1-b}$$

generates the similar-looking cost function

$$C(\mathbf{w}, Q) = k w_1^b w_2^{1-b} Q,$$

where $k \equiv b^{-b} (1 - b)^{b-1}$. Many of the other functional forms commonly used to represent technology are self-dual (e.g., Leontief and Constant Elasticity of Substitution (CES) functions). There are some exceptions, however. Translog functions, in particular, are not self-dual. A translog cost function can be generated from some well-behaved convex technology, but not from a translog production function.

2.1.1.1 INPUT DEMANDS AND FACTOR SUBSTITUTION PATTERNS

The cost function associated with a quasi-concave production function may have kinks or flat segments that preclude differential comparative statics analysis. Under the slightly stronger assumption that the production function is strictly quasi-concave, these curvature problems disappear. Strict quasi-concavity guarantees that the optimal input vector is unique and the cost function continuously differentiable. Moreover, the optimal input choices can be pulled out of the cost function in a single easy step. Recall from the envelope theorem that the adjustment of optimal input demands may be ignored when calculating the impact on the maximand $\mathbf{w}\mathbf{x}$ of small changes in w_i – to a first-order effect, $\mathbf{w}\partial\mathbf{x}/\partial w_i = 0$. Thus,

$$C_i \equiv \partial C / \partial w_i = x_i(\mathbf{w}, Q). \quad (2)$$

The above result, that C_i yields the factor demand x_i , is known as *Shephard's lemma*.

I mentioned earlier that the duality of cost and production functions means that the firm's isoquant map can be reconstructed from its cost function. It should be possible, therefore, to describe the scope for substitution between inputs in terms of the cost function. There are several ways of doing this. The most direct is to assume C is twice differentiable and then define the conditional factor demand elasticities¹

$$\eta_{ij} \equiv \frac{\partial x_i}{\partial w_j} \frac{w_j}{x_i} = \frac{C_{ij} w_j}{C_i}. \quad (3)$$

The conditional elasticities are subject to certain adding-up constraints. Since C is homogeneous of degree one in $\mathbf{w}$, its partial derivatives are homogeneous of degree zero.² Thus $C_i(\mathbf{w}, Q) = C_i(\alpha\mathbf{w}, Q)$, implying that for each input the own- and cross-price elasticities sum to zero:

¹ The *conditional* factor demand elasticity is defined for a given level of output.

² Functions that are homogeneous of degree k have partial derivatives that are homogeneous of degree $k - 1$.

$$\sum_{j=1}^n C_{ij}w_j = 0 \Rightarrow \sum_{j=1}^n C_{ij}w_j / C_i = \sum_{j=1}^n \eta_{ij} = 0, \quad i = 1, \dots, n. \quad (4)$$

Theory does not restrict the sign of η_{ij} , $i \neq j$, when there are more than two inputs. The own-price elasticity η_{ii} , however, cannot be positive. Since C is concave in $\mathbf{w}$, $[C_{ij}]$ is a symmetric, negative, semidefinite matrix having nonpositive elements on the diagonal. The own-price elasticity is of the same sign as C_{ii} , so $\eta_{ii} \leq 0$.

The second method of characterizing the curvature of isoquants relates the cost function to the Allen–Uzawa (AU) partial elasticities of substitution (Allen, 1938; Uzawa, 1962). The AU partial elasticity of substitution between factors i and j is defined as

$$\sigma_{ij} = \frac{C_{ij}C}{C_i C_j}.$$

The cost share for factor j , $\theta_j \equiv w_j x_j / C = w_j C_j / C$, links σ_{ij} and η_{ij} .³

$$\eta_{ij} \equiv \frac{C_{ij}w_j}{C_i} = \left(\frac{C_{ij}C}{C_i C_j} \right) \frac{w_j C_j}{C} = \sigma_{ij} \theta_j. \quad (5)$$

From (5) and the adding-up constraint (4),

$$\sum_{j=1}^n \sigma_{ij} \theta_j = 0, \quad i = 1, \dots, n. \quad (6)$$

Furthermore, it is easily established from concavity of the cost function and the definition of the partial elasticities of substitution that $[\sigma_{ij}]$ is a

³ The AU elasticity of substitution is often criticized as a nonintuitive elasticity that adds little or no information to that already contained in the conditional cross-price elasticity of demand (Chambers, 1988, p. 95; Blackorby and Russel, 1989). Blackorby and Russel argue, in addition, that it is not a natural measure of curvature faithful to Hicks' original concept of the elasticity of substitution.

Both of these criticisms strike me as superficial. The intuition for the AU elasticity is that it generalizes the decomposition of the cross-price elasticity for the two-input case to the case of many inputs. When there are just two inputs, the conditional cross-price elasticity η_{ij} can be expressed as the product of the cost share θ_j and the elasticity of substitution σ . In the general many-input case, σ_{ij} assumes the role of σ . The AU partial elasticity cannot be equated with a single substitution parameter (assuming there is more than one) in the underlying production function. It depends instead on all of the substitution parameters and cost shares that affect the degree to which an increase in w_j causes substitution toward or away from x_i . See the intuitive explanation given in example #2 at the end of the chapter.