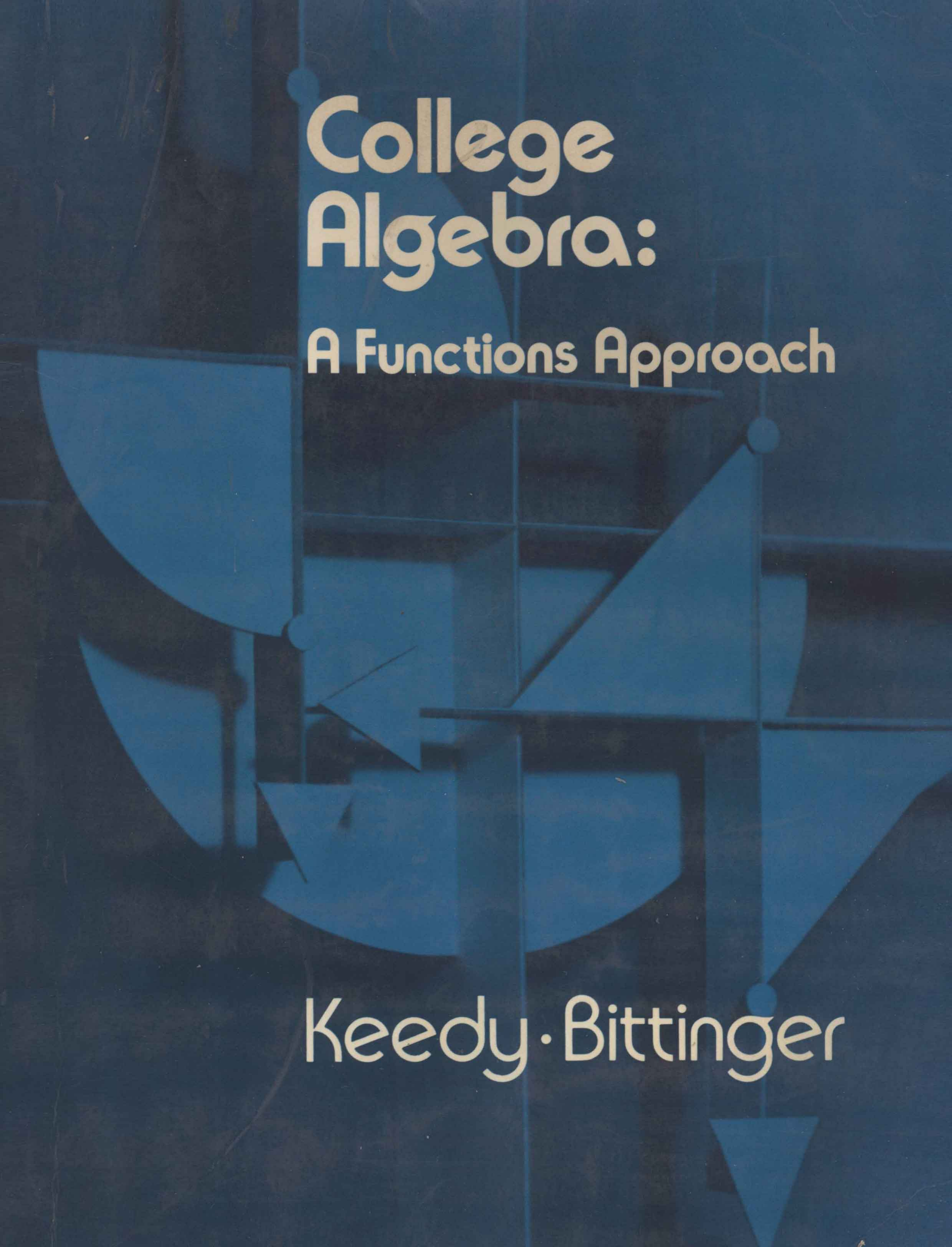

College Algebra: A Functions Approach

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COLLEGE ALGEBRA:
A FUNCTIONS APPROACH

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Preface

This text covers the usual topics of college algebra, but is significantly different from ordinary texts in a number of respects. For example, the development of topics is more gradual and more thorough than usual, the treatment is not highly formal, and the authors have used words sparingly in order to produce a book that students can read.

