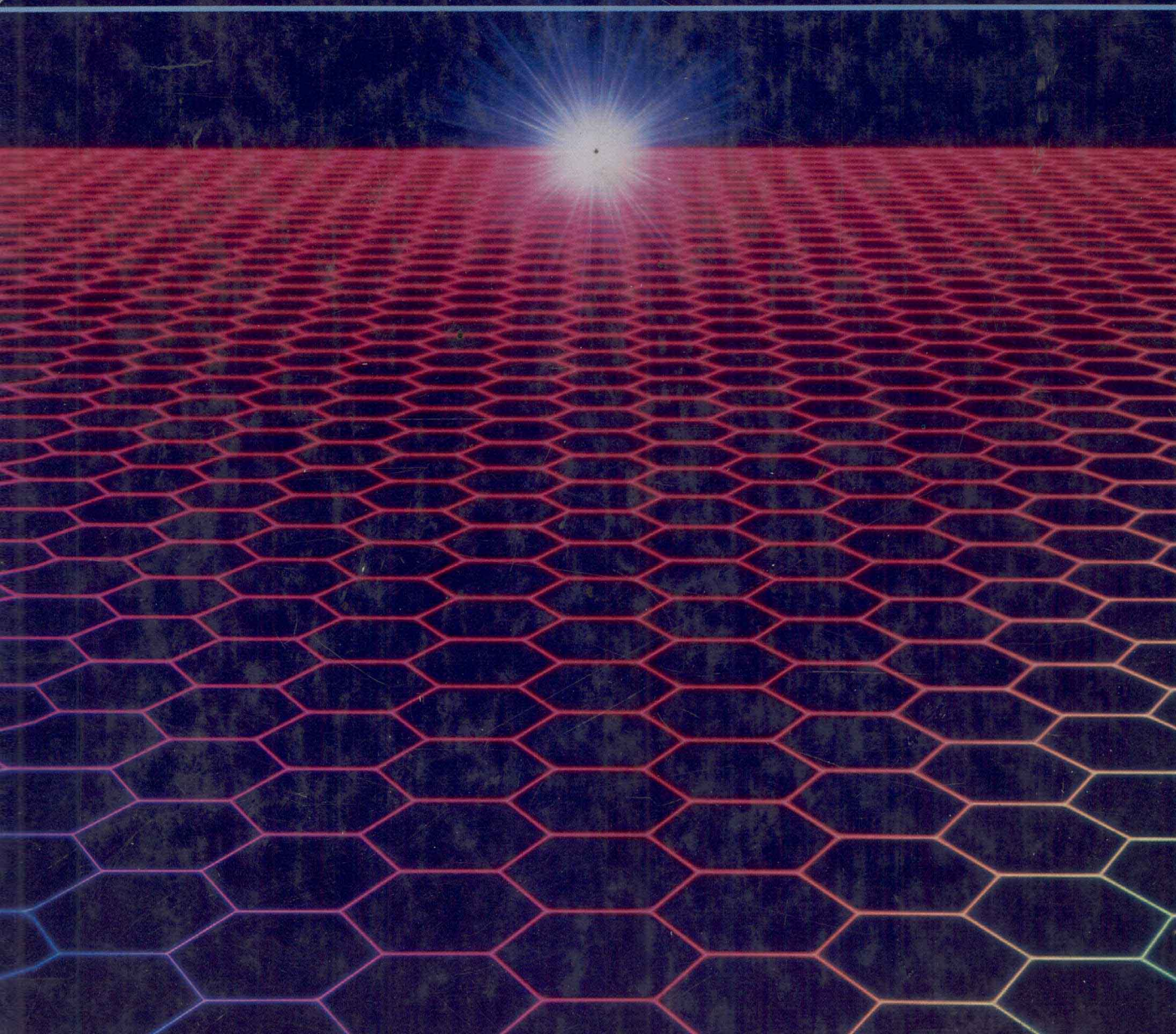


THIRD EDITION

# FINITE MATHEMATICS AND ITS APPLICATIONS

LARRY J. GOLDSTEIN • DAVID I. SCHNEIDER • MARTHA J. SIEGEL







**THIRD EDITION**


# **FINITE MATHEMATICS AND ITS APPLICATIONS**

Larry J. Goldstein  
David I. Schneider

*Department of Mathematics  
University of Maryland*

Martha J. Siegel

*Department of Mathematics  
Towson State University*



*Library of Congress Cataloging-in-Publication Data*

Goldstein, Larry Joel.

