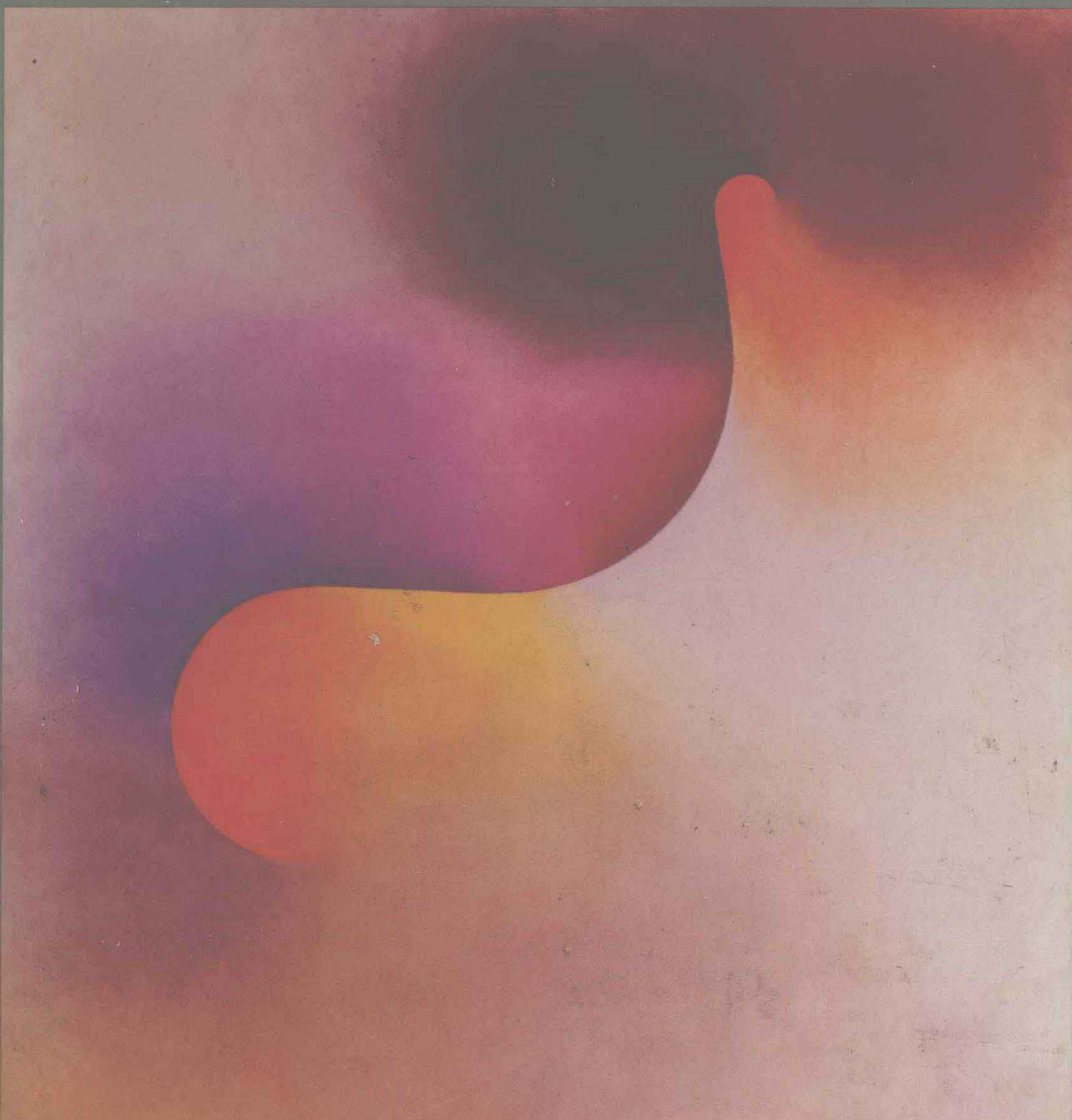
Calculus & Analytic Geometry

THIRD EDITION



MIZRAHI / SULLIVAN

Calculus & Analytic Geometry

THIRD EDITION

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About the Authors

Abe Mizrahi and Michael Sullivan both received Ph.D.'s in mathematics from the Illinois Institute of Technology. Dr. Mizrahi is a professor of mathematics at Indiana University, Northwest Campus, and Dr. Sullivan is a professor of mathematics at Chicago State University. They are also the authors of *Mathematics for Business and Social Sciences, An Applied Approach* (Wiley) and *Finite Mathematics with Applications for Business and Social Sciences* (Wiley), and Sullivan is the author of *Precalculus*, *College Algebra*, *College Algebra and Trigonometry*, *Trigonometry*, and *College Algebra with Review* (Dellen).

