CALCULUS GEMS

Brief Lives and Memorable Mathematics

George F. Simmons

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Professor of Mathematics
Colorado College

With portraits by
Maceo Mitchell

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ABOUT THE AUTHOR

George F. Simmons has the usual academic degrees (CalTech, Chicago, Yale), and taught at several colleges and universities before joining the faculty of Colorado College in 1962, where he is Professor of Mathematics. He is also the author of *Introduction to Topology and Modern Analysis* (McGraw-Hill, 1963), *Differential Equations with Applications and Historical Notes* (McGraw-Hill, 1972, 2d edition 1991), *Precalculus Mathematics in a Nutshell* (Janson Publications, 1981), and *Calculus with Analytic Geometry* (McGraw-Hill, 1985).

When not working or talking or eating or drinking or cooking, Professor Simmons is likely to be traveling (Western and Southern Europe, Turkey, Israel, Egypt, Russia, China, Southeast Asia), trout fishing (Rocky Mountain states), playing pocket billiards, or reading (literature, history, biography and autobiography, science, and enough thrillers to achieve enjoyment without guilt).

For Hope and Nancy,
my wife and daughter,
who *still*
make it all worthwhile

PREFACE

On coming to the end of my work on this book and thinking again about its nature and purpose, I am reminded of W. H. Fowler's Preface to his great *Modern English Usage*: "I think of it as it should have been, with its prolixities docked, its dullnesses enlivened, its fads eliminated, its truths multiplied." And also of W. H. Auden's rueful admission: "A poem is never finished, only abandoned."

Some readers will recognize that this book has been reconstructed out of two massive appendices in my 1985 calculus book, with many additions, rearrangements and minor adjustments. Its direct practical purpose is to provide auxiliary material for students taking calculus courses, or perhaps courses on the history of mathematics. There have been a number of requests that this material be made separately available, and I have been happy to take advantage of this occasion to fill in some gaps and reconsider my opinions. I had a friend who said to me once, "I should probably spend about an hour a week revising my opinions." I treasure the remark and value the opportunity to act upon it.[1]

My overall aims are bound up with the question, "What is mathematics for?" and with its inevitable answer, "To delight the mind and help us understand the world." I hold the naive but logically impeccable view that there are only two kinds of students in our colleges and universities: those who are attracted to mathematics; and those who are not yet attracted, but might be. My intended audience embraces both types.

Part A. This half of the book, entitled Brief Lives, amounts to a biographical history of mathematics from the earliest times to the late nineteenth century. It has two main purposes.

[1] The friend was George S. McCue, late of the Colorado College English Department.

First, I hope in this way to "humanize" the subject, to make it transparently clear that great human beings created it by great efforts of genius, and thereby to increase students' interest in what they are studying. Science—and in particular mathematics—is something that men and women do, and not merely a mass of observed data and abstract theory. The minds of most people turn away from problems—veer off, draw back, avoid contact, change the subject, think of something else at all costs. These people—the great majority of the human race—find solace and comfort in the known and the familiar, and avoid the unknown and unfamiliar as they would deserts and jungles. It is as hard for them to think steadily about a difficult problem as it is to hold together the north poles of two strong magnets. In contrast to this, a tiny minority of men and women are drawn irresistibly to problems: their minds embrace them lovingly and wrestle with them tirelessly until they yield their secrets. It is these who have taught the rest of us most of what we know and can do, from the wheel and the lever to metallurgy and the theory of relativity. I have written about some of these people from our past in the hope of encouraging a few in the next generation.

My second purpose is connected with the fact that many students from the humanities and social sciences are compelled against their will to study calculus as a means of satisfying academic requirements. The profound connections that join mathematics to the history of philosophy, and also to the broader intellectual and social history of Western civilization, are often capable of arousing the passionate interest of these otherwise indifferent students.

Part B. In teaching calculus over a period of many years, I have collected a considerable number of miscellaneous topics from number theory, geometry, science, etc., which I have used for the purpose of opening doors and forging links with other subjects . . . and also for breaking the routine and lifting the spirits. Many of my students have found these "nuggets" interesting and eye-opening. I have collected most of these topics in this part in the hope of making a few more converts to the view that mathematics, while sometimes rather dull and routine, can often be supremely interesting. The English mathematician G. H. Hardy said, "A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas." Part B of this book contains a wide variety of these patterns, arranged in an order roughly corresponding to the order of the ideas in most calculus courses. Some of the sections even have a few problems, to give additional focus to the efforts of students who may read them: Sections A.14, B.1, B.2, B.16, B.21, B.25.

