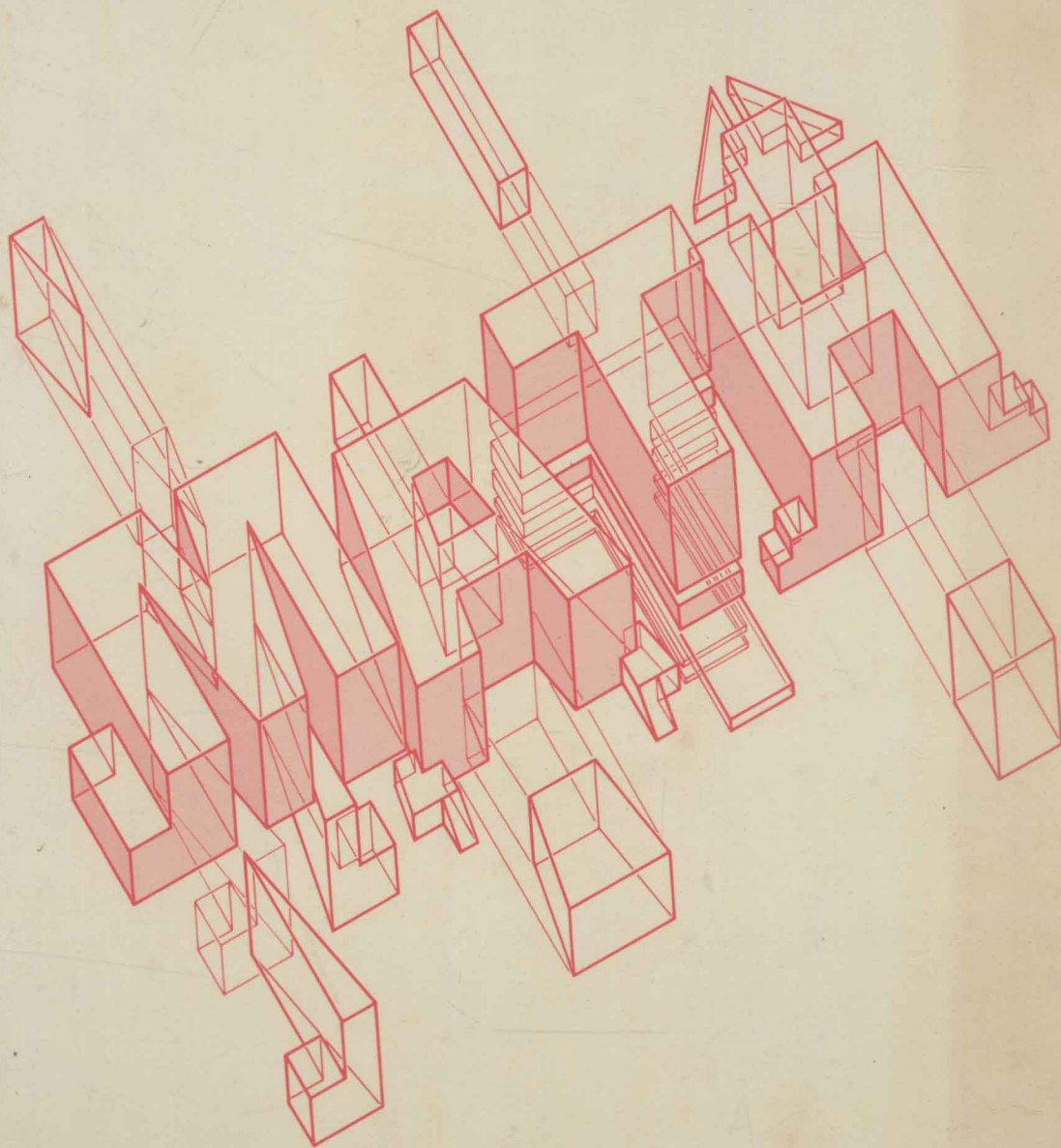


BASIC TECHNICAL MATHEMATICS

CHARLES R. WALL



BASIC TECHNICAL MATHEMATICS

CHARLES R. WALL

Trident Technical College



HARCOURT BRACE JOVANOVIĆ, PUBLISHERS

*San Diego New York Chicago Atlanta Washington, D.C.
London Sydney Toronto*

For my wife, Deborah
sine qua non

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Calculator Keys

Data Entry Keys

$\boxed{0} \dots \boxed{9}$	Digits
$\boxed{\cdot}$	Decimal point
$\boxed{\pi}$	Pi key, enters 3.14159...
$\boxed{+/-}$	Change sign key. (See section 1.2.)
\boxed{EE}	Enter Exponent key. (See section 1.5.)
$\boxed{D/R}$ or \boxed{DRG}	Angle mode key. (See section 5.3.)
\boxed{STO}	Store key. (See section 1.6.)
\boxed{RCL}	Recall key. (See section 1.6.)
\boxed{SUM} or $\boxed{\Sigma}$	Sum key. (See section 1.6.)