Finite mathematics and its applications / Larry J. Goldstein,  
David I. Schneider, Martha J. Siegel.—3rd ed.

p. cm.

Includes index.

ISBN 0-13-317355-0

1. Mathematics—(date) I. Schneider, David I. II. Siegel, Martha  
J. III. Title.

QA39.2.G643 1988

510—dc19

Editorial/production supervision: Maria McColligan  
Interior design: Lee Cohen, Walter A. Behnke  
Chapter opening design: Lorraine Mullaney  
Cover design: Lorraine Mullaney  
Cover photo: FPG International  
Manufacturing buyer: Paula Benevento  
Photo research: Kay Delloso

**Finite Mathematics and Its Applications, third edition**

Larry J. Goldstein/David I. Schneider/Martha J. Siegel



© 1988, 1984, 1980 by Prentice Hall, Inc.  
A Division of Simon & Schuster  
Englewood Cliffs, New Jersey 07632

All rights reserved. No part of this book may be  
reproduced, in any form or by any means,  
without permission in writing from the publisher.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-317355-0 01

Prentice-Hall International (UK) Limited, *London*  
Prentice-Hall of Australia Pty. Limited, *Sydney*  
Prentice-Hall Canada Inc., *Toronto*  
Prentice-Hall Hispanoamericana, S.A., *Mexico*  
Prentice-Hall of India Private Limited, *New Delhi*  
Prentice-Hall of Japan, Inc., *Tokyo*  
Simon & Schuster Asia Pte. Ltd., *Singapore*  
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

### **Chapter Opening Photo Credits**

- 0** FPG International
- 1** George E. Jones III / Photo Researchers, Inc.
- 2** Visicorp
- 3** Ethan Allen
- 4** Rapho / Photo Researchers, Inc.
- 5** J. Schweiker / Photo Researchers, Inc.
- 6** Ken Karp
- 7** IBM Corporation
- 8** Karen Sichieri / Givaudan Corporation
- 9** Hella Hammid / Photo Researchers, Inc.
- 10** Lenone Weber / Taurus Photos
- 11** Stephen Capra

## Mathematics and Its Applications

This volume is one of a collection of texts for freshman and sophomore college mathematics courses. Included in this collection are the following.

*Calculus and Its Applications, fourth edition* by L. Goldstein, D. Lay, and D. Schneider. A text designed for a two-semester course in calculus for students of business and the social and life sciences. Emphasizes an intuitive approach and integrates applications into the development. Much expanded from the highly successful third edition.

*Brief Calculus and Its Applications, fourth edition* by L. Goldstein, D. Lay, and D. Schneider. Consists of the first eight chapters of the above book. Suitable for shorter courses.

*Finite Mathematics and Its Applications, third edition* by L. Goldstein, D. Schneider, and M. Siegel. A traditional finite mathematics text for students of business and the social and life sciences. Allows courses to begin with either linear mathematics (linear programming, matrices) or probability and statistics.

*Mathematics for the Management, Life, and Social Sciences, second edition* by L. Goldstein, D. Lay, and D. Schneider. A text for a two-semester course covering finite mathematics, precalculus, and calculus.

# PREFACE

---

This work is the third edition of our text for the traditional finite mathematics course taught to first- and second-year college students, especially those majoring in business and the social and biological sciences. Finite mathematics courses exhibit tremendous diversity with respect to both content and approach. Therefore, in revising this book, we have incorporated a wide range of topics from which an instructor may design a curriculum, as well as a high degree of flexibility in the order in which the topics may be presented. In the case of the mathematics of finance, we have even allowed for flexibility in the approach of the presentation.

