



COLLEGE ALGEBRA AND TRIGONOMETRY

T H I R D E D I T I O N

JEROME E. KAUFMANN

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COLLEGE ALGEBRA AND TRIGONOMETRY

THIRD EDITION

Jerome E. Kaufmann



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PREFACE

College Algebra and Trigonometry, Third Edition is written for students who need a college algebra and trigonometry course to serve as a prerequisite for the standard calculus sequence or to satisfy a liberal arts requirement.

Four major ideas serve as this text's unifying themes—namely, solving equations and inequalities, solving problems, developing graphing techniques, and understanding the concept of a function.

College Algebra and Trigonometry, Third Edition presents basic concepts of algebra in a simple, straightforward way. A wealth of examples motivates students and reinforces algebraic concepts by the application of these examples to situations with which students can identify. The examples also guide students in organizing their work logically and using meaningful shortcuts whenever appropriate.

In the preparation of the third edition, special effort was made to incorporate improvements suggested by reviewers and by users of the earlier editions, while at the same time preserving the book's many successful features.

New in This Edition

- *Graphics Calculator Examples* incorporated throughout the text enable students to experience the power of this new technology in demonstrating how to visualize solutions to problems.
- *Graphics Calculator Activities*, a new category of problems, were added to give students practice with graphics calculators. About half of the problem sets contain these problems, a total of about 500 new exercises, which are flagged with a graphing utility icon. These exercises were designed to reinforce concepts already presented and lay the groundwork for concepts about to be discussed. They also help students to predict shapes and locations of graphs based on earlier graphing experience and to solve problems that are best expressed graphically with the aid of the graphics calculator. Through working these problems, students become more familiar with the capabilities and limitations of the graphics calculator. See, for example, Problem Sets 3.2, 4.2, 4.3, 5.1, 6.3, 8.1, 8.3, and 9.1.

- *Thoughts into Words*, another new category of problems, are designed to encourage students to express in written form their thoughts about various mathematical ideas. For example, see Problem Sets 1.2, 1.4, 2.1, 2.7, 6.1, 8.1, and 10.5.
- A new section in Chapter 4 introduces transformations (translations, reflections, stretchings, and shrinkings) of some basic curves. These ideas are then used throughout the text when various functions are graphed.
- A more up-to-date coverage of exponents and logarithms in Chapter 5 underscores the important role of the calculator; tables were placed in the appendixes.
- Problem sets were very carefully revised to achieve an even better balance in the level of difficulty.
- Many examples were enhanced or changed to better illustrate the concept under discussion.
- Chapter summaries were revised for maximum use by the student.

Other Special Features

- *Student “Self-Review”* in Chapter 1, a review of intermediate algebra concepts, was written so that students can work through this material with a minimum of assistance from the instructor.
- *Miscellaneous Problems*, an optional set of exercises in many of the problem sets, encompass a variety of ideas: Some are proofs, some exhibit different approaches to topics covered in the text, some bring in supplementary topics and relationships, and some are more challenging problems. Note that, although these problems add variety and flexibility to the exercise sets, they can be omitted entirely without disrupting the continuity of the text.
- *Problem Sets* were constructed on an even/odd basis; that is, all variations of skill-development exercises are contained in the even- and odd-numbered problems.
- *Review Problem Sets* appear at the end of each chapter and are designed to help students pull together all of the ideas presented in the chapter.
- *Cumulative Review Problem Sets* appear at the ends of Chapters 3, 6, and 9.

Additional Comments About Some of the Chapters

- With problem solving as its focus, Chapter 2 pulls together and expands on a variety of approaches to the process of solving equations and inequalities. Polya’s four-phase plan is used as a basis for developing different problem-solving strategies.
- Chapter 3 has as its premise the need for more work with concepts of coordinate geometry—specifically graphing techniques—*before* the notion

of a function is introduced. In this chapter, varying the coefficients of the equation $Ax^2 + By^2 = F$ results in various ellipses and hyperbolas (centers at the origin). In Chapter 12 the standard approach of developing basic forms from the definitions is used.