Function Keys

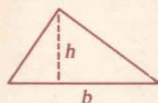
$\boxed{+/-}$	Change sign key. (See section 1.2.)
$\boxed{1/x}$	Reciprocal key. (See section 1.2.)
$\boxed{x^2}$	Square key. (See section 1.3.)
$\boxed{\sqrt{x}}$	Square root key. (See section 1.3.)
\boxed{SIN}	Sine key. (See section 5.3.)
\boxed{COS}	Cosine key. (See section 5.3.)
\boxed{TAN}	Tangent key. (See section 5.3.)
\boxed{LOG}	Common logarithm key. (See section 14.3.)
$\boxed{\ln x}$	Natural logarithm key. (See section 14.3.)

Special Keys

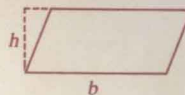
$\boxed{=}$	Equals
\boxed{CE}	Clear Entry
\boxed{C}	Clear
\boxed{INV}	Inverse. (See section 5.3 and section 14.3.)

Operation Keys

$\boxed{+}$	Addition
$\boxed{-}$	Subtraction
$\boxed{\times}$	Multiplication
$\boxed{\div}$	Division
$\boxed{y^x}$	Exponentiation key. (See section 1.4.)



area: $\frac{1}{2}bh$



area: bh

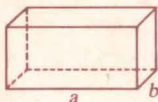


area: πr^2

circumference: $2\pi r$



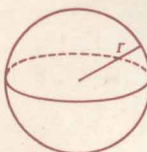
area: $\frac{\theta}{2}r^2$



volume: abc

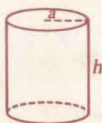
surface area:

$2ab + 2ac + 2bc$



volume: $\frac{4}{3}\pi r^3$

surface area: $4\pi r^2$



volume: $\pi a^2 h$

surface area: $2\pi ah$



volume: $\frac{1}{3}\pi a^2 h$

surface area:

$\pi a \sqrt{a^2 + h^2}$

Lines

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$

Slope-intercept form of the equation of a line: $y = mx + b$

Point-slope form of the equation of a line: $y - y_1 = m(x - x_1)$

Two-point form of

the equation of a line: $y - y_1 = m(x - x_1)$ with $m = \frac{y_2 - y_1}{x_2 - x_1}$

Properties of Inequality

Addition: If $a < b$, then $a + c < b + c$.

Multiplication: $\begin{cases} \text{If } a < b \text{ and } c > 0, \text{ then } ac < bc. \\ \text{If } a < b \text{ and } c < 0, \text{ then } ac > bc. \end{cases}$

Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Quadratic Formula

If $ax^2 + bx + c = 0$ with $a \neq 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(Continued on inside back cover)

Formulas

Fractions

$$\frac{a \cdot x}{b \cdot x} = \frac{a}{b}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Factoring

$$x^2 - y^2 = (x - y)(x + y)$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 + (b + d)x + bd = (x + b)(x + d)$$

$$ax + bx = x(a + b)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Exponents and Radicals

$$x^m \cdot x^n = x^{m+n}$$

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$x^{p/q} = (x^{1/q})^p$$

$$(x^m)^n = x^{mn}$$

$$\sqrt[q]{x} = x^{1/q}$$

$$(xy)^n = x^n \cdot y^n$$

$$\sqrt[q]{xy} = \sqrt[q]{x} \cdot \sqrt[q]{y}$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$\sqrt[q]{\frac{x}{y}} = \frac{\sqrt[q]{x}}{\sqrt[q]{y}}$$

$$x^0 = 1$$

Logarithms

$$y = \log_b x \text{ if and only if } x = b^y$$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$\log_b(mn) = \log_b m + \log_b n$$

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

$$\log_b m^x = x \cdot \log_b m$$

Change of base formula:

