Studies in Applied Electromagnetics and Mechanics 20

# ELECTROMAGNETIC FIELDS, WAVES AND NUMERICAL METHODS

Z. Haznadar and Ž. Štih

IOS Press
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# Electromagnetic Fields, Waves and Numerical Methods

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IOS Press

Ohmsha

Amsterdam • Berlin • Oxford • Tokyo • Washington, DC

ISBN 1 58603 064 7 (IOS Press)
ISBN 4 274 90396 6 C3042 (Ohmsha)
Library of Congress Catalog Card Number: 00-106862

*Publisher*
IOS Press
Nieuwe Hemweg 6B
1013 BG Amsterdam
The Netherlands
fax: +31 20 620 3419
e-mail: order@iospress.nl

*Distributor in the UK and Ireland*
IOS Press/Lavis Marketing
73 Lime Walk
Headington
Oxford OX3 7AD
England
fax: +44 1865 75 0079

*Distributor in the USA and Canada*
IOS Press, Inc.
5795-G Burke Centre Parkway
Burke, VA 22015
USA
fax: +1 703 323 3668
e-mail: iosbooks@iospress.com

*Distributor in Germany*
IOS Press
Spandauer Strasse 2
D-10178 Berlin
Germany
fax: +49 30 242 3113

*Distributor in Japan*
Ohmsha, Ltd.
3-1 Kanda Nishiki-cho
Chiyoda-ku, Tokyo 101
Japan
fax: +81 3 3233 2426



PRINTED IN THE NETHERLANDS

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*Editors*

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Volumes 1-6 have been published by Elsevier Science under the series title "Elsevier Studies in Applied Electromagnetics in Materials".

ISSN: 1383-7281

# INTRODUCTION

The aim of this book is to present classical electromagnetic theory, which includes electromagnetic fields and waves, in a modern way suitable for application in engineering problems and research. It has been written with the intention of serving as a resource to scientists and engineers, and as a textbook and source of knowledge in the study of electrical engineering. This book may be utilised as a textbook in advanced undergraduate and postgraduate courses on electromagnetic fields and waves in engineering universities, especially in electrical engineering faculties.

Electromagnetism is a fundamental topic with a variety of different applications. Its inherent complexity results in the need for usage of sophisticated mathematical procedures. Therefore, we assume that the reader has a knowledge of vector calculus, differential and integral equations and matrix algebra, together with a basic knowledge of electricity and magnetism. The mathematical prerequisites are summarised in the appendices.

Classical electromagnetic theory has been founded on a macroscopic approach that enables the neglecting of the granular structure of matter and charge. We may summarise all knowledge about macroscopic electricity and magnetism using only a few equations and definitions. Accordingly, our approach to problems in electrical engineering has mostly been deductive. Everything originates from Maxwell's equations. Therefore, within this book, we have not followed a historical sequence, but have chosen a logical sequence of subject presentation.

In the first chapter, we start by using Maxwell's equations as postulates upon which the entirety of classical electromagnetic theory has been based. Initially, the study of the basic laws of electromagnetism is restricted to those in free space. After that we adjust these laws to their application in materials.

The introduction of new supplementary functions known as electromagnetic potentials to represent the field is the subject of the second chapter. These mathematical functions, called vector and scalar potentials, are introduced in accordance with Maxwell's equations and their usage significantly simplifies the analysis of electromagnetic fields.

Electromagnetic energy is tackled in the third chapter by analysing the power an electromagnetic field delivers to charge moving in free space. Afterwards, we include the charge on which the macroscopic model of a material has been built. The resulting equation of energetic balance, known as Poynting's theorem, will be our guide in establishing the different forms of energy stored and transmitted by means of an electromagnetic field in both free space and a material.

In continuation, we shall develop the application of this theory to solutions of problems in electromagnetism. The simplest problems involve static fields, where field quantities are independent of time. So, in the fourth chapter, we go on to analyse problems in a static electric field caused by charge at rest. Special attention will be paid to the analytical solution procedures used for solving static electric fields, i.e. the direct solutions of

Laplace's equation by a separation of variables, the solution of problems in unbounded space and a determination of Green's functions by the method of images.

The next logical step in the study of electromagnetism is the analysis of an electromagnetic field caused by charge in uniform motion. The fundamental characteristic of such a field is that it can be split into two independent fields: a static current and static magnetic field. In the fifth chapter, we shall study a static current field in a conductive material and prove its analogy with a static electric field.

The fundamental characteristics of a static magnetic field are described at the beginning of the sixth chapter. In continuation, we look into how to solve problems in current-free regions by means of an antipotential, and how to solve problems relating to given currents in an unbounded space by means of the magnetic vector potential. We shall solve problems associated with given distributions of current in a bounded space by application of the method of images.

Fields in which we may neglect displacement currents are called quasi-static electromagnetic fields. In the seventh chapter, we shall describe a reduced system of Maxwell's equations and derive diffusion equations for field quantities in such a field. Afterwards, we shall introduce a phasor notation which greatly simplifies the solution of sinusoidal fields. The solution of quasi-static field problems is arranged into two classes: problems involving conductors of large thickness and unknown current distribution, and problems involving thin conductors and uniformly distributed currents.

In the eighth chapter, we shall study the propagation of homogeneous transverse plane waves through conductive materials and dielectrics. The phenomenon of dispersion of a wave is analysed in both dielectrics and metals. Afterwards, we go on to analyse the incidence of electromagnetic waves on a plane boundary between two materials. The propagation of guided plane waves is applied in the analysis of transmission lines, waveguides and resonant cavities. Finally, we study the Hertz dipole and a linear antenna in an unbounded space as sources of radiation.