- The concept of a function is introduced in Chapter 4, unclouded by the need to jump back and forth between functions and relations that are not functions.
- Chapter 5 underscores the importance of understanding the concept of an exponent and a logarithm, along with relevant applications.
- The trigonometry material (Chapters 7, 8, and 9) is organized around three central themes: problem solving (in Chapter 7), graphing (in Chapter 8), and solving equations (in Chapter 9). After using angles to introduce the basic trigonometric functions in Chapter 7, we use functions to solve a variety of interesting problems involving right triangles and oblique triangles. The short introduction to vectors in Section 7.7 also focuses on solving problems. Chapter 8 presents variations of all six basic trigonometric curves in a carefully organized manner consistent with the graphing discussions in previous chapters. Section 8.5 introduces the polar coordinate system and establishes some groundwork for graphing polar equations in the next section. Section 8.6 is devoted to the graphing of polar equations and includes some work with symmetry tests. Chapter 9 is centered around solving trigonometric equations and verifying trigonometric identities.
- Chapters 10 and 11 provide the instructor with some flexibility as to choice of topics. Chapter 10 is devoted entirely to solving systems of linear equations, including the use of matrices and determinants. In Chapter 11 the algebra of matrices is the focal point.
- Problem solving is the unifying theme of Chapters 13 and 14. In contrast to many college algebra and trigonometry books, Chapter 14 contains a significant amount of material on probability.

Supplements for Instructors

The following supplements are available to adopters of this text:

- An *Instructor's Edition* of the textbook provides answers to all of the exercises.
- An *Instructor's Solutions Manual* offers detailed solutions to most of the text's exercises.
- *Test Bank with Chapter Tests* may be photocopied by instructors and used by them to test their students. Answers to test questions are provided at the back of the book for the instructor only. There are two multiple-choice and one short-answer test for each text chapter.
- *EXPTest*, a computerized test bank for IBM PCs and compatibles, contains hundreds of problems. Questions are multiple choice, true/false, and open-

THOUGHTS INTO WORDS

39. Discuss whether or not addition, subtraction, multiplication, and division of functions are commutative operations.

40. Explain why the composition of two functions is not a commutative operation.

MISCELLANEOUS PROBLEMS

41. If $f(x) = 3x - 4$ and $g(x) = ax + b$, find conditions on a and b that will guarantee that $f \circ g = g \circ f$.
42. If $f(x) = x^2$ and $g(x) = \sqrt{x}$, with both having as domain the set of nonnegative real numbers, then show that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.
43. If $f(x) = 3x^2 - 2x - 1$ and $g(x) = x$, find $f \circ g$ and $g \circ f$. (Recall that we have previously named $g(x) = x$ the identity function.)

44. In Section 4.1, we defined an *even function* to be a function such that $f(-x) = f(x)$ and an *odd function* to be one such that $f(-x) = -f(x)$. Verify that (a) the sum of two even functions is an even function, and (b) the sum of two odd functions is an odd function.



GRAPHICS CALCULATOR ACTIVITIES

45. For each of the following, (a) predict the general shape and location of the graph, and (b) use your calculator to graph the function to check your prediction. (Your knowledge of the graphs of the basic functions that are being added or subtracted should be helpful when making your predictions.)

- a. $f(x) = x^4 + x^2$ b. $f(x) = x^3 + x^2$
 c. $f(x) = x^4 - x^2$ d. $f(x) = x^2 - x^3$
 e. $f(x) = x^3 - x^2$ f. $f(x) = x^3 - x^2$
 g. $f(x) = |x| + \sqrt{x}$ h. $f(x) = |x| - \sqrt{x}$

46. For each of the following, find the graph of $y = (f \circ g)(x)$ and of $y = (g \circ f)(x)$.

- a. $f(x) = x^2$ and $g(x) = x + 5$
 b. $f(x) = x^3$ and $g(x) = x + 3$
 c. $f(x) = x - 6$ and $g(x) = -x^2$
 d. $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$
 e. $f(x) = \sqrt{x}$ and $g(x) = x^2 + 4$
 f. $f(x) = \sqrt[3]{x}$ and $g(x) = x^3 - 5$

4.6

INVERSE FUNCTIONS

Recall the *vertical line test*: If each vertical line intersects a graph in no more than one point, then the graph represents a function. There is also a useful distinction between two basic types of functions. Consider the graphs of the two functions in Figure 4.54: (a) $f(x) = 2x - 1$ and (b) $g(x) = x^2$. In part (a), any horizontal line will intersect the graph in no more than one point. Therefore, every value of $f(x)$ has only one value of x associated with it. Any function that has this property of

Thoughts into Words encourage students to express in written form their thoughts about various mathematical ideas.