I repeat the fervent hope I have expressed in other books, that any readers who detect flaws or errors of fact or judgment will do me the great kindness of letting me know so that repairs can be made.

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PART A

BRIEF LIVES

Biographical history, as taught in our public schools, is still largely a history of boneheads: ridiculous kings and queens, paranoid political leaders, compulsive voyagers, ignorant generals—the flotsam and jetsam of historical currents. The men who radically altered history, the great creative scientists and mathematicians, are seldom mentioned if at all.

Martin Gardner

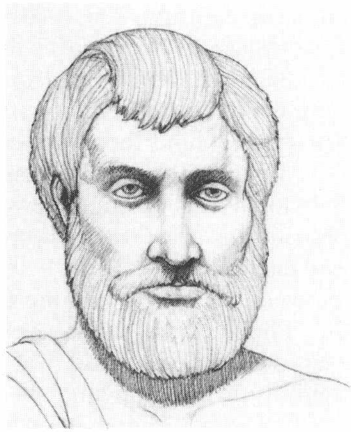
In the index to the six hundred odd pages of Arnold Toynbee's *A Study of History,* abridged version, the names of Copernicus, Galileo, Descartes and Newton do not occur . . . yet their cosmic quest destroyed the mediaeval vision of an immutable social order in a walled-in universe and transformed the European landscape, society, culture, habits and general outlook, as thoroughly as if a new species had arisen on this planet.

Arthur Koestler

Gentlemen, anatomy [read: mathematics] may be likened to a harvest-field. First come the reapers, who, entering upon untrodden ground, cut down great store of corn from all sides of them. These are the early anatomists of modern Europe. Then come the gleaners, who gather up ears enough from the bare ridges to make a few loaves of bread. Such were the anatomists of the last century. Last of all come the geese, who still continue to pick up a few grains scattered here and there among the stubble, and waddle home in the evening, poor things, cackling with joy because of their success. Gentlemen, we are the geese.

Dr. Barclay,
lecturer on anatomy at
Edinburgh University

A.1



THALES
(ca. 625–547 B.C.)

Truth is whatever survives the cleansing fires of skepticism after they have burned away error and superstition. The healthy growth of civilization depends on skepticism more than it does on faith.

Oliver Wendell Holmes

In all history, nothing is so astonishing as the sudden appearance of intellectual civilization in Greece. Many of the other components of civilization—art, religion, complex societies capable of organizing and carrying out enormous projects—had already existed for hundreds or thousands of years, in Egypt, in Mesopotamia, and in China. No one has ever surpassed the Egyptians or Babylonians in monumental stone sculpture, or the Chinese in the production of incomparable works of art in bronze and ceramics. And even today we stand in awe of the organized effort that built the Great Pyramid of Egypt and the Great Wall of China. But what the Greeks achieved in the realm of the intellect is greater still: they invented mathematics, science, and philosophy; they were the first to write genuine analytical history as opposed to mere descriptive annals; and they were the first to speculate freely about the nature of the world and the meaning of life, without being confined by the chains of any stultifying traditional orthodoxy.

It is a good saying that everything has happened at least twice in China, but nevertheless intellectual civilization was born only once, in Greece. It flickered out after several hundred years and was reborn in Western Europe in the 17th century. This greatest creation of the Greek genius has been the powerhouse of Western civilization for more than two thousand years; it has set this civilization apart from all others and has spread over the whole earth, from China to Peru; and it started with Thales and his discovery of skepticism. What lies behind these wonderful events?

In the 7th and 6th centuries B.C. the static older world of Egypt and Babylonia was fading. Newer, more vital peoples, especially the Hebrews, Phoenicians and Greeks, were coming to the center of the stage. The Iron Age was displacing the Bronze Age, and brought with it revolutionary changes in the weapons of war and the tools of agriculture. The alphabet was invented by the Phoenicians, coins were introduced in Lydia, and trade expanded everywhere. The world was ready for a new kind of civilization. This new civilization first appeared in the small trading cities of Ionia on the western coast of Asia Minor, of which Thales' home town, Miletus, was the most important. It later continued its development on the Greek mainland and especially in the Greek colonies on Sicily and the coast of southern Italy.