Preface to the Instructor

A New Era in Teaching Calculus

The third edition of this text is being published at the start of an exciting new era in teaching calculus. Calculus instructors widely agree that this decade will see changes in the way calculus is taught. But leaders in calculus reform are moving in different directions, and it is not clear what the changes will finally be. While experiments are conducted at a number of colleges, the teaching of calculus for the most part adheres to traditional syllabi, with forays in some new directions. While ours is a "traditional" calculus text, we felt it important in this edition to take initial steps into the realm of the "new calculus" by including explorations in the use of graphing calculators and computers.

Continuing Features

In our first edition we set out to write a straightforward, mathematically sound book that would nevertheless capture the interest of students. We felt—and still feel—that there is a place for a mature, mainstream text with the readability and practicality usually associated with lower-level books. The text can readily adapt to either semester or quarter systems. There is flexibility in the order and depth in which the material can be presented.

As before, we present concepts in both theorem–proof style and in the vernacular. We have often found that when students are presented with an alternative explanation, a mental block is unlocked, and better appreciation of the material results. Explanations in plain English also serve to provide the qualitative perspective from which analysis must at times be viewed.

We have retained and augmented our store of interesting applications. These applications are located at the most strategic places—where students are most likely to be fatigued by the presentation of difficult material or where an application can bring to life a particular aspect of mathematics.

We have also kept the historical sections in order to provide perspective on how calculus actually evolved. We have avoided the capsule biography approach to historical notes by including exercises in these optional sections. We want teachers to be able to give their students a taste of what it was like to invent the calculus. This Socratic style of instruction is, in our experience, more instructive than the simple reading of footnotes.

We continue to provide several exercises taken directly from the best-selling texts in physics as well as many exercises that call for the use of a programmable calculator or computer.

Changes Made Throughout the Text

Each chapter concludes with Review Exercises (a set of approximately 40 to 50 exercises representing typical test questions for each chapter) and Challenge Exercises (10 to 20 very difficult, thought-provoking exercises).

More "B" level exercises have been added to the section exercise sets throughout the text. The text now contains over 7,400 exercises and over 700 examples.

Graphing calculator essays with exercises have been added to select chapters in the text.

Computer essays and exercises have been added to select chapters in the text.

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Full color has been added in Chapter 6 and certain other chapters to enhance the three-dimensional effect of the illustrations.

Theorems and definitions are identified with new numbering sequences at the left. Formulas are given a single number on the right. Wherever possible, theorems are identified by names.

A new 8 × 10 inch trim size allows for a more open design.

Chapter-by-Chapter Changes

Chapter 1: The material on trigonometry, formerly in the Appendix, has been rewritten and now appears in Chapter 1.

Chapter 2: A section on the limits of trigonometric functions has been added, parallel to the additional trigonometry material in Chapter 1. Accordingly, more examples and exercises involving trigonometry appear throughout the first seven chapters. We have added a calculator vignette, "Functions and Limits Explored Using Graphing Calculators."

Chapter 3: Four sections from the second edition (3.1, 3.2, 3.3, and 3.12) have been condensed into two sections to improve the flow of material and save time teaching the derivative. The proof for the derivative of x to a rational exponent has been revised. The sections on the Chain Rule and implicit differentiation were rewritten with additional explanation, examples, and problems. We have added a calculator vignette, "Finding Derivatives on the HP-28."

Chapter 4: The material on sketching graphs has been expanded into an entire section to make the difficult subject of curve sketching easier. We have added a calculator vignette, "Exploring Asymptotes and Limits at Infinity Using Graphing Calculators," and a computer vignette, "Using a Microcomputer to Explore Graphs of Functions."

