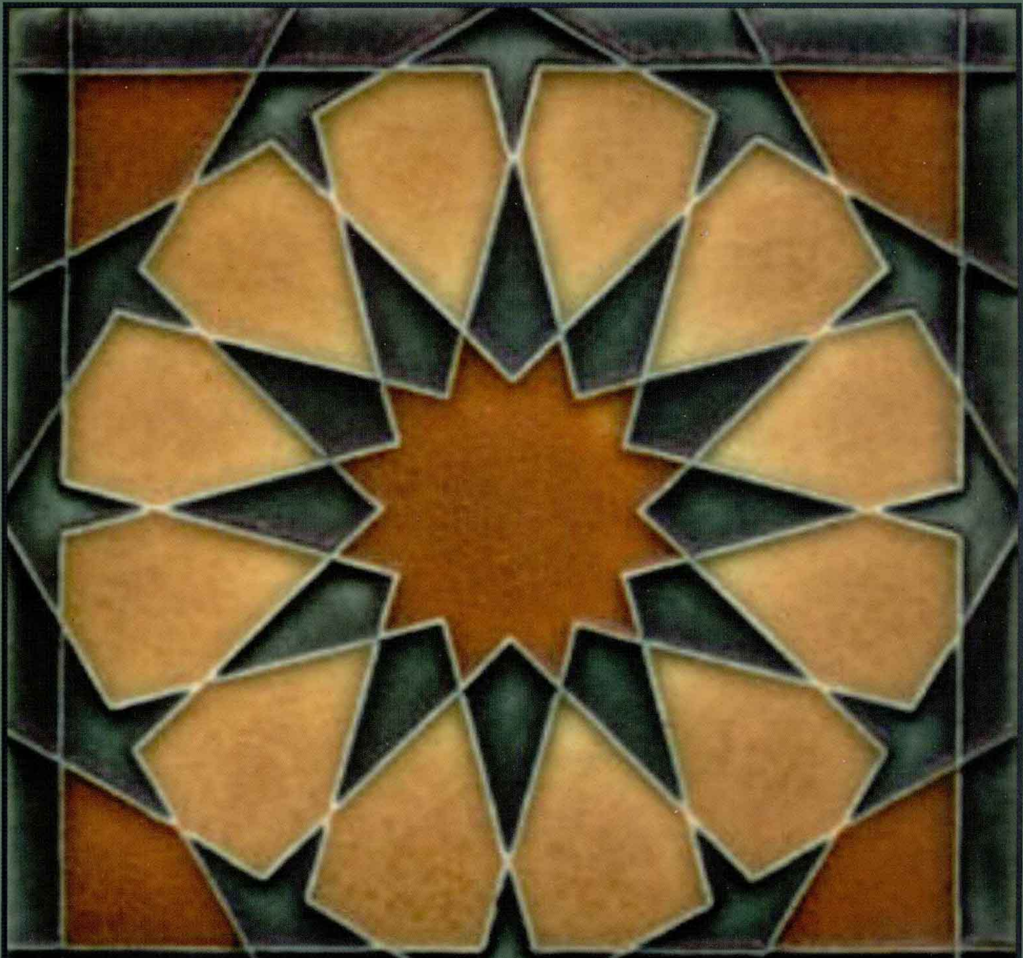
Introduction to Engineering Programming

Solving Problems with Algorithms



James Paul Holloway

INTRODUCTION TO ENGINEERING PROGRAMMING

Solving Problems with Algorithms

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For J. and P. J.

PREFACE

It is clear that the concept of the algorithm fully deserves its place among the supreme accomplishments of human thought. There, in its rightful place with such ideas as calculus and quantum mechanics, the algorithm can be celebrated for its continuing contribution to the advancement of humanity. Although the idea of the algorithm is old, the twentieth-century development of extremely fast electronic algorithm execution machines has catapulted algorithms into the center of our technological culture. The Internet, digital communications, video games, and physical simulation—all are founded on algorithms, and I believe that all educated women and men should be familiar with the basic ideas of algorithmic thought.