In this edition, we have attempted to maintain our popular student-oriented approach. This approach manifests itself throughout and, in particular, in the following features:

- Applications** We provide realistic applications that illustrate the uses of finite mathematics in other disciplines. The reader may survey the variety of applications by turning to the Index of Applications on page xv. Wherever possible, we have attempted to use applications to motivate the mathematics. For example, the idea of linear programming is introduced in Chapter 3 via a discussion of production options for a factory with a labor limitation.
- Examples** We have included many more worked examples than is customary (398). Furthermore, we have included computational details to enhance readability by students whose basic skills are weak.
- Exercises** The more than 1300 exercises comprise about one-quarter of the text—the most important part of the text in our opinion. The exercises at the ends of the sections are usually arranged in the order in which the text proceeds, so that the homework assignments may easily be made after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Supplementary exercises at the end of each chapter expand the

other exercise sets and provide cumulative exercises that require skills from earlier chapters.

#### **Practice Problems**

The practice problems introduced in the previous edition have proved to be a popular and useful feature and are included in the present edition. The practice problems are carefully selected exercises that are located at the end of each section, just before the exercise set. Complete solutions are given following the exercise set. The practice problems often focus on points that are potentially confusing or are likely to be overlooked. We recommend that the reader seriously attempt the practice problems and study their solutions before moving on to the exercises. In effect, the practice problems constitute a built-in workbook.

#### **Minimal Prerequisites**

Because of great variation in student preparation, we have kept the formal prerequisites to a minimum. We assume only a first year of high school algebra. Furthermore, we review, as needed, those topics which are typically weak spots for students.

### **New in This Edition**

Among the many changes in this edition, the following are the most significant.

1. *Additional Examples and Exercises.* The ample stock of exercises and examples in the first and second editions has been significantly expanded. The additional examples introduce new topics and applications and serve to amplify explanations where needed. Some new exercises were designed to reinforce material added to this edition; others were posed to give students thought-provoking questions that challenge their understanding of the underlying concepts. Most of the new exercises require a level of comprehension beyond that needed to mimic the examples and practice problems. Additional word problems allow the student to gain some skill in elementary mathematical modeling.
2. *Notes.* Several notes have been added to this edition to enable students to better understand the subtleties of the mathematical theory.
3. *Simplex Method.* We have added a new section on sensitivity analysis that demonstrates the utility of the final simplex tableau. New examples and exercises have been supplied to illustrate these ideas. Students will find that all the numbers in the final tableau are useful in adding meaning to their interpretation of the models they construct. Linear programming computer software having the capability to display the final tableau can be used effectively with the text.
4. *Duality Theory.* We have greatly extended our treatment of duality in linear programming problems. New sections emphasize the duality theorem as a tool in solving minimization problems and in decisions regarding the profitability of the addition of new items to the industrial production line. The section on the economic interpretation has been revised and expanded.
5. *Probability.* We have changed the order of topics to provide more motivation for the normal distribution. The binomial distribution is introduced earlier in the

chapter. We use the notion of percentiles and other examples familiar to students to explain the use of a normal random variable.

6. *Statistics.* Chapter 7 has been rewritten with emphasis on calculations in both samples and populations, and with an explanation of the use of statistics to estimate parameters. The treatment is kept at the most elementary level, but should prepare students for a bonafide basic statistics course.
7. *Game Theory.* We have expanded Chapter 9 to include more material on the link between the fundamental theorem of game theory and the fundamental theorem of duality.

Note that the book divides itself naturally into three parts. The first part consists of “linear mathematics”—linear equations, matrices, and linear programming (Chapters 1-4); the second part is devoted to probability and statistics (Chapters 5-7); the third part consists of topics utilizing the ideas of each of the other parts (Chapters 8-11). We prefer to begin with linear mathematics since it makes for a smooth transition from high school mathematics and leads rather quickly to interesting applications, especially linear programming. Our preference notwithstanding, the instructor may begin this book with Chapter 5 (Sets and Counting) and then do either the linear mathematics or the probability and statistics.

Answers to the odd-numbered exercises are included at the back of the book. Answers to the even-numbered exercises are contained in the Instructor’s Manual.

If you have comments or suggestions, we would like to hear from you. We hope that you enjoy using this book as much as we have enjoyed writing it.

## Acknowledgments

While writing this book, we have received assistance from many persons. And our heartfelt thanks goes out to them all. Especially, we should like to thank the following reviewers, who took the time and energy to share their ideas, preferences, and often their enthusiasm, with us.