Who was Thales, and what did he think, and why does it matter? He was the first and most interesting of the pre-Socratic philosophers of ancient Greece, that cluster of a half-dozen or so profoundly original minds that blossomed as if by a miracle in the small city-states of the eastern Aegean Sea during the two centuries before the time of Socrates. Even though very little is definitely known about him, he is the center of a vivid anecdotal tradition nourished by many ancient writers, including Herodotus, Aristophanes, Plato, Aristotle, and Plutarch.[1] He seems to have spent the early part of his life as a successful merchant, acquiring enough wealth to provide himself with security and comfort. He then put business aside and devoted the rest of his life to travel and study, as any man of good sense would do.[2]

He is said to have visited Egypt and the Near East, where he talked with the priests and learned much of their lore; and he astounded them in turn by using shadows and similar triangles to calculate the height of the Great Pyramid. He brought back with him to Miletus knowledge—primitive though it

[1] A good account of the various sources and their relations to one another is given by S. Bochner, *The Role of Mathematics in the Rise of Science,* Princeton University Press, 1966, pp. 364–68.

[2] His business acumen is illustrated by the famous story of the olive-presses. (Olive oil was as basic a commodity in Thales' time as wheat and potatoes are in ours.) Aristotle tells it well (*Politics* 1259a): "He was reproached for his poverty, which was supposed to show that philosophy was of no use. According to the story, he knew by his meteorological skill while it was still winter that there would be a great harvest of olives in the coming year; so, having a little money, he gave deposits for the use of all the olive-presses in Chios and Miletus, which he reserved at low prices because no one bid against him. When the harvest-time came, and many were needed all at once, he rented them out at any rate he pleased, and made a quantity of money. Thus he showed the world that philosophers can easily be rich if they like, but that their ambition is of another sort."

was—of Babylonian astronomy and Egyptian geometry. He then vitalized these fragmentary rules of thumb by placing them in the context of the emerging rationality of which he himself was probably the major primary source.

He was apparently the first Greek astronomer, and his studies began the process of establishing this subject as a legitimate science and disentangling it from its oriental associations with astrology. According to Herodotus, writing more than a century later, he astonished all Ionia by successfully predicting an eclipse of the sun that took place (as modern astronomers tell us) on May 28, 585 B.C.[3] Thales' involvement with astronomy gave rise to another famous anecdote that is probably the ancestor of all absent-minded professor stories.[4] This story goes as follows: One night Thales was looking at the stars, and fell into an irrigation ditch. A witty Thracian maidservant pulled him out and chided him by saying, "How do you expect to understand what's happening up in the sky, when you can't even see what's under your own feet?"

He is also said to have diverted the course of the river Halys so that King Croesus' army could pass more easily; to have advised the small Greek cities of Ionia that if they did not unite, they would be absorbed one by one by the Persian empire; to have offered some new practical rules for navigation by using the stars; and to have calculated the distance of a ship at sea by means of observations made at two points along the shore. And then there is the story that once he was in charge of some mules that were heavily loaded with sacks of salt. While crossing a river one of the animals slipped, with the result that the salt dissolved in the water and the load became instantly lighter. This clever beast deliberately rolled over at the next river-crossing to reduce its load, and continued doing so until Thales cured him of this troublesome trick by loading him with sponges instead of salt.

Thales endeared himself to all later generations of Greeks by his many memorable adventures, together with his uniquely personal combination of practical and theoretical wisdom. On being asked what was most difficult, he answered, "To know thyself." And then when asked what was most easy, he replied, "To give advice." To the question "What is God?" he answered, "That which has neither beginning nor end." When he was asked how men might live most virtuously and justly, he answered, "By never doing ourselves what we blame in others."[5] He was versatile, curious, intense, sensible,

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[3] See Book I, Chapter 74, in the *History* of Herodotus. This eclipse was of interest to Herodotus because it stopped a battle between the Medes and the Lydians. For more on the eclipse, see pp. 137–38 in vol. I. of T. L. Heath, *A History of Greek Mathematics*, Oxford University Press, 1921.

[4] Plato, *Theaetetus* 174a.

[5] See pp. 37 and 39 in vol. I of Diogenes Laertius, *Lives of Eminent Philosophers*, Loeb Classical Library, Harvard University Press, 1966.