Graphics Calculator Activities enable students with a graphics calculator to predict shapes and locations of graphs and solve problems that would be difficult or impossible to graph manually.

A new section in Chapter 4 now introduces transformations of some basic curves, so that these concepts can be developed later in the text.

y-axis Reflection

In Figure 4.44 the graph of $f(x) = -\sqrt{x}$ is obtained by reflecting the graph of $f(x) = \sqrt{x}$ through the x -axis. Reflections are sometimes referred to as *mirror images*. Thus, in Figure 4.44, if we think of the x -axis as a mirror, the graphs of $f(x) = \sqrt{x}$ and $f(x) = -\sqrt{x}$ are mirror images of each other.

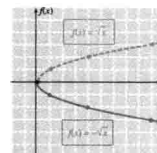


FIGURE 4.44

In Section 4.2 we did not consider a y -axis reflection of the basic parabola $f(x) = x^2$ because it is symmetric with respect to the y -axis. In other words, a y -axis reflection of $f(x) = x^2$ produces the same figure. However, at this time let's describe the general concept of a y -axis reflection.

y-axis Reflection

The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected through the y -axis.

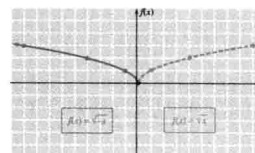


FIGURE 4.45

Miscellaneous Problems add variety to the exercise sets but can be omitted without disrupting the continuity of the text.

40. A motel advertises that they will provide dinner, dancing, and drinks for \$50 per couple for a New Year's Eve party. They must have a guarantee of 30 couples. Furthermore, they will agree that for each couple in excess of 30, they will reduce the price per couple for all attending by \$30. How many couples will it take to maximize the motel's revenue?
41. A cable TV company has 1000 subscribers who each pay \$15 per month. Based on a survey, they feel that for each

decrease of \$25 on the monthly rate, they could obtain 20 additional subscribers. At what rate will maximum revenue be obtained and how many subscribers will it take at that rate?

42. A manufacturer finds that for the first 500 units of his product that are produced and sold, the profit is \$50 per unit. The profit on each of the units beyond 500 is decreased by \$10 times the number of additional units sold. What level of output will maximize profit?

MISCELLANEOUS PROBLEMS

43. Suppose that an arch is shaped like a parabola. It is 20 feet wide at the base and 100 feet high. How wide is the arch 50 feet above the ground? (See Figure 4.37.)

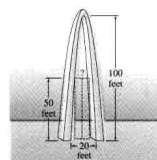


FIGURE 4.37

GRAPHICS CALCULATOR ACTIVITIES

46. Suppose that the viewing window on your graphics calculator is set so that $-15 \leq x \leq 15$ and $-10 \leq y \leq 10$. Now try to graph the function $f(x) = x^2 - 8x + 28$. Since nothing appears on the screen, the parabola must be outside the viewing window. We could arbitrarily expand the window until the parabola appears. However, let's be a little more systematic and use $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ to find the vertex; thus we find the vertex is at (4, 12). So let's change the y-values of the window

so that $0 \leq y \leq 25$. Now we get a good picture of the parabola.

Graph each of the following parabolas, and keep in mind that you may need to change the dimensions of the viewing window to obtain a good picture.

- a. $f(x) = x^2 - 2x + 12$
 b. $f(x) = -x^2 - 4x - 16$
 c. $f(x) = x^2 + 12x + 44$

Now suppose that we want to do a y-axis reflection of $f(x) = \sqrt{x}$. Since $f(x) = \sqrt{x}$ is defined for $x \geq 0$, the y-axis reflection $f(x) = \sqrt{-x}$ is defined for $-x \geq 0$, which is equivalent to $x \leq 0$. Figure 4.45 shows the y-axis reflection of $f(x) = \sqrt{x}$.

Vertical Stretching and Shrinking

Translations and reflections are called rigid transformations because the basic shape of the curve being transformed is not changed. In other words, only the positions of the graphs are changed. Now we want to consider some transformations that distort the shape of the original figure somewhat.

In Section 4.2 we graphed the equation $y = 2x^2$ by doubling the y-coordinates of the ordered pairs that satisfy the equation $y = x^2$. We obtained a parabola with its vertex at the origin, symmetric to the y-axis, but *narrower* than the basic parabola. Likewise, we graphed the equation $y = \frac{1}{2}x^2$ by halving the y-coordinates of the ordered pairs that satisfy $y = x^2$. In this case, we obtained a parabola with its vertex at the origin, symmetric to the y-axis, but *wider* than the basic parabola.

