EMIL WOLF

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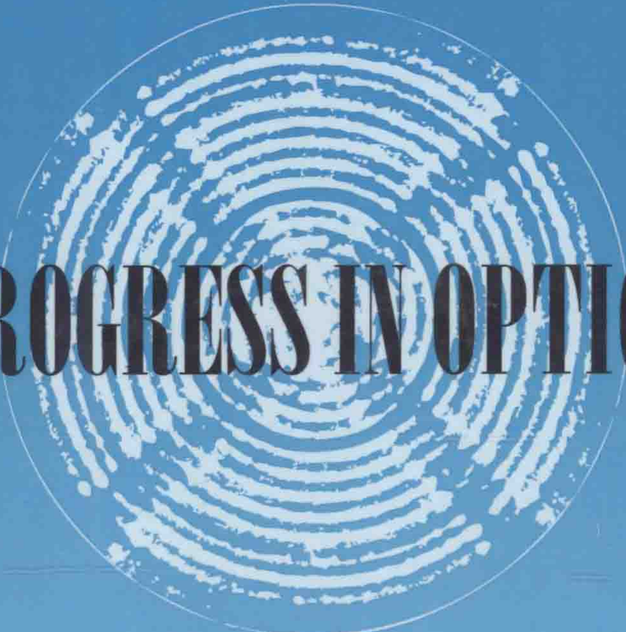

# PROGRESS IN OPTICS

VOLUME 56

CONTRIBUTORS

P. Andrés, K. Banaszek, T. Brown, M. Chekhova, J. Lancis,
W. Leoński, A. Kowalewska-Kudłaszyk, J. Perez Torres,
V. Torres-Company, I. Walmsley

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EDITED BY

E. Wolf
*University of Rochester, N.Y., U.S.A.*

*Contributors*
P. Andrés, K. Banaszek, T. G. Brown,
M. Chekhova, V. Torres-Company,
A. Kowalewska-Kudłaszyk, J. Lancis, W. Leoński,
J. P. Torres, I. A. Walmsley



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# PROGRESS IN OPTICS

VOLUME 56

# PREFACE

This 56th volume of *Progress in Optics* presents five chapters that describe developments in various areas of classical and quantum optics.

In the first chapter, by V. Torres-Company, J. Lancis, and P. Andrés, the mathematical similarities between the paraxial wave equation and formula describing wave propagation in a dispersive medium are used to explore several space-time analogies in optics.

In the second chapter, contributed by T. G. Brown, a novel type of beams, namely beams whose state of polarization varies with position, are reviewed, and their generation and applications are discussed.

In the third chapter, by W. Leonski and A. Kowalewska-Kudlaszyk, various types of so-called quantum scissors that produce output states of finite dimensionality are discussed.

In the fourth chapter, by M. Chekhova, various methods for producing biphotons with prescribed spectral and polarization properties are reviewed. Their applications such as quantum metrology are discussed.

The final chapter, by J. P. Torres, K. Banaszek, and I. A. Walmsley, deals with the engineering of nonlinear sources to enable the control of quantum correlations in various degrees of freedom of the optical field.

Emil Wolf
*Department of Physics and Astronomy*
*The Institute of Optics*
*University of Rochester*
*Rochester, New York*

November 2011

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## 5. Engineering Nonlinear Optic Sources of Photonic Entanglement

CHAPTER 1

# Space-Time Analogies in Optics

**Víctor Torres-Company**[a,b,c], **Jesús Lancis**[c], and **Pedro Andrés**[d]

*[a]Department of Electrical and Computer Engineering, McGill University, H3A 2A7 Montreal, QC, Canada*
*[b]Current address: Department of Electrical and Computer Engineering, Purdue University, IN-47907, USA*
*[c]Departament de Física, Universitat Jaume I, 12007 Castelló de la Plana, Spain*
*[d]Departamento de Óptica, Universitat de València, 46100 Burjassot, Spain*

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## 1. INTRODUCTION

In the last decades, the generation of pulsed beams with pulse durations in the order of pico- and femtosecond has constituted an important topic for the physics and engineering communities, where researchers find themselves continuously pushing the limits to satisfy quite radical premises such as ultrafast, ultrabroad or ultrasmall. The characteristics of this kind of optical radiation, that is, broadband spectrum, enormous temporal resolution, high peak but low average power, potentially high repetition rate, and high spatial coherence make it an indispensable tool to develop many applications in different fields of science and technology (Fermann, Galvanauskas, & Sucha, 2003).