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Angle Measurement

$$\pi \text{ radians} = 180^\circ$$

$$1 \text{ radian} = \left(\frac{180}{\pi}\right)^\circ \approx 57.2958^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ radians} \approx 0.017453 \text{ radians}$$

Complex Trigonometry

$$\text{cis } \theta = \cos \theta + j \sin \theta$$

$$(r_1 \text{ cis } \theta_1)(r_2 \text{ cis } \theta_2) = r_1 r_2 \text{ cis } (\theta_1 + \theta_2)$$

$$\frac{r_1 \text{ cis } \theta_1}{r_2 \text{ cis } \theta_2} = \frac{r_1}{r_2} \text{ cis } (\theta_1 - \theta_2)$$

$$\text{DeMoivre's Theorem: } (r \text{ cis } \theta)^n = r^n \text{ cis } (n\theta)$$

General Trigonometry

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sin(\theta + 2\pi) = \sin \theta$$

$$\cos(\theta + 2\pi) = \cos \theta$$

$$\tan(\theta + \pi) = \tan \theta$$

Sum and Difference Formulas

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

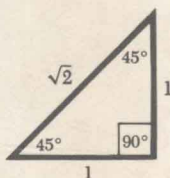
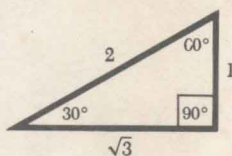
$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

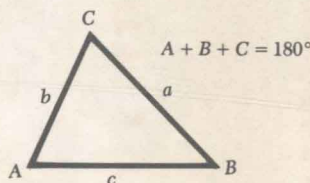
$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Special Triangles



θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—
$\cot \theta$	—	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0
$\sec \theta$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	—
$\csc \theta$	—	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1

Oblique Triangles



Law of sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos A$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Double-Angle Formulas

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Half-Angle Formulas

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$



BASIC TECHNICAL MATHEMATICS

Preface

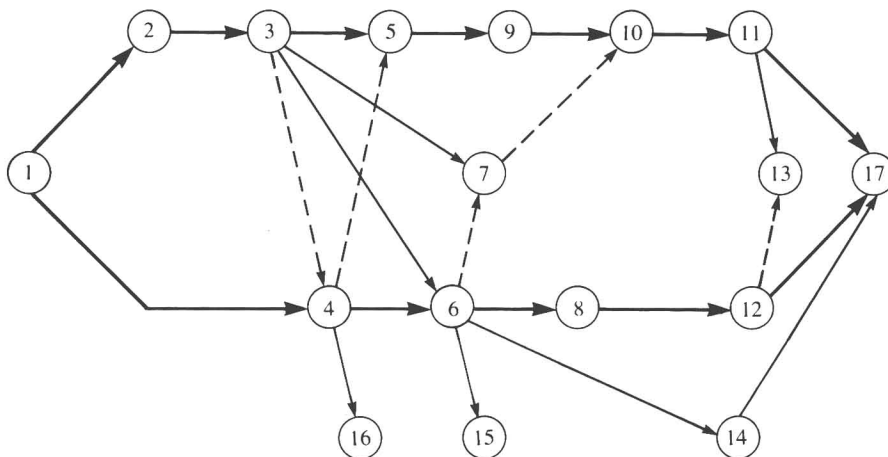
This textbook is designed for students in technical, scientific, and engineering technology programs. It covers the main precalculus topics of algebra, trigonometry, and analytic geometry and includes material on descriptive statistics and an introduction to limits. The prerequisite is two years of high school mathematics including algebra and geometry. However, the discussion begins at a fairly low level to help students brush up on topics partially forgotten.

Main Features

- The book is based on the philosophy that numbers should illuminate rather than mystify—be a path to understanding rather than a roadblock.
- This is a deliberately “user friendly” textbook, written for students rather than for professional mathematicians. The approach is completely intuitive (there are a few formal proofs in the appendixes, but none in the text proper), and the style is informal. However, mathematical correctness has not been sacrificed for informality.
- Calculator usage is stressed throughout, and keystroke sequences appear in examples in

nearly every chapter. There are many benefits to relying heavily on the calculator: if computations need not be done by hand, problems can be less contrived and more realistic; there is no need for trig and log tables; and the student can start thinking algorithmically.