The concepts of *narrower* and *wider* can be used to describe parabolas but cannot be used to accurately describe some other curves. Instead, we use the more general concepts of vertical stretching and shrinking.

Vertical Stretching and Shrinking

The graph of $y = cf(x)$ is obtained from the graph of $y = f(x)$ by multiplying the y-coordinates of $y = f(x)$ by c . If $c > 1$, the graph is said to be *stretched* by a factor of c ; and if $0 < c < 1$, the graph is said to be *shrunk* by a factor of c .

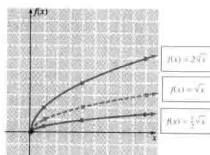


FIGURE 4.46

Vertical Stretching and Shrinking

ended. Instructors can interact with the program by adding to existing questions and producing individual tests. A demonstration disk is available.

- *ExamBuilder*, a computerized test bank for the Macintosh, has features and questions similar to those of EXPTTest. A demonstration disk is available.
- *Transparencies*: 33 full-color acetates provide enlarged versions of illustrations similar to those found in the text.
- *A Quick Reference Card*: Packaged with this edition of the text is a new problem-solving tool—a formula card. This perforated card, found in the back of the book, will help students to master key formulas, equations, and graphs in the course. By serving as a quick reference and minimizing the need for page turning, the formula card reduces the time spent on tedious tasks so that students can focus on the central concepts and principles of the course.

Supplements for Students

- A *Students' Solutions Manual* provides detailed solutions to about one-fourth of the text's exercises.
- *Investigate*, tutorial software for the Macintosh and IBM PCs and compatibles, helps students review college algebra as needed. Questions are presented to the students with full mathematical notation and graphs. When students give incorrect answers, they are stepped through explanations of the problems to give immediate feedback and to correct misunderstandings. The program comes with a pop-up calculator, which instructors may disable at their discretion. *Investigate* may be set up to record individual students' grades and is fully network compatible. Operation of both the MS-DOS and Macintosh versions is identical, allowing easy training and use in labs having both types of computers. Demonstration disks are available for instructors.
- *College Algebra and Trigonometry Videotapes* are available to qualified adopters. Through the departmental or college audiovisual library, students can check out these videos and use them to review material when they need additional help.
- *Precalculus in Context: Functioning in the Real World* by Davis, Moran, and Murphy is a lab manual consisting of twelve projects that encourages students to explore precalculus concepts. Graphics calculators or computer graphing software are required to solve each experiment and its corresponding exercises.
- *Student Edition of Theorist* is software for the Macintosh that combines powerful algebra and graphics capabilities with an intuitive, user-friendly interface. Once they purchase it, students will be able to make use of this software for this course as well as for future mathematics courses.

- *College Algebra and Trigonometry Explorations for the TI-81 and TI-82 Graphics Calculators* by Nancy Hyde, Broward Community College, is designed to complement the Kaufmann precalculus series of texts. The purpose of these explorations is to guide the student into discovering or reinforcing important mathematical concepts through visualization, computation, and programming. This book provides (1) an introduction to the TI-81 and TI-82 graphics calculators; (2) examples to be used in the classroom, in small groups, or by the student working independently; and (3) corresponding exercises. Each college algebra or trigonometry activity in the book includes a brief explanation, relevant graphics calculator techniques, and examples that show the graphical interpretation of the concept. The author demonstrates how these concepts can be explored graphically or how the graphical representation reinforces a concept.

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SOME BASIC CONCEPTS OF ALGEBRA

1.1 Some Basic Ideas

1.2 Exponents

1.3 Polynomials

1.4 Factoring Polynomials

1.5 Rational Expressions

1.6 Radicals

1.7 Relationship Between Exponents and Roots

1.8 Complex Numbers

A

lgebra is often described as generalized arithmetic. That description may not tell the whole story, but it does indicate an important idea, namely, that a good understanding of arithmetic provides a sound basis for the study of algebra. Furthermore, a good understanding of some basic algebra concepts provides an even better basis for the study of more advanced algebraic ideas. Be sure that you can work effectively with the algebraic concepts reviewed in this first chapter.

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