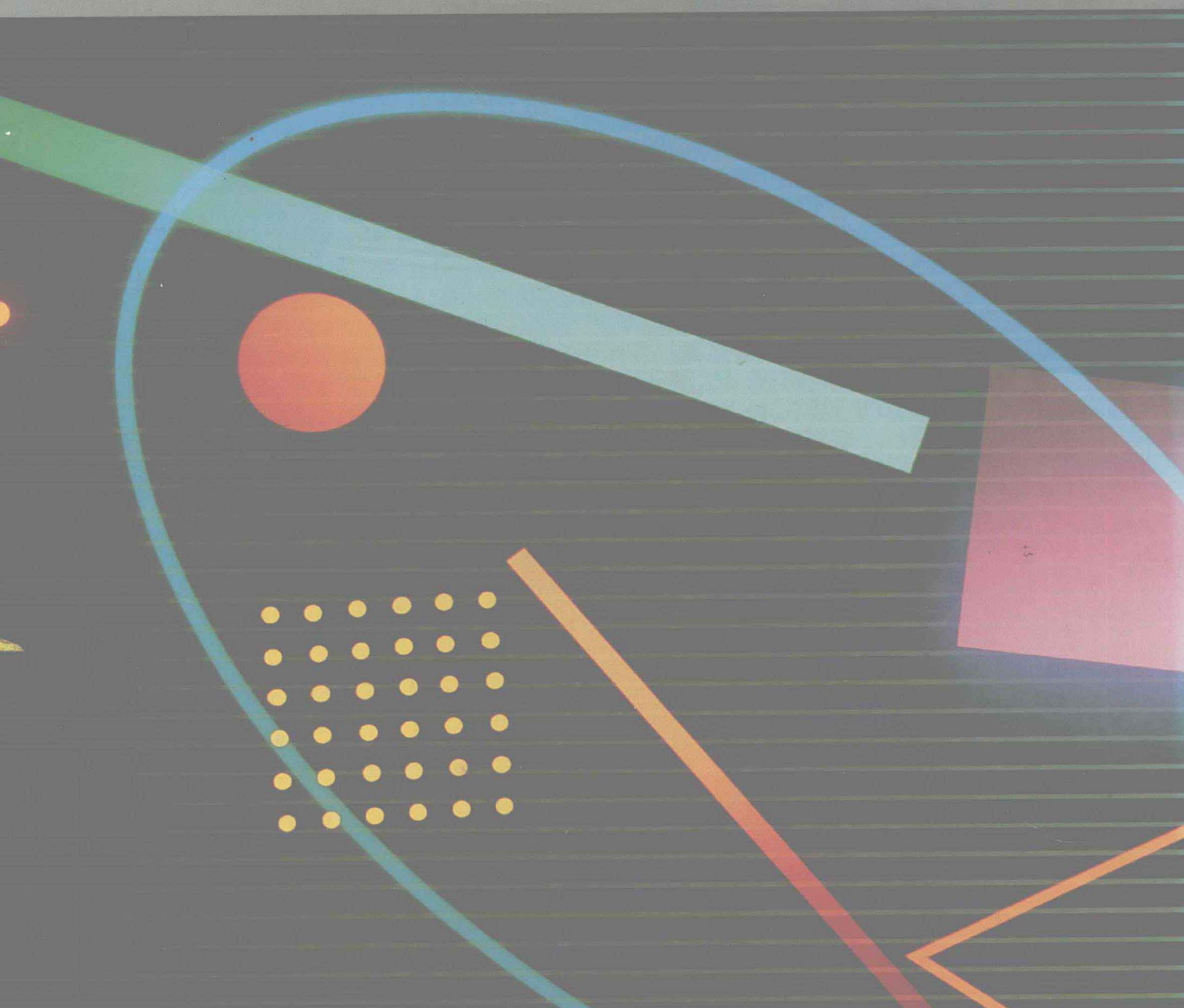
Fourth Edition

ALGEBRA FOR COLLEGE STUDENTS

Max A. Sobel / Norbert Lerner



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Library of Congress Cataloging-in-Publication Data

Sobel, Max A.
Algebra for college students / Max A. Sobel, Norbert Lerner. – 4th ed.
p. cm.
Includes index.
ISBN 0-13-025933-0
1. Algebra. I. Lerner, Norbert. II. Title.
QA154.2.S6 1991
512.9–dc20 90-40847
CIP

Editorial/production supervision: Rachel J. Witty, Letter Perfect, Inc.
Acquisitions editor: Priscilla McGeehon
Interior design: Meryl Poweski
Cover design: Meryl Poweski
Prepress buyer: Paula Massenaro
Manufacturing buyer: Lori Bulwin

A Division of Simon & Schuster
Englewood Cliffs, New Jersey 07632

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

ISBN 0-13-025933-0

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

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PREFACE

TO THE INSTRUCTOR

This fourth edition of *Algebra for College Students* has been written to provide beginning college students with the fundamental algebraic concepts and skills necessary for further study of mathematics. It should also assist students in understanding the applications of mathematics to other subjects.

