MONOGRAPHS IN ELECTRICAL AND ELECTRONIC ENGINEERING No. 21

# Brushless Permanent-Magnet and Reluctance Motor Drives

T. J. E. MILLER



**OXFORD SCIENCE PUBLICATIONS** 

# Brushless Permanent-Magnet and Reluctance Motor Drives

T. J. E. Miller

GEC Titular Professor in Power Electronics University of Glasgow Oxford University Press, Walton Street, Oxford OX2 6DP

Oxford New York Toronto
Delhi Bombay Calcutta Madras Karachi
Petaling Jaya Singapore Hong Kong Tokyo
Nairobi Dar es Salaam Cape Town
Melbourne Auckland

and associated companies in Berlin Ibadan

Oxford is a trade mark of Oxford University Press

Published in the United States by Oxford University Press, New York

©T. J. E. Miller, 1989

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior permission of Oxford University Press

British Library Cataloguing in Publication Data Miller, T. J. E. (Timothy John Eastham), 1947— Brushless permanent-magnet and reluctance motor drives.

> 1. Direct current electric motors I. Title II. Series 621.46'2 ISBN 0-19-859369-4

Library of Congress Cataloging in Publication Data Miller, T. J. E. (Timothy John Eastham), 1947— Brushless permanent-magnet and reluctance motor drives.

(Monographs in electrical and electronic engineering; 21)
Bibliography: p. Includes index.

1. Electric motors, Direct current. 2. Electric motors, Brushless. 3. Reluctance motors. I. Title.

II. Series TK2681.M55 1989 621.46'2 88-23173 ISBN 0-19-859369-4

Typeset by Cotswold Typesetting Ltd, Gloucester Printed in Great Britain by Butler and Tanner Ltd, Frome, Somerset

## Monographs in Electrical and Electronic Engineering 21

Series editors: P. Hammond and R. L. Grimsdale

### Monographs in Electrical and Electronic Engineering

- 10. The theory of linear induction machinery (1980) Michel Poloujadoff
- 12. Energy methods in electromagnetism (1981) P. Hammond
- 13. Low-noise electrical motors (1981) S. J. Yang
- 15. Superconducting rotating electrical machines (1983) J. R. Bumby
- 16. Stepping motors and their microprocessor controls (1984) T. Kenjo
- 17. Machinery noise measurement (1985) S. J. Yang and A. J. Ellison
- 18. Permanent-magnet and brushless DC motors (1985) T. Kenjo and S. Nagamori
- 19. Metal-semiconductor contacts. Second edition (1988) E. H. Rhoderick and R. H. Williams
- 20. Introduction to power electronics (1988) Eiichi Ohno
- 21. Brushless permanent-magnet and reluctance motor drives (1989)
  T. J. E. Miller

### **Preface**

The impulse to write this book was most recently inspired by the publication of Professor Kenjo's books in the same series, the idea being to extend the coverage and provide more detail on synchronous brushless motors and the switched reluctance motor. However, the basic idea of a book in this area goes back several years to a period of particularly interesting developments under the Motor Technology Program at the Corporate Research and Development Center of General Electric in Schenectady, New York. This programme was coupled with exciting developments in semiconductors and power electronics (Baliga 1987), as well as with rapid changes in the technology of motor drives originating in all parts of the developed world. While I was privileged to participate in this programme I also had the benefit of having worked under Professor Peter Lawrenson at Leeds University. The pressures of business prevented any writing until I accepted my present post at Glasgow University, which is supported by GEC, UK. The Scottish Power Electronics and Electric Drives Consortium (SPEED), established in 1986 and modelled on the Wisconsin Electric Machines and Power Electronics Consortium, has provided an environment for further analysis and experimentation, as well as new results and perspectives, and an appreciation of the need for a text in this area. In writing it, I claim no credit for the original inventions or for anything more than a small part in their subsequent development; the book is merely intended as a reasonably organized account of the fundamental principles.

It is hoped that this presentation of the theory of operation of brushless d.c. drives will help engineers to appreciate their potential and apply them more widely, taking advantage of remarkable recent developments in permanent-magnet materials, power semiconductors, electronic control, and motor design (including CAD). The objective is not to 'sell' particular technologies or teach design, but to lay out the basic principles, and it is hoped that this will raise the general credibility and acceptance of new technology that many engineers have striven to establish. It is also hoped that the sections on permanent magnets and magnetic circuits will assist in the exploitation of new PM materials with outstanding properties greatly improved from those of only a few years ago.