*Reviewers of the first edition:* James F. Hurley, University of Connecticut; Sam Councilman, California State University, Long Beach; Carl D. Meyer, Jr., North Carolina State University; Stephen H. Brown, Auburn University; Bart Braden, Northern Kentucky University; James C. Thorpe, University of Missouri, St. Louis; Joseph Stampfli, Indiana University; Martin C. Tangora, University of Illinois, Chicago Circle; William D. Blair, Northern Illinois University; Richard Pellerin, Northern Virginia Community College; Roger Osborn, University of Texas, Austin; Thomas J. Hill, University of Oklahoma, Norman; Donald E. Myers, University of Arizona, Tempe.

*Reviewers of the second edition:* D.R. Dunninger, Michigan State University; Hiram Paley, University of Illinois, Urbana-Champaign; Joan McCarter, Arizona



State University; Robert Carmignani, University of Missouri; Philip Kutzko, University of Iowa; Juan Gatica, University of Iowa; Frank Warner, University of Pennsylvania; Richard Porter, Northeastern University; James Hurley, University of Connecticut; William Ramaley, Fort Lewis College; Donald E. Myers, University of Arizona, Tempe.

*Reviewers of the third edition:* Robin G. Symonds, Indiana University at Kokomo; Elizabeth Teles, Montgomery College; Charles J. Miller, Foothill Community College.

The authors would like to thank the many people at Prentice Hall who have contributed to the success of our books. We appreciate the tremendous efforts of the production, art, manufacturing, marketing, and sales departments. Our sincere thanks go to Maria McColligan for courageously and effectively undertaking the mammoth editorial task posed by our series of books and to Lee Cohen and Lorraine Mullaney for their beautiful design work. An extra special thanks to David Ostrow, Acquisitions Editor at Prentice-Hall, for his help in planning and executing these new editions. His partnership and friendship have added a warm personal dimension to the writing process.

*Larry J. Goldstein  
David I. Schneider  
Martha J. Siegel*

# INDEX OF APPLICATIONS

## Business and Economics

Amortization, 452ff, 457, 460  
Analyzing advertising strategies, 430  
Analyzing business and investment options, 56, 336, 419, 429, 432  
Annuity, 443ff, 440, 448, 449, 492, 493, 494, 502  
Auditing of income tax returns, 302  
Break-even point, 41  
Business competition, strategies, 404, 411  
Changes in stock prices, 200  
Closed Leontieff model, 100  
Comparing car dealerships, 305, 318, 320, 331  
Comparing costs of car rentals, 44  
Compound interest, 435, 436, 438, 439, 440, 441, 442, 460, 469, 470, 472, 473, 477, 494, 502  
Conglomerates, 101, 102  
Consumer habits, 404  
Consumer loans, 456, 457, 458, 460, 469, 470, 473, 474, 477, 488  
Consumer price index, 196  
Cost and revenue curves, 33, 38  
Counting number of ways of assigning employees to tasks, 241, 243  
Counting possibilities for management positions, 218, 223  
Deciding whether to diversify, 187, 192, 193  
Deferred annuity, 450  
Depreciation, 33  
Determining production schedules, 46  
Double-declining balance method of depreciation, 478  
Earning of investment trust, 72  
Effectiveness of advertising campaign, 35, 367  
Fixed cost, 33, 38  
Fluctuations in agricultural prices and production, 488, 498  
Input-output analysis, 96, 97, 99, 101, 102, 104  
Insurance probabilities, 286, 298, 325, 327  
Inventory planning, 355  
Investment planning, 56, 460, 461

- Labor costs, 77
- Linear depreciation, 11, 471
- Marginal cost of production, 33, 38
- Marginal revenue of production, 41
- Maximizing investment returns, 123, 132
- Maximizing profits, 159, 168, 171
- Minimizing transportation costs, 126, 129, 133, 163, 175, 176
- Mortgage, 454, 457, 460, 477, 478, 488, 494, 502
- Optimizing production schedules, 106, 111, 113, 120, 121, 132, 134, 158, 159, 168, 175, 176, 182
- Organizing economic and business data, 196, 197, 201
- Perpetuity, 450
- Quality control, 257, 264, 269, 271, 284, 286, 292, 293, 294, 297, 298, 303, 333, 336, 340, 358, 359, 363, 367, 369, 401
- Revenue of a business, 37, 68, 72
- Sales commission, 40, 43
- Sales performance, 344, 367
- Salvage value, 11, 34
- Savings account transactions, 434, 436, 440, 441, 442, 463, 464, 468, 487
- Shadow prices, 183
- Shipping, 111, 120, 126
- Simple interest, 438, 439, 471, 474
- Sinking fund, 458
- Straight line method of depreciation, 32
- Supply and demand curves, 25, 28, 38
- Unemployment, 197
- Volume of sales, 354, 355, 359

