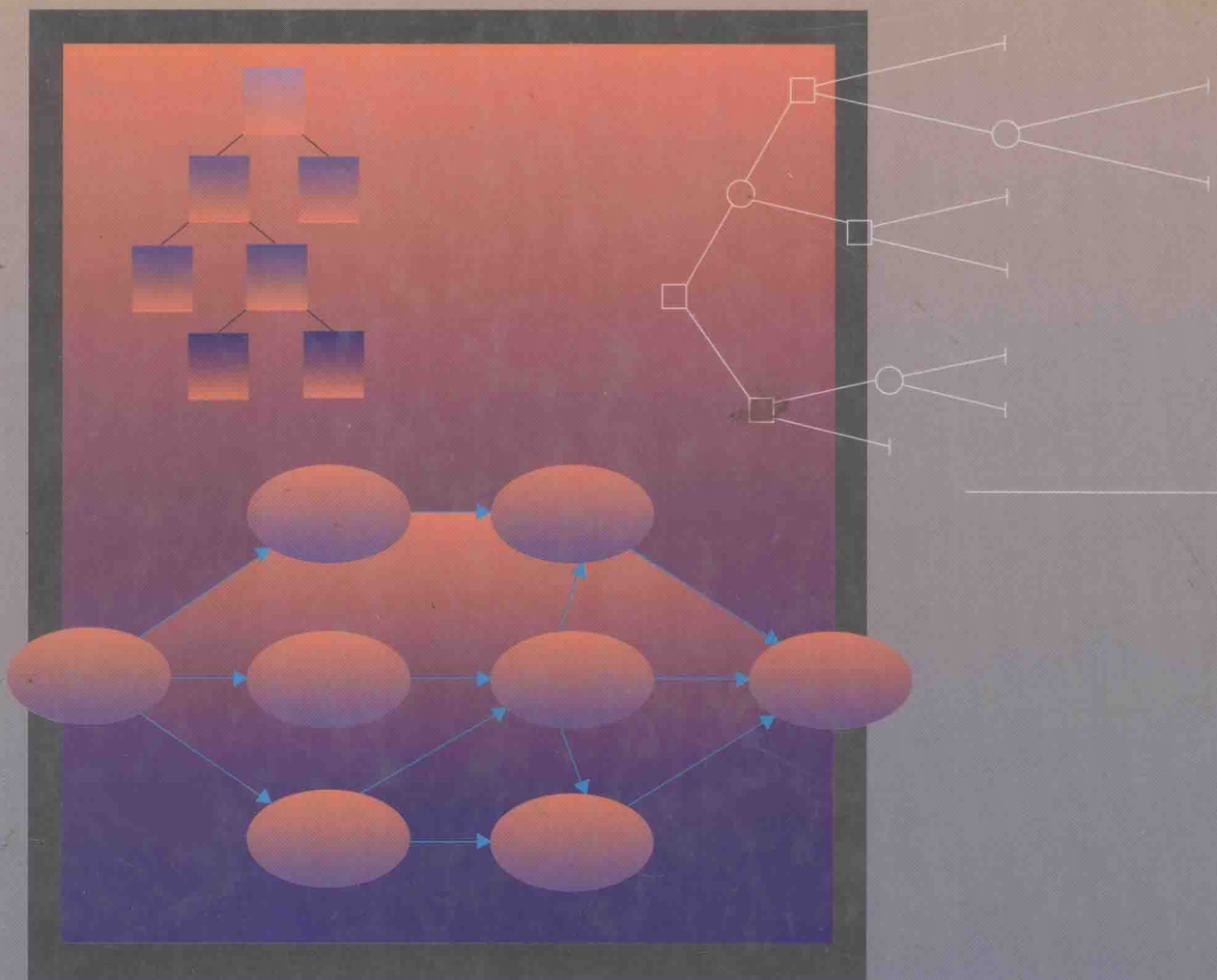


# OPERATIONS RESEARCH

Applications and  
Algorithms

THIRD  
EDITION



WAYNE L. WINSTON

THIRD EDITION

# Operations Research

## Applications and Algorithms

WAYNE L. WINSTON

Indiana University



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## Addressing New Needs

**I**n recent years, operations research software for mainframes and micro-computers has become widely available. Like most tools, however, it is useless unless the user understands its application and purpose. Users must ensure that mathematical input accurately reflects the real-life problems to be solved and that the numerical results are correctly applied to solve them. With this in mind, this book emphasizes model-formulation and model-building skills as well as the interpretation of software output.

