

# SHAPE CLASSIFICATION AND ANALYSIS

*THEORY AND PRACTICE*

SECOND EDITION

Luciano da Fontoura Costa  
Roberto Marcondes Cesar, Jr.



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*To the memory of my grandmother Clorinda, my brightest teacher.  
To my wife, Soraya, my ray of sunlight.*

---

*To the memory of my grandmother Ngair, who showed me how great is being alive.  
To my continuously growing family and, especially, to Adriana and Tainá.*

# Preface - Second Edition

**T**HE SECOND EDITION of *Shape Analysis and Classification: Theory and Practice* incorporates important enhancements from the last reprint version. The book has been graphically reformulated and we hope the reader will like the new design (fonts, equations, figures, and references). We have implemented a number of corrections and improvements, many of which suggested by readers of the first edition, to whom we are very grateful. New material has been added, in particular new sections on topics like graph and complex networks, dimensionality reduction as well as a new chapter on structural pattern recognition and shape representation using graphs. The reference list has been updated generally.

The book now includes a new type of Box called *Additional Resources* indicating useful resources on the World Wide Web (WWW) such as software, databases and videos. In particular, some open-source software projects related to the subjects covered in this book have been initiated since its first edition was issued. Links to these open-source projects are now provided throughout the book (also available from <http://ccsl.ime.usp.br/>). The user is invited to know, use and help to develop all these projects! The second edition of the book has a new URL: <http://www.vision.ime.usp.br/~cesar/shape/>. In addition, slides that may be useful in courses and talks based on the book are now available at: <http://www.vision.ime.usp.br/~cesar/shape/slides>.

In addition to all friends that helped us in the first edition, we are grateful to those colleagues, students and readers that provided important input regarding the book. Additionally to the people already mentioned in the preface of the first edition, we are grateful to: Alejandro Frery, Alexandre Crivellaro, Alexandre Noma, Ana Beatriz, Anselmo Kumazawa, Arnaldo Lara, Carlos H. Morimoto, Celina Takemura, David Martins Junior, Emerson Tozette, Fabrício Lopes, Fátima N. Sombra, Flávia Ost, Giovane Cesar, Heitor Nicolliello, Herbert Jelinek, Isabelle Bloch, Jeferson Silva, Jesus Mena, Jishu Ashimine, João Eduardo Ferreira, João Soares, Jorge Leandro, José Mario Martinez, Junior Barrera, Junior Barrera, Luis Augusto Consularo, Luis Gustavo Nonato, Luiz Henrique de Figueiredo, Luiz Velho, Marcel Claro, Marcel Jackowski, Marcelo Hashimoto, Marcos Moreti, Michael Cree, Nina S. T. Hirata, Paulo José da Silva e Silva, Pete Lestrel, Reinhard Klette, Roberto Andreani, Roberto Hirata Jr., Rogério Féris, Ronaldo F. Hashimoto, Routo Terada, Silvia Pinto, Teófilo Campos, Thiago Paixão, and Yossi Zana, as well as to the many students of undergrad and grad courses taught by the authors. In particular, the second edition would not be possible without the valuable work of David da Silva Pires. David, thank you very, very much.

As always, our friends at Taylor & Francis have provided a fundamental support to our work, and we are especially grateful to Amber Donley and Nora Konopka. We would also like to acknowledge the support provided by the following research funding agencies: FAPESP, CNPq, CAPES, FINEP, COFECUB and NSF.

We have worked hard to provide an improved text about the exciting areas of shape analysis and classification. We hope you will enjoy reading and using this new edition.



