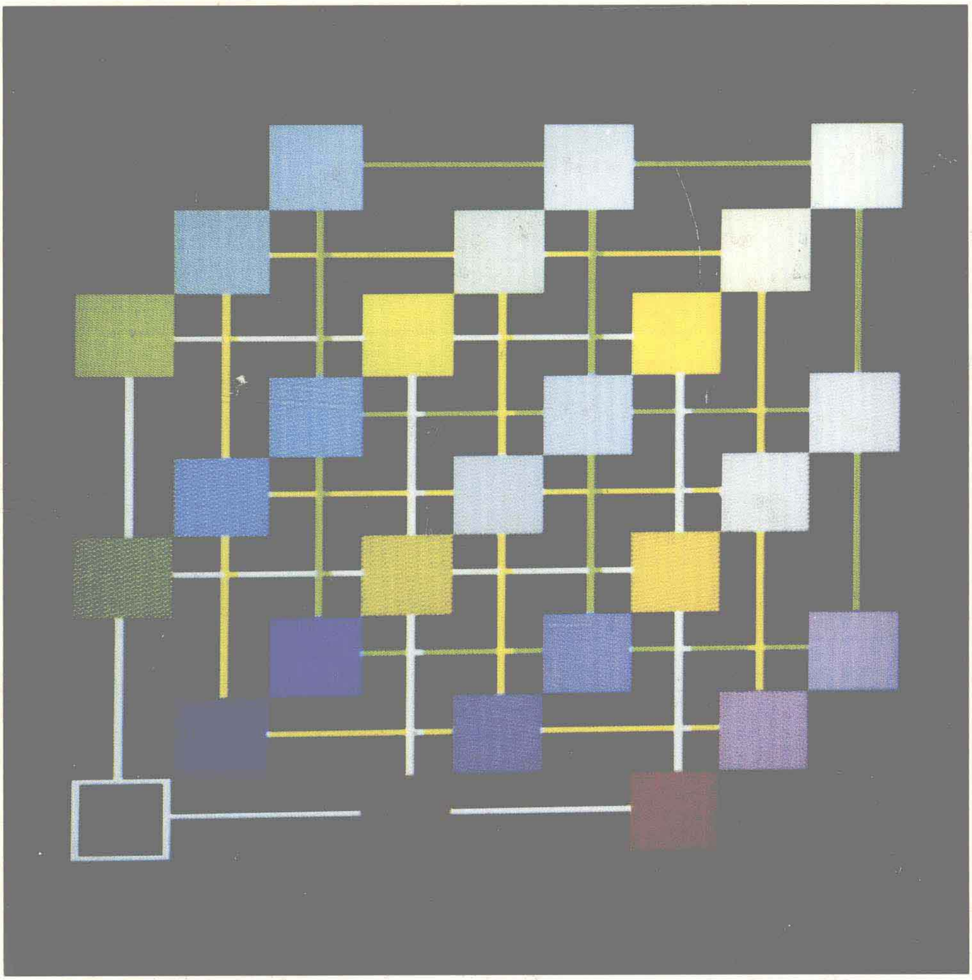
APPLIED FINITE MATHEMATICS



HUNKINS • MUGRIDGE

APPLIED FINITE MATHEMATICS

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Dedicated to our wives

Jeanette and Sandy

for their patience and understanding during the many long hours, late nights, early mornings, and missed meals while this text was being brought to fruition.

Prindle, Weber & Schmidt is a division of Wadsworth, Inc.
Second printing: September, 1981

Library of Congress Cataloging in Publication Data

Hunkins, Dalton R.
Applied finite mathematics.

Includes index.
Mathematics--1961- I. Mugridge, Larry R., joint author. II. Title.
QA39.2.H859 510 80-29563
ISBN 0-87150-306-9

This text was composed in Monophoto Times Roman and Optima by Composition House Limited. It was printed and bound by R.R. Donnelley & Sons Company. The cover was printed by New England Book Components, Inc. The artwork was drawn by Phil Carver & Friends, Inc. Production editor and designer: Nancy Blodget.

PREFACE

Applied Finite Mathematics was written to present important mathematical tools and applications to students with a wide variety of majors. It is our belief that they will be better prepared to understand the use of mathematical concepts in their careers in such areas as business, economics, and life sciences, as well as other courses which are part of their major courses of study.