## Intended Audience

This book is intended to be used as an advanced beginning or an intermediate text in operations research, management science, or mathematical programming. The following groups of students can benefit from using it:

- Undergraduate majors in quantitative methods in business, operations research, management science, or industrial engineering
- MBA or Masters of Public Administration students enrolled in applications-oriented operations research, management science, or mathematical programming courses
- Graduate students who need an overview of the major topics in operations research and management science
- Practitioners who need a comprehensive reference

## Organization

The book contains enough material for a three-semester course; this allows instructors ample flexibility in adapting the text to their individual course plans. For instance, courses in which certain chapters would be applicable include:

- Linear programming: Chapters 2–10 (omitting either Chapter 5 or Chapter 6)
- Deterministic models in operations research: Chapters 2–12, 15–16, and 20 (again omitting either Chapter 5 or Chapter 6)
- Probabilistic models in operations research: Chapters 13–15, 17–19, and 21–24

- An operations research or management science course with a model-building emphasis: Chapter 3; Sections 4.14, 7.1, 7.5, 7.6, 8.2–8.6, 9.1, 9.2, 12.1, and 13.1–13.3; Chapters 14, 16, 17, 19, and 22; Section 23.3; Sections 23.10–23.14; and Chapter 24

## Making Teaching and Learning Easier

The following features help to make this book reader-friendly:

- To provide immediate feedback to students, problems have been placed at the end of each section, and most chapters conclude with review problems. There are more than 1400 problems, grouped according to difficulty: Group A for practice of basic techniques, Group B for underlying concepts, and Group C for mastering the theory independently.
- The book avoids excessive theoretical formulas in favor of word problems and interesting problem applications. Many problems are based on published applications of operations research and management science. The exposition takes great pains, by means of several examples in each chapter, to guide the student step-by-step through even complex topics. Operations research algorithms still receive comprehensive treatment; for instance, Karmarkar's method for solving linear programming problems is explained in detail.
- To help students review for tests, each chapter has a summary of concepts and formulas. An instructor's manual with complete solutions to all problems and advice for instructors is available to adopters of the text. Solutions to many of the problems on disk are also available.
- Each section is written to be as self-contained as possible. This allows the instructor to be extremely flexible in designing a course. The instructor's manual identifies which portions of the book must be covered as prerequisites to each section.
- The book contains instruction in using the popular LINDO, GINO, LINGO, and What's Best! software packages and interpreting their output. PC versions of LINDO, GINO, and linear and nonlinear LINGO come with the book. The nonlinear version of LINGO (which solves linear, integer, and nonlinear programming problems) requires a 386 machine with a math coprocessor. What's Best! may be purchased from the publisher.

The LINDO, GINO, LINGO, and What's Best! sections are completely self-contained, and classes who do not use them can simply skip those sections. Sections marked with a dagger (†) cover optional topics and may be omitted without loss of continuity.

- Six cases are included. Several of these cases integrate topics from several chapters. Most of the cases require the student to run regressions to obtain inputs into a mathematical programming model. These cases should give the student a flavor of what the actual practice of OR is like.
- All computer files referred to in the text (along with the software packages LINDO, GINO, and LINGO) come in a disk in DOS format bound into the book. The text is also available with a Macintosh version.

The linear programming section of the book (Chapters 2–10) is completely self-contained, with all necessary mathematical background given in Chapter 2. Students who are familiar with matrix multiplication should have no problems with Chapters 2–10. Portions of the remaining chapters require a rudimentary knowledge of cal-

culus and probability equivalent to that obtained from a one-semester calculus course and a one-semester business statistics course. All topics in calculus and probability used in Chapters 12–24 are reviewed in Chapter 11.

Since not all students need a full-blown theoretical treatment of sensitivity analysis, there are two chapters on the topic. Chapter 5 is an applied approach to sensitivity analysis, emphasizing the interpretation of computer output. Chapter 6 contains a full discussion of sensitivity analysis, duality, and the dual simplex method. The instructor should cover Chapter 5 *or* Chapter 6, but not both. Classes emphasizing model-building and model-formulation skills should cover Chapter 5. Those paying close attention to the algorithms of mathematical programming (particularly classes in which many students will go on to further operations research courses) should study Chapter 6. If Chapter 5 rather than Chapter 6 is covered, then Chapter 2 may be omitted.

## Changes in the Third Edition

The favorable response to the first two editions of this book has been truly gratifying. Many suggestions from users of the text have been incorporated into the third edition. There are almost 200 new problems (mostly word and formulation problems) and minor changes in many sections of the book. The major change in the third edition is the great increase in material devoted to computer solution of OR problems. ‡ indicates in the table of contents where this material appears. A brief description of the new material devoted to computer solution of OR problems follows.