# Preface - First Edition

**M**OST HUMAN ACTIVITIES—including the reading of this text—and interactions with the environment are performed by visual means, more specifically through the capture of electromagnetic waves and subsequent processing by the human visual system. Given the redundancy underlying most that is visual in nature, evolution has produced visual systems that pay great attention to abrupt variations along the objects, such as their boundaries delimiting uniform regions. Indeed, it was very likely the varied geometry of the outlines of uniform objects in the world that led to the human concept of *shape* and the evolution of visual processing abilities dedicated, or at least related, to the capture, processing, analysis and classification of shapes. Besides, it should be observed that many other natural processes also involve geometric objects, normally interacting through fields and waves. For instance, most biochemical reactions involve the matching between two or more molecules, a process that is governed by their respective shapes and force fields. Similarly, interactions between living beings also involve shape geometry and interactions. For example, the mouth of a whale must be particularly effective for collecting plankton and the wings of birds have to be effective for flying. In brief, the importance of shapes is not only essential to humans, but seems to provide a basic principle underlying most natural processes [Costa et al., 1999].

While the importance of the visual information is now fully acknowledged, defining a scientific area on its own, it was only recently that computer vision came to establish itself as an off-the-shelf tool for both researchers and practitioners. That is to say, it has been the continuous advances in computer technology, including higher processing speed, storage capabilities, better acquisition devices (e.g., cameras and scanners), along with more powerful concepts and algorithms and a progressive cost reduction, that paved the way to the dissemination of imaging techniques through a large number of practical applications in the most diverse areas. Indeed, it is the opinion of the authors of this book that we are undergoing an unprecedented *technological opportunity* for the effective application of image and shape analysis resources to many areas. Yet, while this tendency has been duly reflected in an ever-increasing number of books on imaging, shape processing and analysis, biological and computer vision and pattern recognition, it is felt that only a few (if any) existing textbooks provide an introductory, modern, relatively comprehensive and integrated coverage of shape representation, analysis *and* recognition, which are closely intertwined. In light of the above reasons, we cannot avoid the commonplace of observing that the current book will fill the gap not only between the areas of shape analysis and classification, including one of the most comprehensive lists of practical shape features, but also between theory and practice. While concentrating on 2D shapes, the current book also provides the basis for the treatment of higher dimensional objects.

This book is aimed at serving as a largely self-contained introductory textbook on shape analysis and recognition, including several of the most modern and promising tendencies such as scale-space, analyzing wavelets, fractals, computa-

tional geometry and so on. An important trend that has characterized the evolution of shape analysis is its inter- and multidisciplinary nature, involving an increasing variety of concepts from both mathematics and computer science, including differential geometry, Fourier analysis, signal processing, probability and multivariate calculus, to name a few. This implies that students and researchers alike often experience difficulties while trying to understand many approaches to shape analysis. On the other hand, it is worthy emphasizing that shape analysis is such an important task in so many areas, from biology to material sciences, that it would be highly desirable that experts in all those different areas could understand and apply the techniques explored in this book. As a matter of fact, even those practitioners intending simply to use imaging software should also be acquainted with the basic concepts and techniques in image and shape analysis, in order not only to properly apply the several tools usually bundled into such software, but also interpret the respectively obtained results. As the authors had in mind to create a didactic book that would be accessible to a broad range of readers, a comprehensive and mostly self-contained introduction/review of the involved basic mathematical concepts has been especially prepared and included. In addition, whenever possible the mathematical detail is always preceded by the respective conceptual characterization and discussion and several examples have been included in order to help the assimilation of the presented concepts and techniques. Several special boxes have also been included in the book, being organized according to the following types:

**Examples:** step-by-step examples of the introduced concepts and methods. Such examples can also be treated as exercises, in the sense that the reader should try to solve the presented problems before following up their respective solutions;

**Algorithms:** high-level algorithms that can be straightforwardly implemented in most programming languages, such as C, Java, Delphi®, Pascal and MATLAB® scripts;

**Notes:** additional information on relatively secondary or more advanced topics;

**“To Probe Further” Pointers:** references to more advanced complementary scientific literature, so that the reader can probe further on the state-of-the-art of each specific topic.

In order to increase its readability, this book has been generously illustrated with graphics and images. In addition, a **WWW homepage** (see the box at the end of this preface) has been created as a complement, which will be continuously updated with additional examples, data, programs, errata and URL pointers to related areas. Given the introductory and multidisciplinary nature of the present book, as well as space limitations, several important related works and bibliographical references have certainly been overlooked, for which we apologize in advance. In addition, please observe that although great care has been taken in preparing this book, the authors can accept no responsibility for the direct or indirect consequences of the use of algorithms or any other information in this book.