It is humbling to realize how much scope for innovation remains in the field of motors and drives, even a century and a half after Faraday. Yet in the academic world the subject of motor design and power engineering more generally has fallen into such decline that the demand for power engineers exceeds the supply, and motor designers are scarce. Some of the present material was developed for courses at Drives, Motors, and Controls Conference and at the University of Wisconsin, and this book is addressed to

vi PREFACE

some of industry's educational needs. Examples and problems are included, many of which were developed as tutorial material for (and by!) students at Glasgow University.

The approach taken is essentially academic: theory and calculation predominate, and the really difficult questions of comparisons between different drives, and the design of particular ones, are treated only lightly. It is hoped, however, that most of the basic theory of modern brushless drives will be found here. The treatment of magnetic saturation is given less attention than in classical works on electric motors: in the design of brushless motors, it is important to grasp the first principles, which can be understood, in the main, from linear theory. The widespread availability of finite-element analysis, and its ever-improving capability, make the problems of saturation much more tractable and relieve the need for a more complex analytical approach, which would be exceedingly complex before it could be really useful.

If nothing else, a study of brushless motor drives will lead to a further appreciation of the extraordinary properties of conventional motors, particularly the d.c. commutator motor and the a.c. induction motor, and will throw a little light on the achievements of our forebears. The arrival of silicon power electronics has reopened all the fundamental questions, and added a new dimension to the equation that has for so long been dominated by copper and iron

Glasgow April 1988 T. J. E. M.

## Acknowledgements

Many engineers have contributed to this book through the lessons they have taught me. Most of their original work is in print and referenced throughout the book, but particular acknowledgement is made to those with whom I have worked, including Dr Eike Richter and Dr Edward P. Cornell of GE, together with many others, of whom I would particularly like to mention G. B. Kliman, T. M. Jahns, T. W. Neumann, D. M. Erdman, H. B. Harms, F. L. Forbes, and V. B. Honsinger. Also to be particularly recognized is the work of Professor P. J. Lawrenson and Dr Michael Stephenson and their colleagues at the University of Leeds, where much of the European work on reluctance machines (both synchronous and switched) originated; and of Professor M. R. Harris of the University of Newcastle upon Tyne. Several new ideas and experimental results were contributed by Peter Bower under the Glasgow University SPEED Programme, and acknowledgement is made to Anderson Strathclyde plc, Emerson Electric, Lucas, National Semiconductor, Pacific Scientific, Simmonds Precision, Hoover, Reliance Electric, Smith and Nephew, and GE under this programme. Acknowledgement is also made to Professor J. Lamb and the University Court of the University of Glasgow, and to Professor P. Hammond for his encouragement.

# To my family and friends

# Glossary of symbols

a	no. of parallel		d	duty-cycle (of	
	paths in winding			p.w.m.)	
$\boldsymbol{A}$	area	m <sup>2</sup>	D	stator bore	
$\boldsymbol{A}$	electric loading	A/m		diameter	m
$A_{m}$	magnet pole				
	area	m <sup>2</sup>	$\delta$	phase angle	
α	angular		U	defined on	
	acceleration	rad/s <sup>2</sup>		p. 136	deg or rad
α	phase angle	deg or rad		p. 150	deg of fau
α	overlap angle	deg or rad			
α	pole arc/pole-		e	instantaneous	
	pitch ratio			e.m.f.	V
			$e_{0}$	per-unit value of	
				$E_{0}$	p.u.
β	torque angle	deg or rad	$\boldsymbol{E}$	r.m.s. e.m.f.	p.u.
β	pole arc	deg or rad	$E_{ m dw}$	e.m.f. ascribed to	
$\boldsymbol{B}$	flux density	T		web flux	$V_{r.m.s.}$
$\boldsymbol{B}$	magnetic		$E_{q}$	open-circuit	
	loading (p. 22)	T	4	e.m.f. due to	
$B_{ m g}$	airgap flux-			magnet	$V_{r.m.s.}$
	density (radial)	T	$E_{0}$	value of $E$ or $E_{a}$	1.111.5.
$B_{\rm r}$	remanent flux-		O	at corner-point	
•	density	T		or base	
$B_{\rm s}$	saturation flux-			frequency	$V_{r,m,s}$
	density	T		1	r.m.s.
				1 11	
			$\epsilon$	chording angle	deg or rad
C	commutation		€	step angle	deg or rad
$C_{\mathbf{\Phi}}$	flux			•	
	concentration		$f_{\underline{}}$	frequency	Hz
	factor		F	magnetomotive	
				force (m.m.f.)	A t
01	alat nitah	alaa daa aa	F	mechanical force	N
γ	slot pitch	elec deg or	$F_{m}$	m.m.f. across	
	mboso amala	rad		magnet	A t
γ	phase angle				
	defined on	1	1	<b>0</b>	****
	p. 102	deg or rad	$\phi$	flux	Wb
γ	phase angle		$\phi$	power factor	
	defined on			angle	deg or rad
	p. 136	deg or rad	Φ	flux	Wb
γ	fraction defined		$\Phi_{\mathrm{g}}$	airgap flux (per	
	on p. 113			pole)	Wb