We assume the average student has a high school algebra background. However, for those who need it, a brief review is included in sections 1.1–1.3. A diagnostic test is provided at the beginning of Chaper 1 for students who wish to check their own level of proficiency.

As the material was class tested, we developed a wide variety of "real world" applications which are used to introduce virtually all new concepts throughout the text. We also found that using one or two examples which develop in complexity over several chapters helps the students understand how more complex methods are built on the simpler ones.

Examples, exercises, and applications are the key to understanding. This text includes 330 examples and over 1420 exercises, utilizing a wide variety of applications. A particular effort was made to relate exercises to examples, with the exception of a few challenging problems at the end of most exercise sets. The even-numbered exercises are paired with the odd-numbered ones, with the answers to the latter group given in Appendix D.

The topical coverage can be broken down as follows:

Sections 1.4 and 2.1 give the basic concepts of linear equations.

Sections 2.2–3.7 cover systems of linear equations and matrix algebra, including a linear production model and lines of best fit.

Chapters 4 and 5 cover linear programming problems from the graphical method through the simplex method.

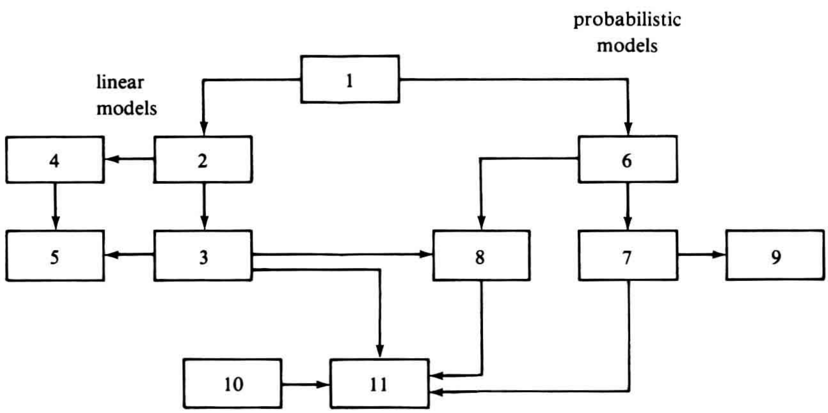
Chapter 6 deals with basic probability concepts, including a review of sets, Venn Diagrams, and counting techniques. This could be followed with Chapter 8, Markov Chains, giving additional applications.

Chapter 7 covers basic statistics and provides applications to such areas as insurance, production control, voting models, and medicine. The concepts of descriptive statistics are related to probability through random variables. This material prepares the student for Chapter 9, Game Theory.

Chapter 10 develops material on simple and compound interest, bank discounts, annuities, and other applications of mathematics to finances.

Chapter 11, Computers and BASIC Language Computing, is included because of the increasingly important role of computers in our world. Students using this chapter should be able to read and make simple modifications in BASIC programs. Should an instructor wish, the programs given in Appendix B could be used to solve problems in Gaussian Elimination or the simplex method included in Chapters 3 and 5, respectively. The use of a computer in problem solving is illustrated in other relevant places in the text as well.

The following flow chart illustrates the chapter dependencies and what we feel is a logical order in which to cover the material.



We are grateful to a number of people who were helpful in bringing this text to fruition. The following reviewers made many valuable suggestions for the improvement of the presentation: Ronald D. Baker, University of Delaware; Barbara J. Bulmahn, Indiana University-Purdue University; William E. Coppage, Wright State University; Sam Councilman, California State University, Long Beach; David Cusick, Marshall University; John J. Dinkel, The Pennsylvania State University; William G. Frederick, Indiana University-Purdue University; Alan Gleit, University of Massachusetts, Amherst; Rose C. Hamm, The College of Charleston; Robert Hathway, Illinois State University; T. L. Herdman, Virginia Polytechnic Institute & State University; J. G. Horne, Jr., Virginia Polytechnic Institute & State University; Edward L. Keller, California State University, Hayward; David E. Kullman, Miami University; William Margulies, California State University, Long Beach; Hal G. Moore, Brigham Young University; Peter J. Nicholls, Northern Illinois University; and W. Wiley Williams, University of Louisville.

Our thanks also go to Rhonda Cranage for her help with photocopying and preparing copies of the manuscript for class use. A special thank you goes to Mary Ann Serio who was invaluable in meeting our deadlines by spending many long hours and late nights typing the manuscript.