Chapter 5: The material on summation notation was rewritten and expanded. The section on the average value of a function was moved here from Chapter 6. A theorem on the integration of even and odd functions now appears in the text. We have added a computer vignette, "Using Microcomputer Graphics to Investigate Approximations to Definite Integrals."

Chapter 6: Applications involving the center of mass and the centroid of an object have been included. Examples and exercises dealing with the revolution of a region about a line other than the x-axis or the y-axis have been added. We have added a calculator vignette, "Finding Arc Length on the HP-28."

Chapter 7: Some of the differential equations material has been postponed and now appears in the new Chapter 20. An alternative derivation of the derivative of $y = x^\alpha$ was added.

Chapter 8: Sections 8.1 and 8.2 have been combined to reflect the emphasis on the inverse trigonometric functions and the interrelatedness of integration and differentiation. New examples on the derivative and integrals of inverse trigonometric hyperbolic functions appear.

Chapter 9: Sections 9.8 and 9.9 involving miscellaneous substitutions have been removed from the text and now appear as exercises.

Chapter 10: A three-step procedure for the use of L'Hospital's rule has been added.

Chapter 11: The discussion of series of positive terms was reorganized and expanded to improve clarity. The Root Test is now covered in the text rather than in the exercises.

Chapter 12: Several new exercises have been added.

Chapter 13: The discussion of curvature has been postponed to Chapter 15. The section on graphing polar equations was entirely rewritten to include more examples. The section on the angle ψ has been removed. We have added a calculator vignette, "Graphing Polar and Parametric Curves on a Graphing Calculator."

Chapter 14: Vectors in the plane and vectors in space are discussed concurrently. The exercises have been upgraded and new applications have been added.

Chapter 15: This chapter was reorganized to improve readability. The topics of curvature in the plane and in space are now discussed concurrently. A discussion of osculating circles appears in the text and a discussion of the binormal vector $\mathbf{B} = \mathbf{T} \times \mathbf{N}$ is given in the exercises. Additional exercises involving radicals and rational functions now appear.

Chapter 16: More epsilon–delta exercises and examples were added. All definitions and theorems have been rewritten to be consistent with earlier material. Additional exercises involving radicals and rational functions now appear. We have added a calculator vignette, "Finding Partial Derivatives on the HP-28."

Chapter 17: A discussion of maximum and minimum problems occurring on the boundary has been added. We have also added a computer vignette, "Graphical Solutions to a Constrained Optimization Problem."

Chapter 18: A new section on Jacobians was added.

Chapter 19: A new section on vector fields appears. The similarities of Green's Theorem with the Divergence Theorem and Green's Theorem with Stokes' Theorem are discussed. The section on surface integrals was rewritten to improve clarity.

Chapter 20: This new chapter provides an introduction to differential equations.

In addition to these substantive changes, all three-dimensional graphics have been redrawn. There are two types of three-dimensional computer-generated graphs: surfaces defined by equations of the $z = f(x, y)$ and quadric surfaces. The graphs of the first type show lines of constant x and lines of constant y on the surface. This is done by drawing the portion of the graph nearest the viewer first, while maintaining a record of the outline of what is presently visible; further portions that are visible are then drawn and added to the outline, whereas new "back" portions that fall within the outline would be hidden and are not drawn. This technique is computationally more efficient than hidden-surface removal algorithms for arbitrary polygonal surfaces. Some of the quadric surfaces are drawn this way also; the rest are generated by "clipping" against the plane that separates the visible and invisible parts of a quadric when viewed from infinity. These graphics have been provided by Douglas Dunham of the University of Minnesota, Duluth.

The three-dimensional airbrushed pieces were done by Ron Kempke of Scientific Illustrators. Each piece was first described mathematically by the appropriate equation, a three-dimensional graph to be used as an accurate template was generated according to the methods described above, and then the airbrush paintings were done. Thus, all three-dimensional artwork is mathematically and dimensionally accurate.

Much of the two-dimensional line art has been redone by Brian Morris, director of computer operations at Scientific Illustrators. All graphs represented by an equation $y = f(x)$ were plotted on a Hewlett-Packard 7550 $x - y$ plotter using the software package PLOTR developed by Scientific Illustrators. In addition, all answer art has been competely replotted to ensure accuracy using the latest technology.