One of the principal and distinguishing features of the text is its design and format. Each page has an outer margin, which is used in two ways: (1) For each lesson the objectives are stated in behavioral terms at the top of the page. These can be seen easily by the student, and when he asks, "What material am I responsible for?" these objectives provide him with an answer. (2) In the margins are sample, or developmental, exercises placed with the text material so that the student can become actively involved in the development of the topic. In addition, as he checks answers to the margin exercises in the back of the book, the student obtains reinforcement or guidance as well as practice on exercises of the type he will be expected to do as homework for the lesson. The text refers the student to these exercises in the margins at appropriate places. The use of the margin exercises has proved to be quite effective, so much so that no *Study Guide* is needed to supplement the text.

The idea of a function is emphasized. After an honest and straightforward review of elementary algebraic skills in Chapter 1 and a treatment of equations in Chapter 2, the concepts of relations, functions, and transformations are introduced in Chapter 3. The treatment of transformations makes this chapter unique, and it sets up the study of later material. In Chapter 4, for example, the idea of transformations is used in connection with quadratic functions, and it provides a new approach to solving inequalities with absolute value, a traditionally difficult topic. The use of transformations also makes for a more efficient treatment of the conic sections in Chapter 9.

There are many ways in which this book can be used. Flexibility is indeed one of its important features. The instructor who wishes to use it as he would use a standard textbook may do so very easily by omitting the exercises in the margins. If an instructor wishes to use the lecture method primarily, but also wishes to introduce some student-centered activity into the class, he can easily do so by merely interrupting his lecture and having students work the exercises in the margins at the appropriate times. On the other hand, the book is well suited for use in audiotutorial or other systems of individualized instruction, or in any approach which is essentially self-study. Because of its design and format it can be used with minimal instructor guidance, yet it retains the flavor of the traditional textbook with the often deadly quality of the programed textbook. It has been found that this minimal need for instructor guidance makes the text particularly effective for use in large classes.

This book contains some other features not usually found in a college text. For each chapter there is a pretest and a chapter test, and in addition there is a final examination. Two alternate forms of the chapter tests and final examination appear in the *Instructor's Manual.* Answers to all tests are given in the *Manual,* where the spacing of the answers matches that on the tests themselves. The tests can thus be removed from the book and scored relatively quickly. Great care has been given to constructing the exercises, all of which are based upon the behavioral objectives, except for an occasional challenge. The first exercises in each set are quite easy, while later ones become progressively more difficult. For the most part, the exercises are in matching pairs; that is, any odd-numbered exercise is very much like the one that immediately follows it.

The material herein is more than enough for a one-term course, so that some choice of topics is possible. There is also a trigonometry text and an integrated text on algebra and trigonometry by the same authors and in the same style: *Trigonometry: A Functions Approach* and *Algebra and Trigonometry: A Functions Approach.*

The authors wish to thank the many people who helped with the development of this book. The material has been class tested, and we especially wish to thank the students involved for their suggestions, their criticisms so freely given, and their patience. Professor Dennis Sorge of Purdue University and Ms. Judy Beecher of Indiana University–Purdue University at Indianapolis also made many suggestions which contributed to the clarity and continuity of the book. In addition, we wish to thank Charles W. Austin, California State University, Long Beach; Wilson E. Brumley, Colorado State University; Allan Claudson, Treasure Valley Community College; Rebecca Crittenden, Virginia Polytechnic Institute; Henry Harmeling, North Shore Community College; Henry Kubo, West Los Angeles College; George J. Langbehn, Salem State College; John A. Mitchem, San Jose State College; David L. Neuhouser, Taylor University; Theron Rockhill, State University College at Brockport; Marion B. Smith, California State University, Bakersfield; and Eugene Spiegel, University of Connecticut, who reviewed the manuscript and made helpful suggestions for its improvement.

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M.L.K.
M.L.B.