## **Social Sciences**

- Accidental nuclear war, 294
- Census statistics, 300, 339
- Commuter trends, 390
- Counting options for Senate committee selection, 232
- Counting ways to cast ballots, 238
- Crime statistics, 299
- Demographic analysis, 85, 87, 339
- Effects of immigration and emigration, 468, 469, 499, 502
- Effects of maternal influence, 374, 376, 384
- Evaluating performance in tests, 344, 350, 366
- Exam scores, 330, 359
- Exam strategies, 370
- I.Q. scores, 359
- Jury decisions, 345
- Learning process, 380, 500
- Marriage trends, 85, 87
- Opinion sampling, 275, 289
- Political polling, 224, 253, 286, 344
- Population growth, 468
- Positions for political candidates, 411
- Psychology experiments, 266, 294, 381, 390
- Social mobility, 401
- Spread of information, 488, 497, 502

Test for discrimination, 344, 369

Voter analysis, 77

Voter patterns, 382, 390

## **Biology and Medicine**

Birth weights of infants, 353, 355

Counting possible outcomes of a medical experiment, 216

Designing an optimum diet, 64, 116, 121, 132, 135, 167, 176, 185

Epidemiology, 77, 88, 393, 399

Frequency of cricket chirps, 39

Genetics, 258, 345, 390, 500

Glucose infusion, 499

Growth of bacteria, 488, 496

Heights of adult males, 369

Heights of elephants, 358

Life expectancy of wildlife, 320

Medical diagnosis, 248, 286

Medical screening, 272, 286, 291, 294, 296

Movement of solutes through a cell membrane, 499

Nutrition planning, 56, 64

Organizing medical data, 248

Recovery from disease, 340, 364, 366

Removal of drug from bloodstream, 499

Which group had a fewer cavities?, 326

## **General Interest**

Absorption of light, 499

Bridge, 242

Carnival games, 314, 316, 327

Coin tossing, 219, 227, 228, 232, 253, 257, 258, 260, 267, 275, 283, 307, 315, 316, 341, 345, 365

Comparing sports performances, 326, 336

Counting options for everyday tasks, 207, 208, 210, 212, 213, 214, 215, 218, 219, 224, 226, 229, 230, 231, 232, 238, 243, 244, 245, 246

Counting possible contest outcomes, 224, 242, 243, 244

Counting possible outcomes of sports events, 214, 220, 231, 232, 245, 246

Counting travel options, 217, 219, 220, 228

Dice, 251, 260, 263, 267, 282, 302, 314, 323, 327, 337, 345, 365

Energy needs, 500

Gambler's ruin, 398

Games, 264, 281, 283, 410, 411, 430

Nobel laureates, 210

Odds, 266

Optimizing exercise routine, 109

Planning political campaign, 168

Poker, 268, 275

Pollution control, 251, 255

Predicting sports performance, 302, 326, 335, 339, 344, 366, 367, 369, 370

Probabilities of everyday occurrences, 251, 256, 257, 258, 263, 267, 268, 273, 274, 275, 276, 277, 278, 279, 283, 286, 288, 294, 302, 344