Ancillaries

1. A *Student Solutions Manual*, prepared by Richard Fritz and Richard Tucker, accompanies this text. We have found the manual an invaluable aid for students who have weak backgrounds in algebra. The manual includes worked-out solutions to every other odd-numbered problem in the text. (The answers to odd-numbered problems appear in the back of the book.)
2. A complete *Solutions Manual* is also available to instructors.
3. A *Student Study Guide*, prepared by Richard St.André, accompanies the text. This study guide tests students on both key concepts and possible algebraic difficulties. It is a strategic aid and is not as comprehensive in solving remedial difficulties as the *Student Solutions Manual*. Our experience has been that the study guide is of great help at urban campuses where students may miss a few classes.
4. A test items booklet, prepared by George Feissner of SUNY Cortland, contains one multiple-choice and one fill-in test per chapter.
5. EXP-Test, a computerized test bank for IBM PCs and compatible hardware, contains all of the test questions in the test items booklet and is available to adopters of the text.
6. Calculus software from *The Math Lab* by Avery, Barker, and Soler is being provided by the publisher to adopters of the text. This software follows an exploratory model that allows students to go far beyond algebraic processes. Concepts such as limits, differentiation, and integration are numerically and graphically performed at the touch of a key. The student focuses on the correct

approach to a problem, and the underlying principles of a problem take precedence over algebraic procedure. This software is available in Apple II and IBM PC ($3\frac{1}{2}$ inch and $5\frac{1}{4}$ inch) versions.

Acknowledgments

Books that are effective teaching tools (for us) and learning tools (for our students) are not so much written as they are developed out of classroom experience. The more experiences of colleagues an author is exposed to, the more the text is thoroughly developed into a reliable teaching tool. This work received an invaluable contribution through the suggestions, criticisms, and encouragement of the following colleagues:

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The Historical Exercises in this text were contributed by Ken Abernethy, from an unpublished paper on the history of the calculus. This outstanding work is unique in that it consistently "reinvents" the calculus, rather than merely talking about the calculus. We hope that some of its contribution is evident in this textbook.

Many professors contributed their time and expertise to making this edition as error-free as possible. In particular, we would like to thank the following: Bobbi J. Barry, David L. Baughman, Steve Blasberg, Thomas Brown, Frank Cheek, George Christopher, Jennifer L. Dydo, Michael Eckers, William Hosch, Kim Hughes, Howard Jones, Maryann Justinger, Eleanor Killam, Patti Fraser Lock, Marie Loftus, Richard P. Savage, Jr., Raymond Southworth, Sarah Wada, Lee Welch, and August Zarcone.

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Finally, we would like to thank the many teachers and students who wrote us unsolicited letters about our book. We have used many of their criticisms, but we would be more than human if we didn't admit we were most gratified by some of the comments of students who told us that their good experiences with the calculus made them eager to take additional mathematics courses. We don't pretend to take much credit for that, but it is a pleasure to imagine that we've had something to do with introducing a new generation to the power and beauty of the calculus.

Using Hand-Held Computers in Calculus and Analytic Geometry*

Computer technology has developed to the point where it can help students learn calculus. Graphing calculators, such as the Casio fx-7000G, fx-7500G, and fx-8000G, Hewlett-Packard 28C and 28S, and Sharp EL-5200, are really hand-held computers. They combine the capabilities of a scientific calculator, a programmable computer, an interactive graphics computer system, and in the case of the Hewlett-Packards, a computer algebra system. These devices can perform many of the computations of calculus and analytic geometry. They can readily produce the graphs of functions and, with some programming, the graphs of polar equations, parametric equations, and conics. These hand-held machines permit the user to explore a wide variety of examples in a relatively short time. They permit interactive experimentation for learning mathematical concepts, solving problems, and generating conjectures. They can make mathematics more oriented toward concept development and problem solving and less oriented toward paper-and-pencil computation. The use of hand-held computers can also be applied to many other areas of undergraduate mathematics: statistics, linear algebra, differential equations, probability, number theory, and mathematical modeling.

In many chapters of this book there are hand-held computer vignettes—brief asides that explain how a pocket computer can be used to provide a different approach to a topic just covered in the text. Only selected applications of supercalculators to calculus are explained, however. Many other uses are possible. For additional ideas, refer to the documentation provided with your machine or read the sources listed in the bibliography at the back of the book. Hewlett-Packard also publishes some applications booklets for the HP-28, which you may find helpful.