Dalton R. Hunkins
Larry R. Mugridge

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Chapter 1

LINEAR EQUATIONS AND APPLICATIONS

The concepts of solving equations and inequalities, graphing linear equations, and writing the equation of a linear function (the equation of a line) make up the content of the first three sections of the chapter. Section 1.4 and the remainder of the text develop from the basic ideas and from your understanding of fundamental properties of real numbers given in appendix A. Some of you may already have sufficient grasp of these concepts and, therefore, can proceed directly to section 1.4. On the other hand, others may not feel as comfortable with them. To determine if you need to spend some time reviewing the first three sections, a pre-test has been designed and appears first. We recommend that you take the pre-test.

The pre-test is divided into three parts that correspond to the first three sections of the chapter.

Part 1 Equations and Inequalities

Part 2 Coordinate Systems: Graphing a Linear Equation

Part 3 Writing the Equation of a Line

The answers are given immediately following the test.

PRE-TEST

PART 1 Equations and Inequalities

Solve the following equations for x.

1 $2x - 5 = 10$

2 $\frac{3}{5}x = 9$

3 $4 - 3(x + 1) = 10$

4 $\frac{1}{2}(4x - 14) = -3x - 2$

5 $3x + \frac{1}{7} = \frac{9}{2}x - \frac{1}{14}$

6 $0.7x - 0.5 = 0.2x + 0.1$

7 $4(x - 1) - 3(x + 2) = 7(x + 2) - 6$

Solve the following inequalities for x and graph the solution set on the number line.

8 $7x + 12 > 26$

9 $6x + 7 \leq 8x - 1$

10 $-2(2x + 1) - 6 \geq 4(x - 1)$

Let x represent the number of cars waiting in line for gasoline. Symbolize the following statements in terms of inequalities.

11 There are at most 15 cars waiting in line.

12 There are at least 15 cars waiting in line.

13 There are no more than 15 cars waiting in line.

14 There are no fewer than 15 cars waiting in line.

PART 2 Coordinate Systems: Graphing a Linear Equation

Plot the following points in the coordinate plane.

1 $(2, 5)$ **2** $(-1, 0)$ **3** $\left(\frac{3}{2}, -4\right)$ **4** $(-4, -6)$

Exercises 5–10 refer to figure 1.1.

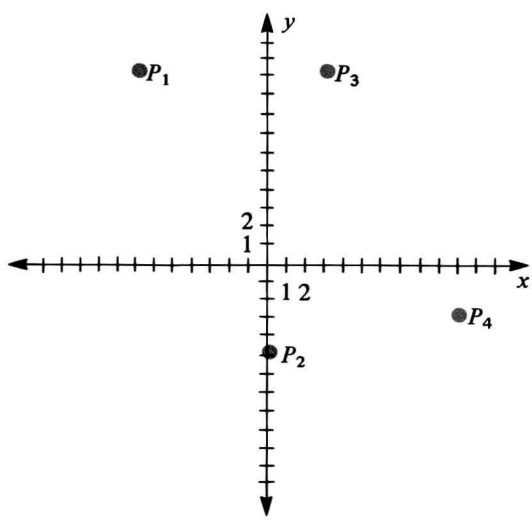

FIGURE 1.1

5 What is the abscissa of P_1?

6 What is the abscissa of P_2?

7 What is the ordinate of P_3?

8 What is the ordinate of P_4?

9 What are the coordinates of the point P_1?

10 What are the coordinates of the point P_4?

Describe where the point (x, y) would be located in the coordinate plane.

11 $x > 0$ and $y < 0$

12 $x = 0$ and $y > 0$

13 $x < 0$ and $y = 0$

14 $x \geq 0$ and $y \geq 0$

Find the slope of the line passing through the two given points.

15 $P(-2, -1)$ and $Q(-3, -1)$

16 $P(4, 9)$ and $Q(9, 4)$

17 $P\left(\frac{1}{2}, 2\right)$ and $Q\left(3, \frac{7}{2}\right)$

18 $P(3, -6)$ and $Q(3, 5)$

Determine whether or not the three given points lie on the same line.