At a systems level, the so-called space-time analogy constitutes a source of inspiration to design and implement new schemes for processing these ultrafast optical signals based on concepts borrowed from the well-established field of Fourier optics (Goodman, 1996), leading to what is now popularly termed as *Temporal Optics*. The key relies on noting the mathematical similarity between the equations that govern the paraxial diffraction of one-dimensional monochromatic light beams and those describing the distortion of plane-wave pulses in a first-order dispersive medium (Akhmanov, Sukhorukov, & Chirkin, 1969; Treacy, 1969). Thanks to the continuous advances in the optoelectronics and optical communication industries, the analogy can be extended to include other elements such as imaginglenses (Kolner, 1994a) or prisms (van Howe & Xu, 2006). These optoelectronic tools have paved an avenue for creating innovative temporal processing systems, bringing the "ultra" appellative to fields as diverse as optical interconnects, optical communications, microwave photonics, biophotonics, or quantum information processing, among others.

In this review, we shall provide the most basic notions for the understanding of the space-time analogy, including the fundamental elements and their possible implementations with state-of-the-art technology. We provide a comprehensive approach, based on a temporal matrix formalism (Nakazawa et al., 1998), to describe the characteristics of some of the most widespread system processing architectures. Special emphasis shall be paid on their applications in the above mentioned fields, highlighting not only their innovative character but offering a comparative study with respect to other, more conventional, solutions. Finally, we will review the extension of the space-time analogy to the noncoherent (Lancis, Torres-Company et al., 2005) and even to the nonclassical (quantum) regime (Tsang & Psaltis, 2006; Torres-Company, Lajunen et al., 2008), where this analogy still offers some potential to design – and sometimes interpret – new physical phenomena with the aid of the notions of classical Fourier optics.

## 2. ULTRASHORT LIGHT PULSE PROPAGATION IN DISPERSIVE HOMOGENEOUS MEDIA

In this section, we introduce the fundamental equations describing the linear distortion of an ultrashort light pulse in a dispersive homogeneous medium. Such a basic physical problem constitutes the cornerstone of ultrafast optical signal processing.

Let us assume a scalar optical field, described by its analytic signal $U(\mathbf{r}, t')$, propagating linearly in a waveguide with translational symmetry through the $z$-direction $z$ and filled with a homogeneous lossless dispersive medium. The propagation constant is $\beta(\omega') = n(\omega')\omega'/c$, where c is the speed of light in vacuum and $n(\omega')$ the frequency-dependent refractive index. We can then write

$$U(\mathbf{r}, t') = A(x, y)\psi(z, t')\exp[-i(\omega_0 t' - \beta_0 z)]. \tag{1}$$

where $\psi(z, t')$ is the pulse envelope, which modulates the monochromatic carrier wave of angular frequency $\omega_0$; the constant $\beta_0 = \beta(\omega_0)$; and $A(x, y)$ is the transversal spatial distribution of the mode, evaluated at $\omega_0$. Once

$A(x, y)$ is calculated, we only need to know $\psi(z, t')$ to determine $U(\mathbf{r}, t')$ at every $z$ position. The evolution of $\psi(z, t')$ is then described by a wave equation.