The book starts by discussing the main operations often involved in shape analysis. Chapter 2 provides a self-contained review of the main mathematical concepts that are fundamental to proper understanding of shape analysis and recognition. Since shape analysis by computers involves acquiring and processing digital images of the objects of interest, an overview of the most typical image processing techniques is provided in Chapter 3. It should be observed that while most students and practitioners from other fields will find Chapters 2 and 3 helpful, computer vision and pattern recognition experts may prefer to directly proceed to Chapter 4, which discusses and formalizes the main concepts in 2D shapes. Chapter 5 covers some of the main techniques for shape representation by digital means (i.e., computers), while Chapters 6 and 7 present several important techniques for extracting information from shapes, the latter concentrating on multiscale techniques, including curvature and wavelets. Chapter 8 presents the main techniques for classifying shapes, with emphasis on the more traditional and sound statistical approaches. Finally, some brief considerations about the future perspectives in shape processing and analysis are presented in Chapter 8.

We would like to express our gratitude to a number of people and institutions. First, we thank Prof. Philip Laplante for the kind invitation to contribute to the *CRC Image Processing Series*. Whatever the merits of the present book, Phil deserves the full credit for the initiative. Also fundamental was the kind support of all CRC staff, especially Ms. Elizabeth Spangenberg, Nora Konopka, Michele Berman, Sara Seltzer and Suzanne Lassandro. We thank Prof. Héctor Terenzi (USP) and Prof. Hernan Chaimovich (USP), through the *Pró-Reitorias* of the University of São Paulo, who made the whole project possible by kindly providing financial support. In addition, many of the reported results and techniques have been developed by the authors under sponsorship of several Brazilian funding agencies, including FAPESP, CNPq, CAPES and CCInt, having also involved several students. We also thank all those researchers who so kindly read and commented on preliminary versions of the chapters in this book, including Andréa G. Campos, Carlos P. Dionísio, Cristian Montagnoli, Jorge J. G. Leandro, Prof. Maria Cristina F. de Oliveira, Prof. Nelson D. A. Mascarenhas, Luiz A. Consularo, Luiz G. Rios, Renato Carmo, Rogério S. Feris and Vital Cruvinel Ferreira. Students who have also contributed in the production of some of the data and results presented in this book are Alexandre Crivellaro, Andréa G. Campos, Franklin C. Flores, Heraldo Madeira, Luiz A. Consularo, Marcel Brun, Nina T. Hirata, Regina C. Coelho, Roberto Hirata and Teófilo Campos. Important discussions and suggestions have been provided by many colleagues, especially Carlos Hitoshi, Damien Barache, Herbert Jelinek, Isabelle Bloch, J.-P. Antoine, João Kogler, Junior Barrera, Pierre Vanderghenst and Roberto Lotufo. A good deal of the diagrams in this book were drawn by Samuel Alvarez and additional technical support was provided by Adenilza Alves, Alexandre Silva, Cláudia Tofanelli, Edna Torres, Marcos Gonçalves and Rita de Cássia, to whom we also extend our acknowledgements. Luciano Costa is especially grateful to his wife, Soraya, for her understanding and help with the revision of this book.

**◆◆◆◆◆◆◆◆ BOOK HOMEPAGE ◆◆◆◆◆◆◆◆**

The reader should not miss the book's homepage, containing a series of additional related information, including related pointers, data files, and errata, to be found at the following address:

[http://www.ime.usp.br/~cesar/shape\\_crc/](http://www.ime.usp.br/~cesar/shape_crc/)

The authors also appreciate receiving comments through e-mail, which should be addressed to [luciano@if.sc.usp.br](mailto:luciano@if.sc.usp.br) or [cesar@ime.usp.br](mailto:cesar@ime.usp.br).

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