Engineers and scientists have long recognized the potential of algorithms to model the physical universe, and thus also to model and even control technological artifacts. With algorithms we can determine the details of the fluid flow about the bow of a ship, or the distribution of neutrons within a nuclear reactor. We can determine the stress on each beam in a bridge, and we can control the performance of a car engine. The algorithms to do such things can be developed without computers, but actually carrying out the detailed computations to execute these algorithms is mind-numbing and, in the end, far beyond human endurance. But the mid-twentieth century invention of electronic algorithm execution machines—now simply called computers—provided us with an escape from the trap of tedium, and then algorithms became an immensely powerful analysis tool.

All engineers will have to assess algorithms as part of their work, and many will actually develop them. But often these algorithms will be created inside special-purpose tools and expressed in special-purpose languages, ranging from the strange language of spreadsheets to the lisp extension language of computer-aided design tools; they might be written in the matrix-oriented languages of SciLab or Matlab, or in an object-oriented scripting language like Python. Often these algorithms will be designed to glue together other, already existing algorithmic tools, and so reforge those tools for a new purpose. Most of these algorithms will be intended to solve an immediate problem quickly and easily, and the tool will be selected with that impending priority in mind. But even here there is a need to create algorithms—to understand the fundamental notions of sequence, iteration, and selection, to understand the dynamic data transformations that algorithms carry out, to understand different ways of organizing data, and to have some sense of what algorithms can do, and what they can't. Because we rely on these programmable tools, we must have some understanding, more deep than superficial, of how they work and how they break.

Beyond this practical utility of getting a computer to solve a problem, learning to think about a problem algorithmically will give you more power over that problem. Even if you don't, in the end, solve a problem by writing a computer code, having an algorithmic perspective on it gives you another way to appreciate the issues involved.

Amazingly, the computer was designed to solve problems that its creators never envisioned and the computer will let you solve problems that no one else has ever even thought of. But causing a computer to carry out this feat requires creating algorithms and implementing them in a way that the computer can interpret. The implementing is comparatively trivial; the creating is very hard. But the ability to create algorithms is important, and the need to do so is inevitable: We have the power of a *programmable* execution machine at our fingertips. How can we choose not to use it?

THIS BOOK

I want you to develop a facility for algorithmic thought as one approach among many to engineering and scientific problems; I want you to think of problems in terms of the steps that can be scripted and then carried out to reach a solution. Once you can do that, the computer will take care of the boring part of actually performing those steps. This is therefore not a book *of* algorithms, but rather a book to make you think *about* algorithms.

This book is an introduction to the idea of the algorithm, and an introduction to creating them. It is aimed at beginning university students in engineering and the sciences. It is not particularly aimed at computer scientists. Although computer science students can certainly learn from this text, a computer science curriculum traditionally begins with a course on programming and builds on it through a whole series of courses that develop particular classical algorithms and data organization techniques. A computer science curriculum seldom focuses on calculus-based problems and basic physical mechanics.

This text, in contrast, is intended to support an engineering curriculum that contains only one first- or second-year course whose focus is primarily on algorithms and programming. Such a curriculum will build on this course by using simulation and computer-aided engineering design tools in later classes, classes whose primary focus is engineering and science rather than computing.

This text invites your exploration of ideas and provides a contemplative means to stimulate your thinking. Beyond this first introduction, you must commit to bringing algorithms to your other work, so that you can practice and build on the ideas introduced here. In our own curriculum at the University of Michigan, we have used this text to support a first-year course in algorithms and programming for engineers, and then built continuously on computing within later technical courses.

In order to think about algorithms, we must have problems to solve. First- and second-year students in engineering are concurrently taking calculus, physics, chemistry, and biology. These simultaneous studies provide us with a ready source of problems for which to build algorithmic treatments, so I will select from this fountain such problems as strike me both interesting and, perhaps with some effort, understandable to first-year university students. There are many wonderful algorithms that students of engineering and the sciences should learn, but this text is not the place to learn them all.