$\Phi_{M1}$	fundamental		k	armature	
	magnet flux per			constant (p. 66)	
	pole	Wb	k	frequency ratio	
$\Phi_{r}$	remanent flux of			defined on	
	magnet	Wb		p. 107	
$\Phi_{y}$	flux in one link	Wb	k	coupling	
				coefficient	
g	airgap length	m	$k_{w1}$	fundamental	
g'	effective airgap			harmonic	
	length $K_c g$	m		winding factor	
g''	effective airgap		$k_{d1}$	fundamental	
	length allowing			distribution	
	for magnet	m		factor	
$g_{\mathtt{d}}''$	effective airgap		$k_{p1}$	fundamental	
• •	length in direct-			pitch (chording)	
	axis, allowing			factor	
	for magnet	m	$k_{\rm s1}$	fundamental	
$g_{q}''$	effective airgap			skew factor	
- 1	length in		$k_{\mathrm{w}}$	winding factor	
	quadrature-axis	m		for inductance	
			$k_{_{f lpha d}}$	defined on	
Н	magnetizing			p. 129	
	force or		$k_{1ad}$	defined on	
	magnetic field			p. 130	
	strength	A t/m	$K_{\rm c}$	Carter's	
$H_{\rm c}$	coercive force	A t/m		coefficient (p. 60)	
$H_{ci}$	intrinsic	,			
Ci	coercivity	A t/m	$l_{\mathbf{k}}$	effective length	
				of core or keeper	m
i	current		$l_{\rm m}$	magnet length	
•	(instantaneous)	Α		(in dir'n of	
I	r.m.s. current or	* *:		magnetization)	m
•	d.c. current	$A_{r.m.s.}$	L	inductance	H
$I_{\rm c}$	controller	^ r.m.s.	$L_{\rm a}$	aligned	
-c	maximum			inductance	H
	current	$A_{r.m.s.}$	$L_{ m u}$	unaligned	
		· ~r.m.s.		inductance	H
J	current density	A/m <sup>2</sup>	l	stack length	m
J	magnetization	T			
J	017-0	1	λ	inductance ratio	
J	polar moment of inertia	ka m²			
$J_{ m m}$	motor inertia	kg m² kg m²	M	mutual	
_	load inertia	kg m <sup>2</sup>	111	inductance	Н
$J_{\mathtt{L}}$	ioau ilicitia	Kg III		madetance	11
k	peak/continuous		μ	permeability	H/m
	torque ratio		$\mu_{r}$	relative	11/111
	(p. 27)		r <sub>r</sub>	permeability	
	*			Permenonity	