Radioactive decay, 488, 496

Roulette, 267, 323, 326, 327, 366

Skydiving, 487

# **FINITE MATHEMATICS AND ITS APPLICATIONS**

---



# CONTENTS

---

Preface xi

Index of Applications xv

Introduction 1

## **1 Linear Equations and Straight Lines** 4

1.1 Coordinate Systems and Graphs 5

1.2 Linear Inequalities 13

1.3 The Intersection Point of a Pair of Lines 22

1.4 The Slope of a Straight Line 28

## **2 Matrices** 45

2.1 Solving Systems of Linear Equations, I 46

2.2 Solving Systems of Linear Equations, II 57

2.3 Arithmetic Operations on Matrices 65

2.4 The Inverse of a Matrix 79

2.5 The Gauss-Jordan Method for Calculating Inverses 90

2.6 Input-Output Analysis 96

## **3 Linear Programming, A Geometric Approach** 105

3.1 A Linear Programming Problem 106

3.2	Linear Programming, I	113
3.3	Linear Programming, II	123
<b>4</b>	<b>The Simplex Method</b>	136
4.1	Slack Variables and the Simplex Tableau	137
4.2	The Simplex Method, I - Maximum Problems	147
4.3	The Simplex Method, II - Minimum Problems	160
4.4	Marginal Analysis and Matrix Formulations of Linear Programming Problems	168
4.5	Duality	177
<b>5</b>	<b>Sets and Counting</b>	194
5.1	Sets	195
5.2	A Fundamental Principle of Counting	203
5.3	Venn Diagrams and Counting	209
5.4	The Multiplication Principle	216
5.5	Permutations and Combinations	221
5.6	Further Counting Problems	227
5.7	The Binomial Theorem	233
5.8	Multinomial Coefficients and Partitions	239
<b>6</b>	<b>Probability</b>	247
6.1	Introduction	248
6.2	Experiments, Outcomes, and Events	249
6.3	Assignment of Probabilities	259
6.4	Calculating Probabilities of Events	269
6.5	Conditional Probability and Independence	277
6.6	Tree Diagrams	288
6.7	Bayes' Theorem	295
<b>7</b>	<b>Probability and Statistics</b>	304
7.1	Frequency and Probability Distributions	305
7.2	The Mean	317

7.3	The Variance and Standard Deviation	328
7.4	Binomial Trials	338
7.5	The Normal Distribution	345
7.6	Normal Approximation to the Binomial Distribution	360
<b>8</b>	<b>Markov Processes</b>	371
8.1	The Transition Matrix	372
8.2	Regular Stochastic Matrices	383
✓ 8.3	Absorbing Stochastic Matrices	391
<b>9</b>	<b>The Theory of Games</b>	403
9.1	Games and Strategies	404
9.2	Mixed Strategies	412
✓ 9.3	Determining Optimal Mixed Strategies	419
<b>10</b>	<b>The Mathematics of Finance</b>	433
10.1	Interest	434
10.2	Annuities	443
10.3	Amortization of Loans	452
<b>11</b>	<b>Difference Equations and Mathematical Models</b>	462
11.1	Introduction of Difference Equations, I	463
11.2	Introduction of Difference Equations, II	470
11.3	Graphing Difference Equations	478
11.4	Mathematics of Personal Finance	490
11.5	Modeling with Difference Equations	495
	<b>Appendix Tables</b>	A1
	Table 1 Areas under the Standard Normal Curve	A1
	Table 2 $(1 + i)^n$ Compound Amount of \$1 Invested for $n$ Interest Periods at Interest Rate $i$ per Period	A2
	Table 3 $\frac{1}{(1 + i)^n}$ Present Value of \$1 Principal That Will Accumulate to \$1 in $n$ Interest Periods at a Compound Rate of $i$ per Period	A3

Table 4  $s_{\overline{n}|i}$  Future Value of an Ordinary Annuity of  $n$  \$1 Payments Each, Immediately after the Last Payment at Compound Interest Rate of  $i$  per Period A4

Table 5  $\frac{1}{s_{\overline{n}|i}}$  Rent per Period for an Ordinary Annuity of  $n$  Payments, Compound Interest Rate  $i$  per Period, and Future Value \$1 A5

Table 6  $a_{\overline{n}|i}$  Present Value of an Ordinary Annuity of  $n$  Payments of \$1 One Period Before the First Payment with Interest Compounded at  $i$  per Period A6

Table 7  $\frac{1}{a_{\overline{n}|i}}$  Rent per Period for an Ordinary Annuity of  $n$  Payments Whose Present Value is \$1 with Interest Compounded at  $i$  per Period A7

**Answers to Odd Numbered Exercises** A8

**Index** A29