Since the functional form of $\beta(\omega')$ is usually unknown, it is very useful to perform a Taylor expansion (Agrawal, 2007)

$$\beta(\omega') = \beta_0 + \beta_1(\omega' - \omega_0) + \frac{\beta_2}{2!}(\omega' - \omega_0)^2 + \frac{\beta_3}{3!}(\omega' - \omega_0)^3 + \cdots \quad (2)$$

where $\beta_n = \mathrm{d}^n\beta(\omega')/\mathrm{d}\omega'^n|_{\omega'=\omega_0}$ is the $n$th-order dispersion coefficient of the waveguide, with $n = 0, 1, 2, 3, \ldots$ From now on, we will express the temporal variations of the pulse in a reference framework moving at the group velocity of the wave packet, $t = t' - \beta_1 z$.

The wave equation describing the envelope distortion is of second order in $z$. In order to reduce this equation to first order, the slowly varying envelope approximation (SVEA) is usually invoked in the multicycle regime (Agrawal, 2007). This approximation requires that the function $\psi(z, t)$ does not change significantly through a distance compared with the carrier wavelength, and the pulse duration to be much larger than the carrier oscillation period. Mathematically, it is translated into $|\partial\psi(z, t)/\partial z| \ll \beta_0|\psi(z, t)|$ and $|\partial\psi(z, t)/\partial t| \ll \omega_0|\psi(z, t)|$. Both inequalities are guaranteed whenever the optical frequency bandwidth, $\Delta\omega$, is much less than the carrier frequency, $\Delta\omega \ll \omega_0$.

Within the SVEA, we have

$$i\frac{\partial\psi(z, t)}{\partial z} = H\psi(z, t), \quad (3)$$

where the (Hamiltonian operator) $H = -\sum_{n=2} \frac{i^n\beta_n}{n!}\frac{\partial^n}{\partial t^n}$. Alternatively, Equation (3) can be rewritten in the frequency domain by Fourier transformation,[1]

$$i\frac{\partial\tilde{\psi}(z, \omega)}{\partial z} = \tilde{H}\tilde{\psi}(z, \omega), \quad (4)$$

where $\tilde{\psi}(z, \omega)$ is the Fourier transform of $\psi(z, t)$ and

$$\tilde{H} = -\sum_{n=2}\frac{\beta_n}{n!}\omega^n. \quad (5)$$

[1] In the following, the angular frequencies are referred at the baseband. They are referred to the optical frequencies, $\omega'$, by a shift $\omega' = \omega + \omega_0$.

This equation can be easily integrated as

$$\tilde{\psi}(z,\omega) = \exp\left[i\sum_{n=2}\frac{\beta_n z}{n!}\omega^n\right]\tilde{\psi}(z=0,\omega). \tag{6}$$

Therefore, the dispersive medium acts as a phase-only spectral filter. Sometimes, it is useful to write the dispersive terms in the more compact form $\Phi_n = \beta_n z$. When $n = 2$, $\beta_2$ is called the group velocity dispersion (GVD) coefficient, and $\Phi_2$ the group delay dispersion (GDD) parameter. Analogously, when $n = 3$, $\beta_3$ is the third-order dispersion (TOD) coefficient.

## 3. FIRST-ORDER APPROXIMATION: SPACE-TIME ANALOGY

We are particularly interested in the case in which only the first term of the operator contributes,

$$i\frac{\partial\psi(z,t)}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2\psi(z,t)}{\partial t^2}. \tag{7}$$

Since it implies a second-order expansion in Equation (2), the medium is said to be parabolic. This first-order approximation is physically plausible whenever $\Delta\omega \ll 3|\beta_2/\beta_3|$. As an example, for standard single-mode fiber (SMF) and a waveform centered in the telecommunication wavelength ($\lambda_0 = 1.5\,\mu$m), the fiber coefficients are $\beta_2 = -21.7\,\text{ps}^2/\text{km}$ and $\beta_3 = 0.1\,\text{ps}^3/\text{km}$. The signal should then have an optical bandwidth shorter than ~100 THz.

Equation (7) is a Schrödinger-like equation for a free particle, ubiquitous in Physics scenarios. In particular, this equation is mathematically identical to that describing the one-dimensional (1D) scalar diffraction of a paraxial monochromatic beam propagating in the $z$-direction (Goodman, 1996)

$$i\frac{\partial U_e(z,x)}{\partial z} = -\frac{1}{2k_0}\frac{\partial^2 U_e(z,x)}{\partial x^2}, \tag{8}$$

where $U_e(z,x)$ denotes the transversal profile of the 1D beam and $k_0$ the wave number.