$\mu_{ m rec}$	relative recoil permeability		$\theta$	angular coordinate; rotor position	deg or rad
n	gear ratio		$\theta_{\mathbf{D}}$	dwell angle;	
n	speed	rev/sec		conduction	
n	no. of phases or			angle (of main	
	phaselegs (Ch. 7)			switches)	deg or rad
N	speed	r.p.m.			
N	no. of turns		$r_0$	rotor slot-	
$N_{\rm p}$	no. of turns per			bottom radius	m
**	pole		$r_1$	rotor outside	
$N_{ m ph}$	no. of series			radius (Ch. 7)	m
3.7	turns per phase		$r_1$	stator bore	
$N_{\rm r}$	no. of rotor			radius (Ch. 4–6)	m
A.T	poles		$r_2$	stator slot-	
$N_{\rm s}$	no. of stator			bottom radius	m
λI	poles no. of series		$r_3$	stator outside radius	<b></b>
$N_{\rm s}$	turns per phase		R	resistance (of	m
	of sine-		Λ	phase winding)	Ohm
	distributed		R	reluctance	A t/Wb
	winding or		$R_{\rm g}$	airgap	A t/ W b
	equivalent sine-		Λ' <sub>g</sub>	reluctance	A t/Wb
	distributed			retuctance	11 0/110
	winding		S	split ratio (p. 23)	
$N_1$	no. of turns in		$\sigma$	airgap shear	kN/m2 or
•	coil 1, etc.			stress	p.s.i.
			$\sigma$	half the skew	mech deg
p	no. of pole pairs			angle	or rad
P	permeance	Wb/A t		-	
$P_{\mathbf{m}0}$	magnet		t	time	S
	permeance	Wb/A t	T	temperature	deg C
$P_{m}$	magnet		T	torque	Nm
	permeance		$T_{\rm a}$	average torque	Nm
	including rotor		TRV	torque per rotor	
	leakage	*****		volume	$Nm/m^3$
	permeance	Wb/A t			
$P_{\rm r1}$	rotor leakage	XX71 / A .	и	magnetic	
n	permeance	Wb/A t		potential	A t
P	power	W		P	
PC	permeance coefficient		v	linear velocity	m/s
	coemcient		v	voltage	V
a	no. of slots per		V	(instantaneous)	v
q	pole per phase		r	voltage (d.c. or r.m.s. d.c.)	V
	(Ch. 5)		$V_{\rm c}$	controller max.	<b>y</b>
q	no. of phases		c	voltage	V
4	(Ch. 7)		$V_{\rm c}$	copper volume	V <sub>r.m.s.</sub> m <sup>3</sup>
	( · )		c	copper volume	***

$V_{\rm m}$ $V_{\rm r}$ $W$ $W_{\rm m}$ $W$ $W$	magnet volume rotor volume web width magnet width energy conversion	m <sup>3</sup> m <sup>3</sup> m J	$\omega_{ extsf{b}}$	'base' or 'cornerpoint' speed rad/s mech. angular velocity $\omega/p$ mech (Ch. 5, 6) rad/s
$W'$ $W_{ m f}$	energy per stroke coenergy stored field energy	J J	a a	armature-reaction aligned
X X X,	per-unit reactance reactance synchronous	p.u. Ohm	a a, b, d e g L	average c phases a, b, c direct-axis electrical, electromagnetic airgap; airgap flux load
$X_{s0}$ $X_{d}$	reactance value of $X_s$ at corner-point direct-axis synchronous	Ohm Ohm	m m, M mc mp	mechanical  M magnet  maximum continuous  peak rated
$X_{q}$	reactance quadrature-axis synchronous reactance	Ohm	ph q r s	phase quadrature rotor stator saturated
$X_{\sigma}$	armature leakage reactance link width	Ohm m	u w 0 1	unaligned winding factor corner-point or 'base' value fundamental harmonic
y V	flux-linkage (in Ch. 7, of phase winding)	V s or Wb t	1, 2 1, 2,	component pertaining to coils 1, 2 (Ch. 4) 3 phases
ω	elec. angular velocity $= 2\pi f$ (in	elec rad/s	Supe	erscripts
ω	Ch. 5-6) mech. angular velocity (Ch. 7)	rad/s mech rad/s	u ~	unsaturated phasor (complex quantity) peak (of sinewave)

# Contents

G	LOSSARY OF SYMBOLS	xiii
1	INTRODUCTION	1
	1.1 Motion control systems	1
	1.2 Why adjustable speed?	1
	1.2.1 Large versus small drives	4
	1.3 Structure of drive systems	5
	1.4 New technology	7
	1.4.1 Digital electronics	7
	1.4.2 Power integrated circuits	8
	1.4.3 Power semiconductor devices	9
	1.4.4 New magnetic materials	9
	1.4.5 CAD and numerical analysis in design	9
	1.4.6 Other contributing technologies	10
	1.5 Which motor?	11
	1.5.1 Evolution of motors	11
	1.5.2 The d.c. commutator motor	13
	1.5.3 The PM d.c. commutator motor	13
	1.5.4 The induction motor drive	16
	1.5.5 The brushless d.c. PM motor	17
	1.5.6 The brushless PM a.c. synchronous motor	17
2	PRINCIPLES OF SIZING, GEARING, AND	20
	TORQUE PRODUCTION	
	2.1 Sizing an electric motor	20
	2.1.1 Airgap shear stress	20
	2.1.2 Torque per unit stator volume	23
	2.2 Choice of gear ratio in geared drives	24
	2.2.1 Simple acceleration of pure inertia load	25
	2.2.2 Acceleration of inertia with fixed load torque	26
	2.2.3 Peak/continuous torque ratio of motor	26
	2.2.4 General speed and position profiles	27