Many sections have been rewritten to improve the exposition. This includes a variety of new applications and additional display examples. Most exercise sets have been enhanced with the introduction of additional questions of various degrees of difficulty.

Emphasis has been given throughout the text to the use of a scientific calculator, especially in the chapter on exponential and logarithmic functions. Calculator displays have been included, and exercises that should be solved by a calculator have been designated by the logo  , although the instructor still maintains the option of not allowing their use in class or on tests.

Many users of previous editions have contributed valuable suggestions that have been incorporated into this fourth edition. Please feel free to communicate with the authors and provide your input for future revisions. Some of the changes that have been incorporated into this fourth edition are included in the following summary:

Chapter 1. Section 1.5, Operations with Real Numbers, is new. This section extends the fundamental operations to include rational and irrational numbers. The calculator is used to obtain approximations when using irrational numbers.

Chapter 2. This chapter reviews the fundamental algebraic concepts and skills required for the remaining course of study. Negative exponents are developed in Section 2.2 (rather than in Chapter 5, as in the third edition). Scientific notation is now covered in this section. The work on Pascal's triangle has been extended, but has been placed in Chapter 12. However, this work can be extracted from Chapter 12 and covered earlier, if needed.

Chapter 3. The introductory work on quadratic equations is included as Section 3.6. This is a more natural placement than in the preceding editions, where it was located in the chapter on fundamental operations. More extensive work on quadratic equations is done in Chapter 5.

Chapter 4. Synthetic division has been moved into Chapter 8, preceding the work on the factor, remainder, and rational root theorems.

Chapter 5. This chapter contains the same subject matter as in the third edition. Also, the algebraic work dealing with the quadratic formula and applications of quadratic equations has been included here.

Chapter 6. This chapter is similar to Chapter 6 in the third edition.

Chapter 7. The work from Chapter 8 of the third edition on quadratic functions and inequalities is covered here. In addition a more extensive treatment of the conic section is included. Each conic section is covered in an individual section.

Chapter 8. This is essentially a new chapter. It includes sections on graphing polynomial, rational, and radical functions. The method of shifting (translating) fundamental curves to obtain graphs of more complicated functions is a unifying concept. The work on synthetic division and the factor, remainder and rational root theorems are also included.

Chapter 9. This chapter has been completely rewritten. After some introductory work on inverse functions, exponential functions of the form $y = b^x$ are introduced. Then the logarithmic functions are developed as the inverses of the exponential functions. The work with the laws of logarithms follows, as do applications of exponential growth and decay. Common logarithms are included as an optional section.

Chapter 10. The subject matter from Chapter 7 in the third edition on systems of linear equations is covered here. In addition, some work on systems of linear inequalities, followed by an introduction to linear programming, is included. There is also a section on solving nonlinear systems of equations.

Chapter 11. This chapter is similar to Chapter 10 in the third edition on Sequences and Series.

Chapter 12. This is a new chapter and introduces the student to permutations, combinations, and probability. A section on the binomial expansion is also included. The binomial formula is developed in two ways, one of which uses binomial coefficients. However, the work on this formula, without using the binomial coefficients, can be extraced and used earlier in the course, if needed.

It is important to note that this book was written with the expectation that students would read it, and not just refer to it as a source of exercises. Thus, we have worked carefully to make the exposition clear. To this end, we have used an informal, yet precise, approach in presenting basic mathematical concepts throughout.

You may find it helpful to note the following pedagogical features, which have been included in this edition to assist your students in learning the fundamentals of algebra. Some have been successfully used in past editions, and others have been added for this current edition, as noted.

Margin Notes

Throughout the text margin notes have been inserted to enhance the exposition, raise questions, point out interesting facts, show alternate procedures, give references and reminders, and caution the student to avoid errors.

Test Your Understanding

These short sets of exercises are found within most sections of the text and encourage students to *think carefully*. Students can use them to test their knowledge of new material, before attempting to solve the section exercises. Answers to all of these are given at the end of each chapter, providing an excellent means of self-study.

Caution Items

Where appropriate, caution items appear in the text or in the margin notes to alert students to typical errors that are to be avoided. Students can see both the correct and the incorrect solution to an exercise.

Illustrative Examples

The text contains more than 500 illustrative examples with detailed solutions. These appear in every section and allow students to see sample solutions prior to working on the end-of-section exercises. They can refer to these while working on the exercises, if they experience any difficulties. These sample solutions are especially helpful as review material, when students have been absent from class.

Challenges

This is a new feature. In each chapter, we have included challenging problems that encourage students to *think creatively*. These can be used to test the students' skill in solving a more difficult problem or one that has an unusual twist to it.