We have devoted much of the new edition to a description of the structured modeling package LINGO. Real world practitioners of OR do most of their work with structured modeling packages, and it is time that textbooks caught up! We begin our discussion of LINGO in Section 4.8. Other sections in which LINGO is discussed include:

- Use of LINGO to solve transportation problems (Section 7.1)
- Use of LINGO to solve maximum flow problems (Section 8.3)
- Use of LINGO in critical path problems (Section 8.4)
- Use of LINGO to solve minimum cost network flow problems (Section 8.5)
- Use of LINGO to solve integer programming problems (Section 9.2)
- Use of LINGO to solve traveling salesperson problems (Section 9.6)
- Use of LINGO to solve nonlinear programming problems (Section 12.9)
- Use of LINGO to determine posterior probabilities (Section 13.4)
- Use of LINGO to solve two-person zero-sum games (Section 15.3)
- Use of LINGO in inventory problems (Section 17.7)
- Use of LINGO to determine steady-state probabilities and mean first passage times (Section 19.5)
- Use of LINGO to determine a steady-state census in a work force planning model (Section 19.7)
- Use of LINGO in queuing problems (Sections 22.6, 22.9, and 22.11)

Throughout Chapter 12 we have expanded our coverage of GINO. In Section 17.7 we have described how GINO can be used to solve inventory problems. In Sections 22.6, 22.9, and 22.11 we have described how GINO can be used to solve queuing problems.

The spreadsheet is a very useful and important tool for solving many OR problems. In the following sections we discuss spreadsheet solutions of many OR problems:

- In Section 4.14 we show how the spreadsheet add-in What's Best! can be used to solve LPs.
- In Section 7.1 we show how What's Best! can be used to solve transportation problems.
- In Section 12.4 we show how a spreadsheet can be used to implement Golden Section Search.
- In Section 12.9 we explain how What's Best! can be used to solve nonlinear programming problems.
- In Section 14.3 we show how to use a spreadsheet to implement the Analytic Hierarchy Process (AHP).
- In Sections 16.2–16.5 we show how to use spreadsheets to perform the basic calculations for deterministic EOQ models.
- In Section 19.6 we show how to use a spreadsheet to perform the calculations needed to analyze an absorbing Markov chain.
- In Section 20.9 we show how a spreadsheet can be used to solve dynamic programming problems.
- In Section 22.3 we show how a spreadsheet can be used to determine steady-state probabilities for birth–death models.
- In Sections 22.4–22.6, 22.9, and 22.11, we show how a spreadsheet can be used to solve queuing problems.
- In Sections 23.10–23.14 we show how a spreadsheet can be used to perform simulations.
- In Section 24.3 we show how to use a spreadsheet to do exponential smoothing.
- In Sections 24.6–24.8 we show how to use a spreadsheet to perform regression analysis.

In addition to the previously discussed changes, the third edition contains several other changes:

- Six full-length cases have been added. These cases vary in difficulty, but they will impart a real-world flavor to any operations research and management science class.
- In Sections 5.2 and 6.8 we discuss how shadow prices should be interpreted in the presence of degeneracy.
- Section 6.12 discusses the widely used Data Envelopment Analysis.
- Chapter 12 has been completely rewritten. It now contains much more discussion of the art of formulating nonlinear programming problems. The discussion of Lagrange multipliers (Section 12.7) and Kuhn–Tucker conditions (Section 12.8) has also been expanded to include a graphical interpretation of each.
- Section 14.4 discusses the use of tradeoff curves in multiattribute decision making.

## Using the Computer Software

A disk bound with the book contains the install program, LINDO (with LINDO files referred to in text), GINO (with GINO files referred to in text), linear LINGO (with LINGO files referred to in text), the nonlinear LINGO program, and Lotus 123 files



referred to in the text. The files are stored in compressed form. To write the files to a hard drive or a floppy disk, insert the disk into your A drive and type “install” in response to the A prompt. Then follow the on-screen instructions. If the files have been written to a hard drive, you may copy them back to a floppy disk so that you can have a portable version of LINDO, LINGO, or GINO to carry with you. The installation process will also result in several sample files (created by LINDO systems) for LINDO, LINGO, and GINO (indicated by a .LND extension) being copied to each program’s directory.

*Operations Research: Applications and Algorithms* seeks to cover the important topics in operations research and management science at a beginning-to-intermediate level. It is hoped that after reading the book’s discussions of particular topics students will be able to (and will want to) delve into the more specialized references at the end of the chapters, which take up the topics in more detail.

## Acknowledgments

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The LINDO, GINO, LINGO, and What’s Best! printouts appear courtesy of Professor Linus Schrage. I would like to thank all the people at LINDO systems (particularly Mark Wiley, Linus Schrage, and Kevin Cunningham) for their cooperation with the project.

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I retain responsibility for all errors and would love to hear from users of the book.  
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