19 $P(2, 6)$, $Q(-1, -3)$, and $R(0, 0)$

20 $P(-2, 7)$, $Q(0, 1)$, and $R(6, -13)$

21 Consider the equation $2x - 5y = 20$ and determine the pair (x, y) that satisfies the equation when:

(a) $x = 0$ (b) $y = 0$ (c) $x = -5$ (d) $y = -1$

22 Consider the equation $\frac{2}{3}x + \frac{5}{4}y - 1 = 0$ and determine the pair (x, y) that satisfies the equation when:

(a) $x = 0$ (b) $y = 12$ (c) $x = 1$ (d) $y = -6$

Find the slope m and y-intercept b for each of the following equations. Draw the graph of the equation.

23 $y = -2x + 7$

24 $4x - 3y + 5 = 0$

25 $y + 3 = 0$

26 $\frac{2}{3}x + \frac{5}{4}y - 1 = 0$

PART 3 Writing the Equation of a Line

Write the equation of the line for the given slope m and the given y-intercept b.

1 $m = 3$ and $b = -1$

2 $m = -\frac{1}{2}$ and $b = 0$

Write the equation of the line for the given slope m and the given point P.

3 $m = 1$ and $P = (0, 2)$

4 $m = -3$ and $P = (-1, -3)$

Write the equation of the line passing through the two given points.

5 $P(-1, 3)$ and $Q(3, 5)$ **6** $P(2, -1)$ and $Q(2, 5)$

7 $P(1, -5)$ and $Q(-3, -5)$ **8** $P(-2, 7)$ and $Q(2, -3)$

9 Bob and Judy plan to drive over the same route from their home in separate cars. Suppose that Bob leaves at 8:00 A.M. and travels at 45 miles per hour. Judy leaves at 8:20 A.M. and drives at 50 miles per hour. Express the distance between them as a function of time (in hours).

10 In a particular state, the income tax is a fixed percentage of the gross income; the rate is 2.3%. Express the amount of tax owed as a function of the gross income.

11 For a certain taxi company the fare is \$1.00 plus 50¢ per each half mile of the trip. Express the total fare of a trip in terms of the number of miles of the trip.

12 A leak was found in a storage tank which is 50 feet high. At 11:00 A.M. the tank was full; however, at 12:30 P.M. the level of fluid in the tank had dropped 1 foot. Assuming that the fluid is leaking from the tank at a constant rate, express the depth of the fluid in the tank as a function of time (in hours).

13 The Consumer Price Index was 181.5 in 1977 and 191.9 in 1978. Assuming that the Consumer Price Index is linearly dependent on time, express this dependency with an equation.

ANSWERS TO PRE-TEST

PART 1

1 $\frac{15}{2}$ **2** 15 **3** -3 **4** 1

5 $\frac{1}{7}$ **6** 1.2 **7** -3

8 $x > 2$

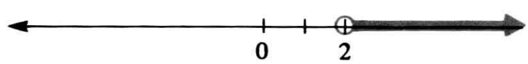

FIGURE 1.2

9 $x \geq 4$

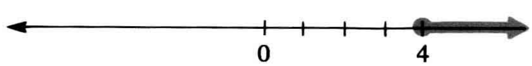

FIGURE 1.3

10 $x \leq -\frac{1}{2}$

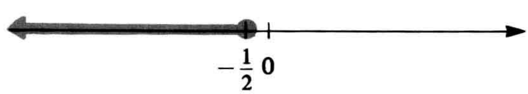

FIGURE 1.4

11 $x \leq 15$ **12** $x \geq 15$ **13** $x \leq 15$ **14** $x \geq 15$

PART 2

For exercises 1 through 4, see figure 1.5.

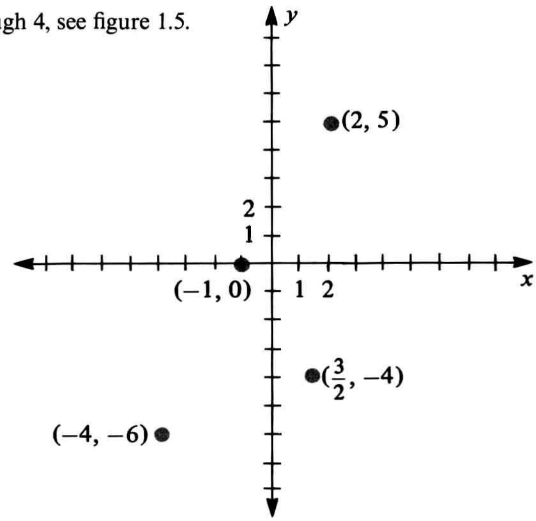

FIGURE 1.5

5 -7 **6** 0 **7** 10 **8** -3

9 $(-7, 10)$ **10** $(10, -3)$ **11** quadrant IV **12** positive y-axis

13 negative x-axis **14** quadrant I, positive x-axis, positive y-axis, or (0, 0)