The mathematical similarity between the Equations (7) and (8) is what we know as the space-time analogy, mentioned in the Introduction. Table 1 summarizes the transfer rules connecting both domains. This connection between diffraction and dispersion was found independently by two different groups at the end of the sixties (Akhmanov, Sukhorukov, & Chirkin, 1969; Treacy, 1969). On the other hand, Papoulis had pointed out a formal

**TABLE 1** Space-Time analogy transfer rules

| Space | | Time | |
|---|---|---|---|
| Description | Variable/Parameter | Variable/Parameter | Description |
| Position | $x$ | $t$ | Proper time |
| Spatial frequency | $2\pi u$ | $\omega$ | (Baseband) angular frequency |
| Wave number$^{-1}$ | $1/k_0$ | $-\beta_2$ | GVD coefficient |
| Paraxial propagation factor (distance $z$) | $\exp[-i2\pi^2 zu^2/k_0]$ | $\exp[i\Phi_2\omega^2/2]$ | First-order dispersion (GDD parameter $\Phi_2$) |
| Spatial lens factor (focal length $f$) | $\exp[-ik_0x^2/(2f)]$ | $\exp[iKt^2/2]$ | Time lens factor (chirping rate $K$) |

similarity between diffraction and chirp radar, including the equivalent of spatial lenses (Papoulis, 1968a). Later on, we found in the literature some theoretical research work performed by Saleh and Irshid (1982) about an extension into the temporal domain of the Collet-Wolf equivalent theorem regarding spatially partially coherent light (Mandel & Wolf, 1995). In the same decade, it is worth mentioning the temporal equivalents of the Talbot effect proposed by Jannson and Jannson (1981) and spatial Fourier transformation (Jannson, 1983). However, it was not until the pioneering work of Kolner and Nazarathy (1989) who, inspired on developing pulse compression techniques based on electro-optic phase modulators (Kolner, 1988), developed a formal treatment of this analogy to include what we know today as "time lenses". It became then evident that a huge avenue of temporal equivalent systems for ultrafast signal processing was feasible to build, given the instrumentation available at that time (see e.g., Lohman & Mendlovic, 1992; Kolner, 1994a; Godil, Auld, & Bloom, 1994; Papoulis, 1994; Mendlovic, Melamed, & Ozaktas, 1995).

## 4. ELEMENTS AND THEIR IMPLEMENTATIONS

### 4.1. Temporal ABCD Matrices

In the previous section, we have introduced the formalism of the space-time analogy and advanced that it can be extended to include other photonic components apart from first-order dispersive media, such as time lenses. In this section, we provide a unified formalism to describe the linear distortion of the pulse envelope in a system composed by concatenating different elements that are susceptible to be described within the framework of this analogy: the so-called "Gaussian" systems. Each element in a Gaussian system is mathematically characterized by a unitary $2 \times 2$ matrix. The whole system is quantified by a matrix calculated by the multiplication, in the right order, of each of the elements that compose it. This formalism is well known in spatial first-order Fourier optics, which works in the paraxial regime (Siegman, 1985; Collins, 1970; Palma & Bagini, 1997), and has been adapted into the temporal domain (Dijaili, Dienes, & Smith, 1990; Nakazawa et al., 1998; Mookherjea & Yariv, 2001).