Contents

Chapter 1

REVIEW OF ELEMENTARY ALGEBRA

NAME ____________________

CHAPTER 1 PRETEST

CLASS ________ SCORE ________ GRADE ________

Add.

1. $5 + (-9)$
2. $-2.5 + |-2.5|$

Multiply.

3. $(4)(-9)$

Divide.

4. $\frac{18}{-3}$

Subtract.

5. $22 - {}^{-}8$

Convert to decimal notation.

6. 3.261×10^6
7. 4.1×10^{-2}

Convert to scientific notation.

8. 63.21
9. .01432

Simplify.

10. $(7a^2b^4)(-2a^{-4}b^2)$
11. $\frac{54x^6y^{-4}z^2}{9x^{-3}y^2z^{-4}}$
12. $\sqrt[3]{64}$
13. $\sqrt[4]{81}$
14. $\dfrac{\frac{x}{y} + \frac{y}{x}}{\frac{x^2}{y^2} - \frac{y^2}{x^2}}$
15. $(\sqrt{3} - \sqrt{7})(\sqrt{3} + \sqrt{7})$

ANSWERS

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________
13. ________
14. ________
15. ________

Write an expression containing a single radical.

16. $\sqrt{y^5} \cdot \sqrt[3]{y^2}$

16. ____________

Convert to radical notation.

17. $b^{\frac{7}{5}}$

Write rational exponents and simplify.

18. $\sqrt[6]{\dfrac{r^{12}s^{18}}{2^6}}$

17. ____________

Solve.

18. ____________

19. $(z^2 - 1) + z = 14 - z$

20. $14 - 4y < 22$

21. $a^2 + 21 = 10a$

19. ____________

22. Divide and simplify.

$$\frac{r^2 - s^2}{2r + s} \div \frac{r + s}{2r^2 - rs - s^2}$$

23. Subtract and simplify.

$$\frac{3}{x + y} - \frac{x - 5y}{x^2 - y^2}$$

20. ____________

21. ____________

22. ____________

24. Rationalize the denominator.

$$\frac{\sqrt{y} - 2\sqrt{x}}{\sqrt{x} - \sqrt{y}}$$

25. Change $100 \dfrac{\text{ft}}{\text{sec}}$ to $\dfrac{\text{yd}}{\text{min}}$.

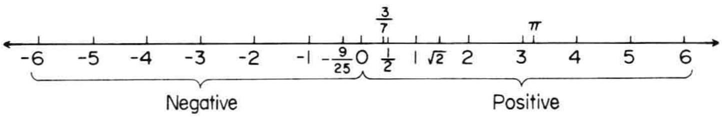
23. ____________

24. ____________

25. ____________

1.1 OPERATIONS ON THE REAL NUMBERS

The set of real numbers corresponds to the set of points on a line. On this number line, the numbers to the right of (greater than) 0 are called *positive*.



Those to the left of (less than) 0 are called *negative*. The real numbers, 0, 1, −1, 2, −2, 3, −3, and so on, are called *integers*. Those real numbers that can be named by a fractional symbol with integer numerator and denominator are called *rational*. Those that cannot be so named are called *irrational*. The numbers 3/7, −9/25, and 5 are rational. The numbers $\sqrt{2}$ and π are irrational. The distance of a number from 0 is called its *absolute value*. The absolute value of a number x is denoted $|x|$. Thus $|3| = 3$ and $|-7| = 7$. Absolute values are never negative.

Do exercises 1 through 3. (Exercises are in the margin.)

ADDITION

To add two negative numbers we can add their absolute values, but remember that the sum is negative.

Example. Add $-5 + (-6)$.

The absolute values are 5 and 6. We add them: $5 + 6 = 11$. The answer will be negative, so $-5 + (-6) = -11$.

To add a negative and a positive number, we find the difference of their absolute values. The answer will have the sign of the addend with the larger absolute value. If the absolute values are the same, then the sum is 0.