15 0 **16** -1 **17** $\frac{3}{5}$ **18** not defined

19 on the same line **20** not on the same line

21 (a) $(0, -4)$ (b) $(10, 0)$ (c) $(-5, -6)$ (d) $\left(\frac{15}{2}, -1\right)$

22 (a) $\left(0, \frac{4}{5}\right)$ (b) $(-21, 12)$ (c) $\left(1, \frac{4}{15}\right)$ (d) $\left(\frac{51}{4}, -6\right)$

23 $m = -2; b = 7$

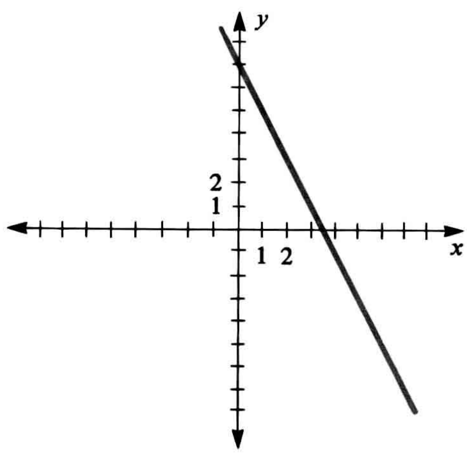

FIGURE 1.6

24 $m = \frac{4}{3}; b = \frac{5}{3}$

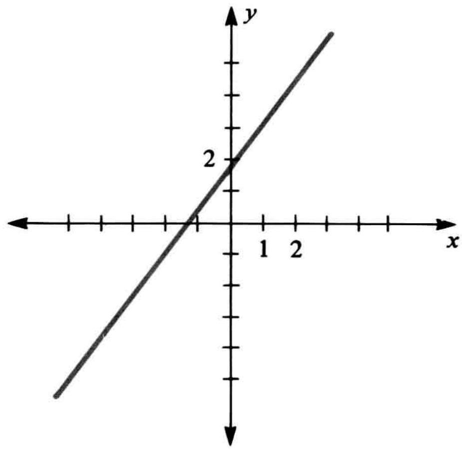

FIGURE 1.7

25 $m = 0; b = -3$

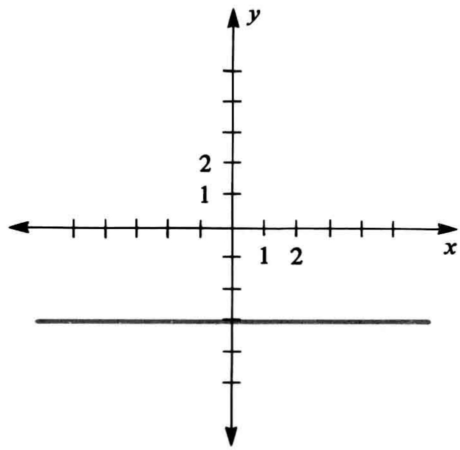

FIGURE 1.8

26 $m = -\frac{8}{15}; b = \frac{4}{5}$

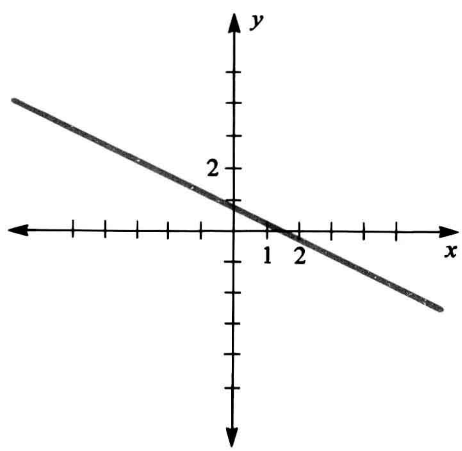

FIGURE 1.9

PART 3

1 $y = 3x - 1$

2 $y = -\frac{1}{2}x$

3 $y = x + 2$

4 $y = -3x - 6$

5 $y = \frac{1}{2}x + \frac{7}{2}$

6 $x = 2$

7 $y = -5$

8 $y = -\frac{5}{2}x + 2$

9 $d = -5t + 15$

10 $T = 0.023I$

11 $F = m + 1$

12 $d = -\frac{2}{3}t + 50$

13 $I = 10.4t + 181.5$ ($t = 0$ means 1977)