The finite element method, which is a wide-spread numerical tool for solving field problems, is described in the ninth chapter. We have applied a step-by-step approach based on describing the solving of actual electromagnetic problems. We begin with a detailed description of all the essential steps needed to be taken in the solution of simple, two-dimensional static problems. In continuation, we outline non-linear problems and finally we describe the solving of dynamic quasi-static fields.

The method of moments is introduced at the beginning of the tenth chapter. Afterwards, we describe the application of this method to the solution of three-dimensional static electric field problems and the scattering of electromagnetic waves. Finally, we go on to describe thin-wire structures, introduce Pocklington's equation and finally show the application of the method of moments to the solution of radiation problems.

Throughout this book, we have presented a lot of elaborate examples that explain the introduced methods and concepts in detail. These examples were solved and graphical presentations were generated using the programming system *Mathematica*®.

Zagreb, June 2000.

Zijad Haznadar
Željko Štih

*To*:

*Biserka* and *Leila*
*Alemka* and *Vilim*

# CONTENTS

# 1. THE ELECTROMAGNETIC FIELD

The basic laws of electromagnetic theory were founded at the beginning of the nineteenth century. In 1862, James Clerk Maxwell (1831 – 1879) expressed them in the form of a harmoniously coupled set of equations. Since then we have used *Maxwell's equations* for the precise mathematical modelling of macroscopic electromagnetic phenomena. These fundamental equations of classical electromagnetic theory describe the physical properties of an electromagnetic field and connect them with the charges and currents which represent sources.

The approach to the theory of electromagnetic fields applied in this book is a deductive one, contrary to the usual historical approach. It starts with a presentation of the fundamental postulates focussed upon in Maxwell's equations that completely covers the results of experimental research over many years, as well as research of electromagnetic phenomena. Subsequently, using these postulates as laws of electromagnetism, we shall derive and thus describe the mathematical and physical properties of electromagnetic fields and waves.

Originally, studies of electromagnetic theory tended to develop an "acting at a distance" concept, meaning that the forces acting on a particular charge or current element were taken to be directly caused by forces on that charge or current element resulting from all other sources. By contrast, Maxwell accepted Faraday's ideas. Michael Faraday (1791 – 1867) introduced the idea of an electric field that exists at all points of the media surrounding the source charge. He presented a field in analogy to stresses in elastic materials and called the media aether. By the introduction of "displacement currents", Maxwell completed and mathematically formulated a unique theory of electromagnetic fields. He wrote his classic equations in 1862.

In this chapter, we shall define electric and magnetic fields by means of the law for Lorentz force (Hendrik Antoon Lorentz, 1853 – 1928) separately from the fundamental postulates of electromagnetic fields. Initially, the study of the basic laws of electromagnetism will be restricted to those in free space. After that we shall adjust these laws to their application in materials. We shall analyse the macroscopic behaviour of fields in conductors, dielectric and magnetic materials, thus postulating that the influence of polarised and magnetised materials on the laws of electromagnetic fields can be modelled by the addition of polarized and magnetized charge and currents to free charge and currents already existing within the fundamental laws.

# 1.1. A PRESENTATION OF AN ELECTROMAGNETIC FIELD

For our purpose, we can define a *field* as a set of values associated to every point of a specific region that defines the acting of a particular quantity in the region. Accordingly, if a quantity is associated with every point of the region and if the changing in time and space of this quantity is defined by laws expressed as functions of space and time coordinates, that field is associated with the region.

In the study of classical electromagnetism in this book, we shall analyse scalar and vector fields in three-dimensional Euclidian space. We shall study static and time-dependent dynamic fields.

It is very important to note that an electromagnetic field is defined by a set of fundamental laws of physics that can be expressed in mathematical form by differential equations in three-dimensional space and time. Differential equations join the character of a field at a particular point to those of other fields and sources at the same point independently of causes placed everywhere else. Hence the fact that the character of an electromagnetic field depends directly upon the sort and properties of the material at a given point and in its enclosure. Consequently we can analyse the general features of various fields in some bounded region of space without any interest in the state in space outside of the chosen region. This is one of the fundamental characteristics of Faraday's and Maxwell's approach to electromagnetism.

## 1.1.1. Definitions of the fields *E* and *B*

We associate the forces observed in electromagnetic experiments with the existence of an electric charge and its motion. We can represent the distribution of charge and its motion as either a cause or a source, and an observed force as a consequence or an effect. The existence of charge and its motion produces electromagnetic field in a space. There are a few laws of forces that describe the physical properties of every electromagnetic system and associated electric field. In order to start the study of an electromagnetic field and its character directly, we shall first of all define certain field quantities. Hence we do not start with *Coulomb's law of force* (Charles Auguste de Coulomb, 1736 – 1806) as has been historically established. We shall define an electric and magnetic field using the *Lorentz force law* separately from other postulates. A number of experiments with electromagnetic forces show agreement with a few fundamental postulates:

***1. The existence of an electric charge***

According to the atomic-electron theory of electricity, there are in existence two types of electric charges: *positive* and *negative*. All charges in nature are multiples of the charge of the electron, whose present-day accepted value is $q_e = (1.6021892 \pm 0.0000007)\cdot 10^{-19}$ coulombs. The unit of electric charge is defined by means of Coulomb's law and we measure it in *coulombs* (C), which is equivalent to ampere-seconds (As). In this book, we