Examples $8 + (-5) = 3$, $8.6 + (-4.2) = 4.4$,

$4 + (-10) = -6$, $\frac{3}{5} + \left(-\frac{9}{5}\right) = -\frac{6}{5}$,

$-5 + 3 = -2$, $\pi + (-\pi) = 0$.

Do exercises 4 through 9.

MULTIPLICATION

The product of two negative numbers is positive. The product of a negative and a positive number is negative.

Examples $3 \cdot (-4) = -12$, $1.5 \cdot (-3.8) = -5.7$,

$-5 \cdot (-4) = 20$, $-\frac{2}{3} \cdot \left(-\frac{4}{5}\right) = \frac{8}{15}$,

$-3 \cdot (-2) \cdot (-4) = -24$.

Do exercises 10 through 12.

OBJECTIVES

You should be able to:

a) Find the absolute value of a real number.
b) Find the additive inverse of a real number.
c) Find the reciprocal of a real number.
d) Add, subtract, multiply, and divide positive and negative real numbers.

Simplify.

1. $|43|$

2. $|-17|$

3. $|0|$

Add.

4. $-5 + (-7)$

5. $-1.2 + (-3.5)$

6. $-\frac{6}{5} + \frac{2}{5}$

7. $.5 + (-.7)$

8. $8 + (-3)$

9. $\frac{14}{3} + \left(-\frac{14}{3}\right)$

Multiply.

10. $4 \cdot (-6)$

11. $-\frac{7}{5} \cdot \left(-\frac{3}{5}\right)$

12. $(-2)(-3)(-5)$

Divide.

13. $\frac{-20}{-5}$

14. $\frac{4.5}{-1.5}$

15. $\frac{-12}{-36}$

Find the additive inverse of each and symbolize it in two or more ways.

16. 6

17. −12.

18. 0

19. y

20. $(2x + 3y)$

Subtract.

21. $2.5 - 1.2$

22. $12 - {}^{-}5$

23. $-\frac{8}{5} - \frac{3}{5}$

24. $-20 - {}^{-}7$

DIVISION

The quotient of two negative numbers is positive. The quotient of a positive and a negative number is negative.

Examples

$$\frac{-8}{-4} = 2, \quad \frac{-4.8}{-2.4} = 2,$$

$$\frac{-12}{4} = -3, \quad \frac{-5.5}{5} = -1.1,$$

$$\frac{5}{-10} = -\frac{1}{2}, \quad \frac{3}{4} \div \left(-\frac{2}{3}\right) = -\frac{9}{8}.$$

Do exercises 13 through 15.

RECIPROCALS

If the product of two numbers is 1, they are *reciprocals* of each other.

Examples

The reciprocal of $\frac{1}{2}$ is 2.

The reciprocal of $-\frac{2}{3}$ is $-\frac{3}{2}$.

The reciprocal of 0.16 is 6.25.

When fractional notation for a number is given, its reciprocal can be found by inverting. To divide by a number is the same as multiplying by its reciprocal. Every real number except 0 has a reciprocal.

ADDITIVE INVERSES

The *additive inverse* of a number is the number opposite to it on the number line. When a number and its additive inverse are added, the result is always 0. The additive inverse of a number x is symbolized $-x$ or ${}^{-}x$. Every real number has an additive inverse.

Examples

The additive inverse of 3 is negative 3, symbolized -3 or ${}^{-}3$.

The additive inverse of -5 is 5, symbolized $-(-5)$ or ${}^{-}(-5)$, or 5.

The additive inverse of 0 is 0, symbolized -0 or ${}^{-}0$ or 0.

Do exercises 16 through 20.

SUBTRACTION

To subtract one number from another, we can add its additive inverse.

Examples

$$8 - 5 = 8 + (-5) = 3,$$

$$10 - (-4) = 10 + 4 = 14,$$

$$8.6 - (-2.3) = 8.6 + 2.3 = 10.9,$$

$$-15 - (-5) = -15 + 5 = -10.$$

Do exercises 21 through 24.