1.1 EQUATIONS AND INEQUALITIES

Equations occur frequently in applications. For example, Al, who owns the Pizza Place, figures that if he makes x pizzas, then his cost (in dollars) is $1.20x + 98$. He also knows that his income from selling x pizzas is $4x$. Al wants to find the number of pizzas he must sell in order to break even; that is, he wants to determine x so that his cost equals his income. Therefore, setting cost equal to income, he arrives at the equation

$$1.20x + 98 = 4x.$$

By using rules of equations, he finds that $x = 35$. Therefore, Al must make and sell 35 pizzas to break even.

In section 1.3 we consider how to write expressions such as Al's cost equation. But for now, let us review the properties and rules used to arrive at Al's answer.

A **solution** to an equation containing one variable x is a number that can be substituted for x such that the statement is true. For example, the solution to the equation

$$x + 3 = 7$$

is clearly

$$x = 4$$

since

$$4 + 3 = 7.$$

As another example, the solution to

$$2x = -12$$

is

$$x = -6$$

since

$$2(-6) = -12.$$

For simple equations like the preceding ones, we can give the solution from our knowledge of adding and multiplying real numbers. However, if we cannot see the solution to an equation immediately, we can derive simpler **equivalent equations** from the original equation. An equivalent equation is obtained by rewriting expressions using properties of real numbers and the following rules of equality.

RULES OF EQUALITY

1. We can add or subtract the same quantity on both sides of an equation. That is,

$$a = b \text{ is equivalent to } a + c = b + c$$

and

$$a = b \text{ is equivalent to } a - c = b - c.$$

2. We can multiply or divide both sides of an equation by the same nonzero quantity. That is,

$$a = b \text{ is equivalent to } ac = bc, \text{ when } c \neq 0$$

and

$$a = b \text{ is equivalent to } \frac{a}{c} = \frac{b}{c}, \text{ when } c \neq 0.$$

In the following examples we solve equations by deriving equivalent equations.

EXAMPLE 1

$$5x + 5 = 3x + 19$$
$$5x + (-3x) + 5 + (-5) = 3x + (-3x) + 19 + (-5) \qquad \text{(rule 1)}$$
$$2x = 14$$
$$x = 7$$

■

EXAMPLE 2

$$2(x + 1) = 3(x + 6) - 4$$
$$2x + 2 = 3x + 18 - 4$$
$$-x = 12$$
$$x = -12$$

■

EXAMPLE 3

$$0.7x + 0.35 = 0.65x - 1$$
$$70x + 35 = 65x - 100$$
$$5x = -135$$
$$\frac{5x}{5} = \frac{-135}{5} \qquad \text{(rule 2)}$$
$$x = -27$$

■

EXAMPLE 4

$$2(3x - 5) = 15 - \frac{1}{2}(3x - 4)$$

$$6x - 10 = 15 - \frac{3}{2}x + 2$$

$$\frac{15}{2}x = 27$$

$$x = \frac{2}{15}(27) = \frac{18}{5}$$

■

EXAMPLE 5

$$0.1x - 0.3[2x - (x + 0.1)] = -0.8x + 0.33$$

$$0.1x - 0.6x + 0.3(x + 0.1) = -0.8x + 0.33$$

$$-0.2x + 0.03 = -0.8x + 0.33$$

$$0.6x = 0.3$$

$$x = 0.5$$

■

The next two examples illustrate how equations can be used to solve word problems.

EXAMPLE 6 Let x be the number of electronic slot machines produced on production line A. Suppose that a second production line B can produce three-fourths as many slot machines as line A. Determine x if the total output from both lines is 280 slot machines.

If x slot machines are produced on line A, then $\frac{3}{4}x$ slot machines are produced on line B. Thus, the total output from both lines is represented by

$$x + \frac{3}{4}x.$$

Since the total output is 280, we have

$$x + \frac{3}{4}x = 280.$$

Solving for x, we obtain

$$\frac{7}{4}x = 280$$

$$x = 160.$$

■

EXAMPLE 7 The developer of the Buckeye Industrial Park plans to divide 572 acres of land into 8-acre and 12-acre plots. If there are to be three times as many 12-acre plots as 8-acre plots, find the number of each type of plot that can be in the industrial park if the entire 572 acres are developed.