Concretely, the action of any linear system on the input complex envelope of a short light pulse, $\psi_{\text{in}}(t)$, can be characterized as a linear superposition

$$\psi_{\text{out}}(t) = \int \psi_{\text{in}}(t')K(t,t')dt', \tag{9}$$

where the system is described by the kernel $K(t,t')$ and $\psi_{\text{out}}(t)$ denotes the output complex envelope. In the case in which the linear system is Gaussian, the Kernel takes the following form

$$K(t,t') = \begin{cases} \sqrt{\frac{i}{2\pi B}} \exp\left[\frac{-i}{2B}\left(At'^2 + Dt^2 - 2tt'\right)\right] & \text{if } B \neq 0 \\ \sqrt{\frac{1}{A}} \exp\left[\frac{-iCt^2}{2A}\right] \delta(t' - t/A) & \text{if } B = 0 \end{cases}. \tag{10}$$

Here the constants $A$, $B$, $C$, and $D$ account for the system's matrix coefficients. The reason why these systems are called Gaussian is because of the quadratic dependence in the exponential term.

## 4.2. Spectral Dual Formalism

In temporal optics, there are some situations in which the Fourier transform of the envelope, $\tilde{\psi}(\omega)$, may be the physical magnitude of interest, rather than $\psi(t)$. Of course, they are connected each other by a Fourier transform relation and both carry the same quantity of information. Since any linear system in time is linear in frequency too, it is useful to provide a similar analysis of Equations (9) and (10) in the dual space. By dual we mean that these devices behave identically from a mathematical point of view, but their action is performed in the Fourier domain (Papoulis, 1968b). Thus, we can write

$$\tilde{\psi}_{\text{out}}(\omega) = \int \tilde{\psi}_{\text{in}}(\omega')\tilde{K}(\omega,\omega')\text{d}\omega', \tag{11}$$

where the Kernel in the frequency domain forms a Fourier transform pair with the Kernel in time, that is,

$$\tilde{K}(\omega,\omega') = \iint K(t',t'') \exp[i(\omega t' - \omega' t'')]\text{d}t'\text{d}t''. \tag{12}$$

For the particular case in which the system is Gaussian, it becomes Gaussian in the spectral domain too and therefore a similar structural form of Equation (10) holds,

$$\tilde{K}(\omega,\omega') = \begin{cases} \sqrt{\frac{i}{2\pi B_\omega}} \exp\left[\frac{-i}{2B_\omega}\left(A_\omega \omega'^2 + D_\omega \omega^2 - 2\omega\omega'\right)\right] & \text{if } B_\omega \neq 0 \\ \sqrt{\frac{1}{A_\omega}} \exp\left[\frac{-iC_\omega \omega^2}{2A_\omega}\right] \delta(\omega' - \omega/A_\omega) & \text{if } B_\omega = 0 \end{cases}. \quad (13)$$

Of course, the dual coefficients are related to the matrix parameters $A$, $B$, $C$, and $D$ in the time domain. By inserting Equation (10) into (12), we easily obtain

$$\begin{pmatrix} A_\omega & B_\omega \\ C_\omega & D_\omega \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} D & -C \\ -B & A \end{pmatrix}. \quad (14)$$

Equations (12) and (14) imply that the action of a Gaussian system in the spectral domain is mathematically identical to the action of a Gaussian system in the temporal domain whose matrix elements are provided by the inverse matrix. The implications of this statement will become clearer in the following sections.

## 4.3. Basic Photonic Components

Within the previous matrix formalism, we now proceed to describe the mathematical structure of some devices that will probe very useful in temporal optics applications and describe briefly their implementation with current technology.

### 4.3.1. Group Delay Dispersion (GDD) Circuit

As previously advanced, a GDD circuit is an element designed to introduce a quadratic phase factor in the spectral domain, $\tilde{\psi}_{\text{out}}(\omega) = \exp[i\Phi_2\omega^2/2]\tilde{\psi}_{\text{in}}(\omega)$, where $\Phi_2$ is the GDD coefficient. The corresponding matrix is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \Phi_2 \\ 0 & 1 \end{pmatrix}. \quad (15)$$

These photonic components, as highlighted in Table 1, constitute the temporal equivalent of the paraxial diffraction. However, we must note that while diffraction only takes place for positive wave numbers in temporal optics, the GDD parameter can be positive or negative, depending on the material component, waveguide structure, and the signal's carrier frequency. This subtleness certainly opens exciting new possibilities for ultrafast signal processing.