CHANGING SIGNS

The rule for subtraction is sometimes stated "change the sign of the subtrahend and then add." To change a sign means to replace a number by its additive inverse. When we change the sign of a positive number, we get a negative number. When we change the sign of a negative number, we get a positive number. When we change the sign of 0, we get 0.

PROPERTIES OF REAL NUMBERS

Addition and multiplication of real numbers are both *commutative*. This means that the order is unimportant. For example. $3 + 2 = 2 + 3$ and $5 \cdot 4 = 4 \cdot 5$.

For any real numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$

Addition and multiplication of real numbers are both *associative*. This means that we can add or multiply, choosing the grouping as we please. For example, $4 + (5 + 1.2) = (4 + 5) + 1.2$ and $5 \cdot (4 \cdot 2) = (5 \cdot 4) \cdot 2$.

For any real numbers a, b, and c, $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.

When a number is to be multiplied by a sum, we can either add and then multiply or multiply and then add. This is known as the *distributive property*. It is often stated as follows:

For any real numbers a, b, and c, $a(b + c) = ab + ac$.

There is also a distributive property for multiplication over subtraction.

For any real numbers a, b, and c, $a(b - c) = ab - ac$.

The number 0 is called the *additive identity* because when it is added to any number n, the result is n. The number 1 is called the *multiplicative* identity, because when it is multiplied by any number n, the result is n. The foregoing properties are the basis of arithmetic and algebraic manipulations.

EXERCISE SET 1.1

Find the additive inverse of each and symbolize it two ways.

1. 8 **2.** 1.4 **3.** $-\frac{10}{3}$ **4.** -17

5. $3x$ **6.** $4y$ **7.** $(a + b)$ **8.** $(2a + b)$

Simplify.

9. $|12|$ **10.** $|2.56|$ **11.** $|-47|$ **12.** $|-5.6|$

Find the reciprocal of each.

13. 3 **14.** 5 **15.** $-\frac{2}{3}$ **16.** $-\frac{5}{9}$

Add.

17. $-3.1 + {}^{-}7.2$
18. $-5.2 + {}^{-}8.3$
19. $-34 + {}^{-}76$
20. $-42 + {}^{-}64$
21. $-\frac{7}{5} + \frac{3}{5}$
22. $-\frac{8}{3} + \frac{5}{3}$
23. $-412 + 186$
24. $-735 + 319$
25. $6 + {}^{-}10$
26. $8 + {}^{-}12$
27. $17.5 + {}^{-}31.2$
28. $3.17 + {}^{-}10.46$
29. $\frac{9}{2} + \frac{-3}{5}$
30. $\frac{11}{3} + \frac{-5}{2}$
31. $4.73 + {}^{-}1.42$
32. $8.97 + {}^{-}2.95$
33. $-5 + {}^{-}7 + {}^{-}12$
34. $-6 + {}^{-}4 + {}^{-}10$
35. $-14.12 + {}^{-}3.74 + {}^{-}7.43$
36. ${}^{-}105.8 + {}^{-}41.3 + {}^{-}83.4$
37. $-8 + 12 + {}^{-}5$
38. $-10 + 13 + {}^{-}6$

Multiply.

39. $5 \cdot (-6)$
40. $7 \cdot (-4)$
41. $\frac{12}{5} \cdot \left(-\frac{17}{7}\right)$
42. $\frac{13}{2} \cdot \left(-\frac{15}{4}\right)$
43. $(-7.1) \cdot 8$
44. $(-8.2) \cdot 6$
45. $(-1.4) \cdot (1.8)$
46. $(-1.3)(1.7)$
47. $-\frac{8}{3} \cdot \left(-\frac{5}{2}\right)$
48. $-\frac{7}{5} \cdot \left(-\frac{6}{7}\right)$
49. $-1.4 \cdot (-1.8)$
50. $-1.6(-1.7)$
51. $-12.3\,(-4.12)$
52. $-14.3(-4.21)$
53. $-2 \cdot (-4)(-5)$
54. $-3(-4)(-6)$
55. $-7(-2)(-3)$
56. $-6(-2)(-4)$
57. $-\frac{14}{3}\left(-\frac{17}{5}\right)\left(-\frac{21}{2}\right)$
58. $-\frac{13}{4}\left(-\frac{16}{5}\right)\left(-\frac{23}{2}\right)$

Divide.