Throughout the text we will discuss alternatives. I will write many pieces of code more than once, and discuss the trade-offs and aesthetic issues involved in the various versions. You should similarly do so in your own creation of algorithms. I realize that you are reluctant to consider alternative ways of accomplishing a goal. After all, once something works, why look for another way, especially when you have a Russian exam tomorrow? But there is more to accomplishing a goal than simply obtaining something that works. We should consider aesthetics, flexibility, our certainty of correctness, ease of use, and robustness.

At the end of each section you will find questions; some of these are simple review questions, asking you to recall something of the recently read text. But some are intended to make you think about what you just read; indeed, some of these have no right answer, but ask you to consider or comment on choices. At the end of each chapter you will find projects, of varying length and complexity; most of these projects ask you to take up your keyboard and create codes. There is no better way to understand and appreciate algorithms than to write them, and execute them on a computer. And there is no better way to understand a problem than to develop an algorithm to solve it.

As you approach these end-of-chapter projects, you will note that often I don't lay out all the details; frequently the projects ask you to write functions or procedures, but do not specify an entire code. I want you to think about each assignment, formulate questions about it, and thereby take some intellectual ownership of it. I also want *you* to develop some means of testing your code. Further, I don't know how your instructor will prefer to work, so I try not to restrict her with details that may not be appropriate.[1]

THE EXPRESSION OF ALGORITHMS

When developing an algorithm, we must have some way to write it down. A few texts introduce only pseudocode, which is some means of algorithm expression intended only to be written and read by humans. The weakness of this approach is that it does not allow the algorithm to be tested on a real computer. Some other texts introduce pseudocode alongside a real programming language that can be used to control a real computer. I do this, but I use pseudocode sparingly, and use it less and less as the text progresses. My expectation is that you will become more proficient at simply reading code written in a real programming language.

Many practicing engineers create algorithms by first sketching out their ideas in a computer language, but they leave out details so as to concentrate on the big issues first. This is rather like making a rough draft of a story without worrying about the spelling or correctness of grammar, but focusing instead on the flow of plot and character. Despite this common practice, some textbook authors insist on scoping out all algorithms in pseudocode before writing them in a real language. I used to follow this practice, but after teaching algorithms and programming for a few years, I have come to see this "translation of pseudocode to real language" as counterproductive, especially for beginners. It is rather like requiring the rough draft in Latin, and then translating it into English to get the grammar right. Few students new to programming find their logic errors in pseudocode. They find their errors when testing their code. So I think it is

[1]In my own class I often give students a handout specifying the function and procedure interfaces quite precisely, because I often check assignments by linking their compiled code to my own test harness code. But other instructors will have their own preferences.

important for students to get their thoughts promptly into real code and executing on a computer. So I deemphasize pseudocode. My own observation is that students will naturally use this technique of code and test, no matter how much I might insist they do otherwise. Upon reflection, they are right to do so.

When developing algorithms, it is important to have a way to get those algorithms executed on that unimaginative, tyrannical hunk of doped silicon affectionately known as a computer's central processing unit. To do so, we must express our algorithms in a real programming language, not pseudocode. In this text I use C++. There are many reasons for this choice, not all of them objective. C++ is a good intermediate-level language, and is widely used. It is easy to find the software tools needed to use C++ on most any computer you may have. And it is easy to find, just down the hall, a member of the C++ literati whose brain you can pick at 1 A.M. in exchange for a slice of pizza and a coke.

The downside of C++ is that it can take years to fully know and understand the whole language. Fortunately, the language can be used, and used well, without knowing all there is to know about it. All the C++ that I will use in this book is presented in this book. But C++ is a huge language; it is fully, if incomprehensibly, described in ISO/IEC 14882, the 776-page tome that defines the language. It is also fully and somewhat more delicately described in the 1,040 pages of *The C++ Programming Language* by Bjarne Stroustrup, who is primarily responsible for creating the language. But I have no reason to create another 800-page book to bind between these covers. Indeed, to do so in the name of fully describing the language would be a confusing distraction from my primary aim. So I introduce only as much of the language as I need to describe the fundamental algorithmic and data organization ideas that I want to present.