X CONTENTS

	2.3		Production of smooth torque	28	
	Pro	blems fo	r Chapter 2	32	
3		RMAN RCUIT	ENT-MAGNET MATERIALS AND S	34	
	3.1	Perman	ent-magnet materials and characteristics	34	
	3.2	B-H loo	p and demagnetization characteristics	35	
	3.3	3.3.1 3.3.2	ature effects: reversible and irreversible losses High-temperature effects Reversible losses Irreversible losses recoverable by remagnetization	41 41 41 43	
	3.4	Mechan	ical properties, handling, and magnetization	44	
٢	3.5	3.5.1 3.5.2	tion of permanent magnets in motors  Power density  Operating temperature range  Severity of operational duty	46 46 47 47	
	Pro	blems fo	r Chapter 3	50	
4			WAVE PERMANENT-MAGNET BRUSH- TOR DRIVES	54	
	4.1	Why br	ushless d.c.?	54	
	4.2	Magnet	ic circuit analysis on open-circuit	58	
	4.3	Squarev	vave brushless motor: torque and e.m.f. equations	63	
	4.4	Torque/	speed characteristic: performance and efficiency	66	
	4.5	Alternat	tive formulations for torque and e.m.f.	68	
	4.6	Motors	with 120° and 180° magnet arcs: commutation	70	
	4.7	Squarew reaction	vave motor: winding inductances and armature	76	
	4.8	Control	lers	80	
	4.9	Comput	er simulation	83	
Problems for Chapter 4					

	CONTENTS	xi
5	SINEWAVE PERMANENT-MAGNET BRUSHLESS MOTOR DRIVES	88
	<ul> <li>5.1 Ideal sinewave motor: torque, e.m.f., and reactance</li> <li>5.1.1 Torque</li> <li>5.1.2 E.m.f.</li> <li>5.1.3 Inductance of phase winding</li> <li>5.1.4 Synchronous reactance</li> </ul>	89 89 92 94 96
	5.2 Sinewave motor with practical windings	96
	5.3 Phasor diagram	100
	5.4 Sinewave motor: circle diagram and torque/speed characteristic	103
	5.5 Torque per ampere and kVA/kW of squarewave and sinewave motors	109
	5.6 Permanent magnet versus electromagnetic excitation	112
	5.7 Slotless motors	115
	5.8 Ripple torque in sinewave motors	116
	Problems for Chapter 5	117
6	ALTERNATING-CURRENT DRIVES WITH PM AND SYNCHRONOUS-RELUCTANCE HYBRID MOTORS	118
	6.1 Rotors	118
	<ul> <li>6.2 A.c. windings and inductances</li> <li>6.2.1 Open-circuit e.m.f.</li> <li>6.2.2 Synchronous reactance (d-axis)</li> <li>6.2.3 Synchronous reactance (q-axis)</li> <li>6.2.4 Magnet flux-density and operating point</li> </ul>	122 122 128 131 134
	<ul><li>6.3 Steady-state phasor diagram</li><li>6.3.1 Converter volt-ampere requirements</li></ul>	135 144
	6.4 Circle diagram and torque/speed characteristic	145
	6.5 Cage-type motors	147
	Problems for Chapter 6	148
7	SWITCHED RELUCTANCE DRIVES	149
	7.1. The switched reluctance motor	149

7.2 Poles, phases, and windings	156
<ul><li>7.3 Static torque production</li><li>7.3.1 Energy conversion loop</li></ul>	158 164
7.4 Partition of energy and the effects of saturation	168
7.5 Dynamic torque production	172
7.6 Converter circuits	173
<ul><li>7.7 Control: current regulation, commutation</li><li>7.7.1 Torque/speed characteristic</li><li>7.7.2 Shaft position sensing</li></ul>	180 183 188
7.8 Solid rotors	188
Problems for Chapter 7	190
REFERENCES AND FURTHER READING	192
ANSWERS TO THE PROBLEMS	200
INDEX	202