59. $\frac{-20}{-4}$
60. $\frac{-30}{-5}$
61. $\frac{-18.9}{-2.7}$
62. $\frac{-24.8}{-3.1}$
63. $\frac{36}{-6}$
64. $\frac{49}{-7}$
65. $\frac{2.96}{-3.7}$
66. $\frac{2.73}{-3.9}$
67. $\frac{-35}{7}$
68. $\frac{-40}{8}$
69. $\frac{-11}{55}$
70. $\frac{-10}{70}$

Subtract.

71. $11 - 15$
72. $12 - 17$
73. $9{,}435 - 12{,}486$
74. $8{,}931 - 11{,}874$
75. $12 - {}^{-}6$
76. $13 - {}^{-}4$
77. $17.5 - {}^{-}21.2$
78. $18.3 - {}^{-}41.3$
79. $-\frac{10}{3} - \frac{-17}{3}$
80. $-\frac{11}{5} - \frac{-18}{5}$
81. $-1.45 - {}^{-}2.14$
82. $-1.54 - {}^{-}2.41$
83. $-12 - 8$
84. $-14 - 5$
85. $-276 - 188$
86. $-342 - {}^{-}177$

1.2 EXPONENTIAL NOTATION

INTEGERS AS EXPONENTS

The set of integers is as follows:

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

The braces indicate that a *set* is being named. When an integer greater than 1 is used as an exponent, the integer gives the number of times the base is used as a factor. For example, 5^3 means $5 \cdot 5 \cdot 5$. An exponent of 1 does not change the meaning of an expression. For example, $(-3)^1 = -3$. When 0 occurs as the exponent of a nonzero expression, we agree that the expression is equal to 1. For example, $37^0 = 1$. When a minus sign occurs with exponential notation, a certain caution is in order. For example, $(-4)^2$ means that -4 is to be raised to the second power. Hence $(-4)^2 = (-4)(-4) = 16$. On the other hand, -4^2 represents the additive inverse of 4^2. Thus $-4^2 = -16$.

Do exercises 25 through 34.

Negative integers as exponents have meaning as follows:

Definition. If n is any positive integer, then a^{-n} means $\frac{1}{a^n}$ for $a \neq 0$. In other words, a^n and a^{-n} are reciprocals of each other.

Examples

$$\frac{1}{5^2} = 5^{-2},$$

$$7^{-3} = \frac{1}{7^3},$$

$$5^{-4} = \frac{1}{5^4} = \frac{1}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{1}{625}$$

Do exercises 35 through 37.

PROPERTIES OF EXPONENTS

Let us consider an example involving multiplication.

$$b^5 \cdot b^{-2} = (b \cdot b \cdot b \cdot b \cdot b) \cdot \frac{1}{b \cdot b} = \frac{b \cdot b}{b \cdot b} \cdot (b \cdot b \cdot b)$$

$$= 1 \cdot (b \cdot b \cdot b) = b^3$$

Note that the result can be obtained by adding the exponents. This is true in general.

OBJECTIVES

You should be able to:

a) Explain the meaning of an integer exponent, whether the exponent is positive, negative, or zero.
b) Multiply, using exponents.
c) Divide, using exponents.
d) Raise a power to a power.
e) Convert from decimal notation to scientific notation, and conversely.

Rename with exponents.

25. $8 \cdot 8 \cdot 8 \cdot 8$

26. xxx

27. $4y \cdot 4y \cdot 4y \cdot 4y$

Rename without exponents.

28. 3^4

29. $(5x)^4$

30. $(-5)^4$

31. -5^4

32. $(3x)^0$

Simplify.

33. $(5y)^2$

34. $(-2x)^3$

35. Rename $\frac{1}{4^3}$ using a negative exponent

36. Rename 10^{-4} without a negative exponent.

37. Write three other symbols for 4^{-3}.