The Casios

The Casio fx-7000G, fx-7500G, and fx-8000G are all quite similar. In each case, the screen displays eight lines of numbers, words, and symbols in the text window. The graphics display is 95 × 63 pixels. The 8000 can be linked to a printer or tape recorder (to save programs on tape) using special interface equipment. The 7500 has the fastest graphics and the most memory. They all have the following features that are useful in calculus and analytic geometry:

1. *Big Screen Computation.* You can see both problems and answers on the screen at the same time. The screen is large enough to display up to four computations, both inputs and outputs, at once. If an answer doesn't make sense or the machine displays an error message, you can edit your input without having to reenter the entire command and then reexecute the problem.
2. *Interactive Graphics.* You can create virtually any mathematical graph: functions, relations, geometric figures, even three-dimensional graphs. The viewing rectangle and the scales are set using the **Range** feature.

* Gregory D. Foley
The Ohio State University

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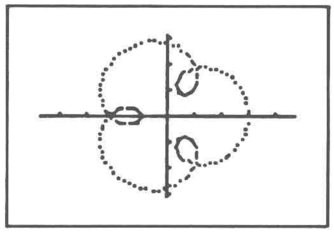
Trace allows pixel-to-pixel movement along the most recently drawn graph, with the computer displaying the x- or y-coordinate associated with each pixel along the way. It is easy to stop, reverse direction, and switch from x-readout to y or vice versa. The automatic zoom feature or the **Factor** command can be used to zoom in or zoom out about a plotted or traced-to point, or as a default, about the center of the current viewing rectangle. Early versions of the 7000 did not have automatic zoom, but now all models have this important feature.

3. *On-Screen Programming.* The Casio programming language is simple and easy to learn. On-screen editing makes programming on this little machine like programming on a microcomputer rather than on past generations of programmable calculators. A few fundamentals will carry you a long way. Programming is useful for repeated calculations and for polar, parametric, conic, and three-dimensional graphing. As an example of combining programming and graphics, an epitrochoid as it would appear on the Casio is shown here.



The Sharp

The Sharp EL-5200 has four lines of textual display and 96 × 32 pixels of graphics display. The graphics are slower than on the Casios, and the user does *not* see the graph being drawn. The Sharp, however, has a scrolling screen, a **SOLVE** key, and built-in matrix capabilities. The SOLVE key is especially helpful for many problems in calculus. The programming is awkward compared to the Casio, but the memory capacity is larger even than the Casio fx-7500G.

The Hewlett-Packard

The HP-28C, which is very similar to the 28S but with a smaller memory, is no longer manufactured. Like the Sharp, the HP-28S has only four lines of text. The graphics screen is 137 × 32 pixels, and graphs are shown as they are being drawn. The 28S can solve equations and operate on matrices and, in addition, can find derivatives and definite integrals, generate Taylor series, and determine antiderivatives of polynomials. The 28S can also handle complex numbers. The programming is nice for experienced programmers, permitting the commands of BASIC, FORTRAN, and Pascal. It is the most powerful, and the most expensive, of the machines described here.

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Using Computing in Teaching Calculus*

Why Do It?

A colleague and I were having a conversation. My colleague said, "I don't use the computer at all in my calculus course. I want my students to think." My reply was that I use computing extensively for exactly the same reason. This brief conversation contains the extreme positions held within the mathematical community. As the warden said to Luke in the movie *Cool Hand Luke*, "What we have here is a failure to communicate." Both my colleague and I have the same goal—to have our students understand and appreciate one of the premier achievements of human intellectual activity. Both of us are sincere in our desire to achieve this goal. The communications failure is a result of our concept of the role that technology can play in achieving the

* L. Carl Leinbach
Gettysburg College

goal. Obviously, there are many, quite possibly the majority of, calculus teachers who hold positions somewhere between that of my colleague and me. The computing sections of this text are directed to this group.