Explorations

A set of Exploration exercises in each chapter encourages students to *think creatively*. These demand higher-order thinking skills and do not depend solely on applying routine skills.

Written Assignments

This is a new feature. Throughout the book, written exercises are featured at the end of many of the standard sets of exercises. These generally ask for a written explanation or description rather than only an algebraic solution of an exercise. It is generally agreed that we should encourage practice in writing in mathematics courses, and it is hoped that this experience will help students with their other college courses as well.

Summary

This is a new feature. Each chapter contains a concise chapter review, which includes key terms and basic concepts and formulas.

Review Exercises

At the end of each chapter, there is a set of review exercises identical to some of the illustrative examples developed in the text. Students can use them as a self-study review of the chapter by comparing their answers with the worked-out solutions found in the body of the text.

Chapter Tests

Each chapter concludes with two forms of a chapter test, standard answer and multiple choice. Answers to both tests are provided at the end of the book. These enable students to test their knowledge of the work of the chapter in preparation for an instructor's test on the material.

Cumulative Reviews

This is a new feature. At the end of every three chapters cumulative review questions test the work of the course to date. More than any other subject, the study of mathematics is cumulative, and students need to be certain that they have not forgotten previously learned skills. The answers are included in the back of the book.

Inside Covers

Inside the front and back covers, summaries of useful information, including basic graphs and important algebraic and geometric formulas are presented for easy reference.

SUPPLEMENTARY MATERIALS

The following supplementary materials are available for this book.

1. *Instructor's Solution Manual* This manual provides the insructor with completely worked-out solutions for every even-numbered exercise in the text.
2. *Student Solutions Manual* This manual contains completely worked-out solutions for the odd-numbered exercises found at the end of each section, as well as for each of the chapter tests.
3. *Instructor's Manual with Tests* Includes testing material for classroom use.
4. *Interactive Algebra Tutor Software* Provides tutorial assistance and drill problems for students.

ACKNOWLEDGMENTS

The preparation of this fourth edition of *Algebra for College Students* has been guided by many people. First, we wish to thank the many students and instructors who used the first three editions and contributed their comments and suggestions.

For their detailed reviews, criticisms, and suggestions we thank

Martin Brown
Jefferson Community College, Louisville, KY

Laurene V. Fause
Florida Institute of Technology, Melbourne, FL

Roy Fraser
Skyline College, San Bruno, CA

Julie D. Horne
University of Georgia, Athens, GA

Merwin J. Lyng
Mayville State College, Mayville, ND

Thomas A. McCready
California State University, Chico, CA

Chantal Shafroth
North Carolina Central University, Durham, NC

Nancy Joe Shaw
Elizabethtown College, Elizabethtown, PA

Lee-Ing Tong
Southeastern Massachusetts University, North Dartmouth, MA

For her thorough and careful writing of the manuscript for the *Student Solutions Manual,* as well as for her many helpful comments and contributions, we thank Margaret Hall Babcock.

For their assistance in the preparation of supplementary materials, we also acknowledge:

Joe May
North Hennepin Community College, Brooklyn Park, MN

William Radulovich
Florida Community College

Deborah White
College of the Redwoods, Mendocino Coast, CA

For their valuable comments we are grateful to

Andrew N. Aheart
West Virginia State College, Institute, WV

Sonya M. Oetzel
Madison Area Technical College, Madison WI

Roger Riveland
Bismarck Jr. College, Bismarck, ND

For her outstanding editing, constant patience, and cooperation, we express our sincere appreciation to our production editor, Rachel J. Witty, Letter Perfect, Inc.

For providing us with guidance and direction, and the occasional prodding we needed throughout the development of the text, we thank our editor at Prentice Hall, Priscilla McGeehon.

For their patience and endurance throughout the many months that we devoted to manuscripts, galleys, page proofs, and endless telephone conversations, we offer our thanks to our wives Manya and Karin.

We hope that you and your students will enjoy using this book. We invite your reactions and welcome your suggestions.

Max A. Sobel
Norbert Lemer

SUGGESTIONS FOR THE STUDENT

We have prepared this book for you to read and to enjoy as well. Every effort has been made to make each topic meaningful, with clear exposition and numerous worked-out examples to serve as models for the exercises that you will attempt. Nevertheless, we recognize that many students enter this course with weak backgrounds, and often with a distaste for mathematics due to prior experiences.