ORGANIZATION OF THE TEXT

If you try to view this as simply a programming language text, and mistakenly compare it to others of that breed, you will note some unusual ordering of material. In Chapter 1, I introduce the idea of an algorithm as a set of steps that transform data from input to output, and I also give a whirlwind introduction to the organization of algorithms, and how a static description of an algorithm must control its later dynamic execution. The next three chapters cover the three pillars of algorithms: sequence (Chapter 2), iteration (Chapter 3), and selection (Chapter 4).

The most unusual feature here is the coverage of iteration before selection. Selection is often claimed to be simpler in concept than iteration, so conventional wisdom would mistakenly assert that I should discuss selection first. But to do so is to postpone the time when we can begin to take on really interesting problems and algorithms. Without iteration we can engage only in glorified formula evaluation exercises. My aim is to get you into the good stuff early, so you have time to consider it, to practice it, and to appreciate it.

Too much of the early engineering curriculum already presents a false picture of engineering as the practice of plugging numbers into formulas. Engineering is about creative design within the constraints imposed by nature, need, and society. Significant algorithms allow us to find creative solutions to engineering problems, but significant algorithms *always* exploit iteration.

Just as Chapters 2 through 4 describe the key concepts in organizing algorithms, Chapters 5 through 7 describe the organization of data. Part of Chapter 5 more fully describes the fundamental scalar data types of C++, a topic that would be explored earlier in a language-oriented text. But we really did not need that information earlier; earlier it would have been stuff to plow through only because we need it later, and it would not help us understand harder, more central concepts. Better, I think, to discuss such matters after you have the more difficult ideas of algorithmic organization fermenting in your thoughts.

The level of sophistication required to fully comprehend the examples and exercises varies. Some are quite straightforward, and some might greatly stretch your intellect. Example problems often involve discretization, time stepping of differential equations, solution of nonlinear equations, estimation of integrals, or the solution of a system of linear equations. We have taught all of this material in our first-year course at the University of Michigan, although we have never taught all of it in a single term. I expect your instructor will select material appropriate for your particular course.

Chapter 8 provides some introduction to the limitations of computers and of algorithms. Because algorithms must be executed on a finite computer with limited memory, there are limits to the accuracy and range of information that might be represented on the computer. Chapter 8 contains an extensive discussion of the representation of floating point numbers and floating point arithmetic. Although this material is easily understood with only an understanding of numbers and algebra, it does take some time to appreciate, and might easily be omitted from a course using the text. Chapter 8 also briefly discusses discretization and truncation errors, and estimating the time complexity of algorithms.

Two appendices provide a brief overview of some of the key C++ language constructs and library facilities used in the book. They also contain a few language constructs that were not used in the main body of the text. I encourage you to skim these appendices early on, and to then refer to them often. There are useful details to be found in them, but these are details that would be outside the main stream of the text, or else are scattered throughout the text yet gathered more conveniently together in the appendices. Think of the appendices not as optional supplements to your reading, but as critical material that needs to be read asynchronously.

ACKNOWLEDGMENTS

The choices made in creating this text are mine, but they have been influenced by conversations and a long association with many others. I must especially acknowledge the contributions of my fellow Engineering 101 instructors, especially Robert Beck, Alex Bielajew, and Kenneth G. Powell. I must also thank the many graduate student teaching assistants who have worked on the course, most especially Dan Osborne. Dan was born to teach, and has probably had more influence on this text than any other colleague.

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ALGORITHMS AND ENGINEERING

I think that algorithms are important. I think you should know how to make *your* computer solve *your* problems, rather than the problems that some distant programmer thinks you should solve. I also think algorithms are insanely fun to create. When we create an algorithm, we start with a problem to solve and travel through the whole process of engineering: We design a solution, we implement it, we test it, we refine it, and we seek to make it beautiful. I hope that you see this beauty while reading this text.

James Paul Holloway
Chelsea, Michigan
December 31, 2002

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