The fundamental objects for study of the calculus are functions. Students of the calculus spend their time examining functions and the effects of certain operations on these functions. A student who successfully completes a calculus course should be able to understand at a deep level the information conveyed by any function, not just those presented in a closed algebraic form. The medical practitioner should know what it means for the concentration of a drug to decay at an exponential rate. The economist should understand what is meant when one is talking about economic "forces." The social scientist should understand trends and the limitations of linear projections. The physicist and chemist should be able to use a knowledge of present states to make meaningful predictions about future states of a system. The mathematician should have an intimacy with a very important class of mathematical objects. This knowledge is gained not by the inclusion of specific examples in a book (no matter how carefully chosen) but by a thorough understanding of functions independent of the manner in which they are presented.

Unfortunately, students come to us with very little intuition about functions. They have concentrated on algebraic formulas and see, for example, the replacement of the variable x by the expression $(x + a)$ in an equation as an algebraic operation and not a translation of the solution set. The exercises that are assigned in most calculus courses perpetuate this perspective. This is especially unfortunate, since it is from the exercises that students create their impressions about what is really important in the subject. The differentiation and integration operators become means of transforming, sometimes rather painfully, one algebraic expression into another. Seldom does a student spend time studying the graph of a function and deciphering the information that is contained in that object. Even more seldom does the student come to the realization that this information is valuable in understanding the given relationships.

By no means am I against algebra! What is needed is a balance between the algebraic knowledge and the knowledge that is gained by visualization. The computing packages provide us with an opportunity to provide that balance. It is easy to generate data in the form of graphs of functions. The student can study functions presented in this form, make conjectures, and test those conjectures with more data. In this mode, the student acts like a laboratory scientist. Herein lies the beauty of using computing in this way; the student then submits conjectures, supported by laboratory observation, for mathematical verification. If we have done our job correctly, some of these conjectures will fail, but for the most part they will succeed. The result is that the student, in addition to developing some intuition about functions, has developed a vested interest in the class presentation. This fact alone makes computer use important.

How Can It Be Done?

This text contains four sections specifically devoted to using the computer to explore concepts presented in the more traditional manner. The first on Newton's method is rather conventional in its approach. The method is used to find a numerical solution to an equation that would be very difficult to solve by algebraic methods. The second section illustrates the problems that can be encountered by a strict reliance on the evidence of a computer-generated graph. The subsequent investigation of that graph is designed to enforce the idea that the analysis done in calculus can give us insights that are not necessarily obvious to the observer of a computer-generated graph. In the third section, the student will use the graphical and numerical capabilities of the computing package to anticipate the fundamental theorem of calculus. The approximating sum for the definite integral is used to investigate the very nature of what is being approximated, not just to make a numerical approximation. The student is encouraged to make a hypothesis that is verified by the proof of the fundamental theorem. The fourth section deals with the explanation of the LaGrange multiplier method for constrained optimization of a function of two variables. An estimate made using graphical evidence is enhanced with numerical techniques.

These four sections are designed to guide students through the investigation and lead them to an understanding of the analysis done in class. As such, the sections need to be read interactively. The student should be in front of a computer generating the data that are presented in the text. The exercises following the section should also be done in front of a computer. Students should be encouraged to generate their own exercises. It is vital that the student become involved in the process of generating data and giving an initial test to hypotheses.

Software Packages

The original intent was to write an essay on packages that are available for use in a calculus course. Actually there are too many to mention. The philosophy of use is much more important than the particular product that is used. My personal choice and the one that was used to generate the four sections in this book is *MicroCalc* by Harley Flanders of the University of Michigan. This package contains a rich assortment of routines that are easily accessed by the user. The package contains both the graphical and numerical routines that are required for the types of investigations done in this book. It even contains a rudimentary symbolic differentiation routine that proves useful in Newton's method and Lagrange multiplier investigations. Another option is to use a computer algebra system such as DERIVE from the Soft Warehouse in Honolulu, Hawaii, Maple from the Symbolic Computation Group at Waterloo University, or Mathematica from Wolfram Enterprises. These packages contain good numerical and graphics routines as well as very good routines for symbolically solving equations, symbolic differentiation, and symbolic integration among other mathematical operations. The possession of one of these packages can expand one's horizon far beyond the traditional calculus with computing course, but that is another story.

Preface to the Student

I hear . . . . . and I forget
I see . . . . . and I remember
I do . . . . . and I understand

CHINESE SAYING

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