We urge you to be patient, and suggest that if you are willing to devote the necessary time and effort to the course, then you should become successful. Based on the authors' many years of teaching experience, we would like to offer a short list of suggestions that should help to ensure this success:

1. Make every possible effort to keep up to date. Set aside regular time periods for the work in this course, and stick to your plan.
2. Be patient! Always try to reason things out on your own first, despite any difficulties that you may encounter. Even a modest effort along these lines will prove to be rewarding in the long run.
3. Read each section with paper and pencil at hand. Try to solve the illustrative examples before looking at the worked-out solutions provided.
4. Attempt each of the Test Your Understanding exercises, whether or not they are assigned. Reread the section if you have difficulty with any of these.
5. Try as many exercises as possible at the end of each section. Complete the odd-numbered ones first and check your answers with those given at the back of the book. Keep in mind that sometimes your answer may be correct, even though it is not in the same form as the given answer. If you miss very many, reread and study the section again.
6. Prior to a test, make use of the review exercises at the end of each chapter. You can check your results by referring to the designated section from which these are taken where you will find the completely worked-out solution for each one.
7. Complete each of the chapter tests under testing conditions. That is, do not refer to the text as you complete these, and set a fixed period of time for your work, usually an hour. The answers are given at the back of the book.
8. If convenient, find time to study and solve problems cooperatively with a classmate. Such efforts can be beneficial, as you explain ideas to one another.

We are convinced that even if you have had a negative attitude towards mathematics in the past, an honest attempt to learn it properly will not only result in greater success, but will also lead to a self-awareness that you have more ability and talent than you ever gave yourself credit for! Please feel free to let us know, if you have any comments, criticisms, or suggestions. We look forward to hearing from you, as you continue with your study of *Algebra for College Students*. Good luck!

Max A. Sobel
Norbert Lerner

CONTENTS

CHAPTER

1 THE SET OF REAL NUMBERS

1.1 SETS OF NUMBERS

Welcome to the *set* of students who will spend the semester using this text. The word **set** is an important one in mathematics and, as in its everyday usage, it means a collection of things. Many words are commonly used to denote sets:

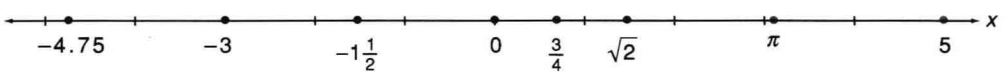
A *flock* of geese
A *swarm* of bees
A *squadron* of planes
A *team* of players
A *herd* of cattle

Can you find other such words that denote sets?

In mathematics we are often concerned with different *sets of numbers*. In most of your past work you have been dealing with the **set of real numbers.** Here are some examples of real numbers:

$$5 \quad -3 \quad 0 \quad \tfrac{3}{4} \quad \sqrt{2} \quad -4.75 \quad -1\tfrac{1}{2} \quad \pi$$

We can illustrate these numbers on a **number line,** where each number is the **coordinate** of some point on the line.



WATCH THE MARGINS! We will use these for special notes, explanations, challenges, and hints throughout the book.

Sometimes we will only need to use a **subset,** or part, of the real numbers. For example, the set N of **natural numbers** is a subset of the set of real numbers:

$$N = \{1, 2, 3, \ldots\}$$

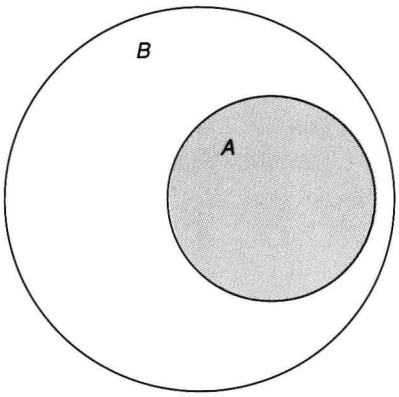
Every natural number is a real number, but not every real number is a natural number. Each of the numbers 1, 2, 3, . . . is said to be a **member** of or an **element** of set N.

According to this definition, every set is a subset of itself.

DEFINITION OF SUBSET

Set A is a **subset** of set B if every element of A is also an element of B. To state this in symbols, we write

$A \subseteq B$ Set A is a subset of set B.



$A \subseteq B$

The set of natural numbers is also referred to as the set of **counting numbers.** We name a set with a capital letter and include the members of the set within a pair of braces. The three dots are used here to indicate that the set goes on without end; that is, N is an example of an **infinite set.**

Note that 0 is *not* a member of the set of natural numbers. However, if we add the number 0 to the set of natural numbers, we produce another infinite set called the set of **whole numbers,** W:

$$W = \{0, 1, 2, 3, \ldots\}$$

The symbol $\not\subseteq$ is read "is not a subset of."

Every natural number is also a whole number, but not every whole number is a natural number. Thus $N \subseteq W$, but $W \not\subseteq N$ because 0 is an element in W that is not in N.

Both sets that we have just introduced are examples of infinite sets. However, some of the sets we will use will be **finite sets.** The members of a finite set can be listed and counted, and there is an end to this counting. For example, the set of whole numbers that are less than 5 is an example of a finite set:

$$\{0, 1, 2, 3, 4\}$$