- There is more emphasis on functions and their graphs than in most technical mathematics books, providing a solid background for future work in calculus.
- The treatment of systems of equations in Chapter 7 is unified, allowing the student to see what these problems have in common and to use the same problem-solving strategy.
- Chapter 11 features a novel treatment of trigonometric identities as equations which happen always to be true. This avoids the sudden shift of emphasis that occurs in other books. If traditional proofs are not needed elsewhere in the book, they are not needed here.
- The organization of the book allows for flexibility in organizing courses. The following chapter dependency chart allows for several sequences of topics. In the chart, solid lines indicate direct dependency, while dashed lines indicate a weaker dependence that can be worked around.



- There are 758 carefully selected examples. In general, early examples in a section illustrate routine cases, while later examples show some of the “twists” that can occur.
- There are 3,980 problems. Working problems is far more important to gaining understanding than even the best examples and the most lucid explanations. Proficiency comes with practice, and there is ample opportunity here for practice. Answers appear at the end of the book for odd-numbered section problems and for all of the end-of-chapter review problems.
- There are numerous applications. The goal of this book, and indeed of technical mathematics in general, is not so much to teach applied math as to teach mathematics that can be applied. However, the best indication of how topics may be applied is an assortment of examples and problems from diverse areas. The List of Applications that follows this Preface shows the variety of uses appearing in the text.
- All but three sections within the chapters end with a set of exercises, and each chapter concludes with a Progress Check collection of review problems.

Supplementary Materials

The Instructor’s Manual includes answers to even-numbered exercises as well as comments and teaching tips for each chapter. It also contains reproducible copies of some text figures.

Content Review

Chapter 1 covers the basic details of calculator use. Instructors may skip this chapter or make it a reading assignment.

There are two major threads in algebra: manipulating expressions and solving equations. Algebraic manipulation is the topic of Chapter 2, while Chapter 3 covers elementary equations and inequalities. Inequalities are included early because of their importance in describing functions. Much of the material in these chapters may be skipped if student backgrounds are sufficiently strong.

Chapter 4 begins the study of functions, and right triangle trigonometry is taken up in Chapter 5 for those students pursuing a concurrent physics course.

There are two logical ways to continue the study of first degree equations after Chapter 3. One way is to consider equations of second degree, as in Chapter 6, and another is to take up several first degree equations in several unknowns, which is done in Chapter 7. Also, Chapter 6 includes “disguised quadratic equations,” and Chapter 7 includes systems of non-linear equations.

In Chapter 8, we return to functions and their graphs, studying linear and quadratic graphs, functions with “split definitions,” and some topics (such as periodicity) that are required in later chapters.

In Chapter 9, the trigonometric functions are defined for arbitrary angles, and the graphs of the trig functions are examined. Applications of the trig functions are covered in Chapter 10, while the subject of Chapter 11 is trigonometric equations and identities.

Polynomials of higher degree are taken up in Chapter 12, along with rational functions and several optional topics that might be needed later.

Chapter 13 covers complex numbers and their applications. Exponential and logarithmic functions are studied in Chapter 14.

Chapter 15, on conic sections, may be studied any time after Chapter 6, but the topic appears late in the book because many instructors prefer to use it as a lead-in to calculus. Similarly, Chapter 16 on statistics may be taken up any time after Chapter 4.

Chapter 17 offers an introduction to limits. One goal of this material is to prepare students for calculus, but another goal is to provide a powerful method for obtaining good approximate solutions to equations that cannot be solved exactly.

Acknowledgments

Only one name appears on the title page, but this book is nevertheless the result of work by many

people. It is a pleasure to acknowledge the contributions of the following people.

My colleagues at Trident Technical College, especially Henry Berry, Elizabeth Bliss, Jon Craig, and Henry Graves. Collectively and individually they made many fine suggestions and acted as superb sounding boards for ideas.

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thors often gripe privately about reviewers not reading their work in detail, but I can make no such complaint. The reviewers did uniformly excellent work, and I appreciate their efforts, comments, and suggestions.

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List of Applications

Aeronautics

airplane 5.5
rocketry 3.1, 10.3

Architecture

bridge 5.5, 15.3
building measurements 5.5, 6.4
ramp design 5.5
seating capacity 6.4
suspension bridge 15.5
tunnel 15.5
whisper chamber 15.3

Astronomy and space

moon radius 5.5
satellite orbit 10.4, 15.3
solar system 14.7, 15.3, 16.3
telescope 15.4

Automotive

antifreeze 7.7
headlights 15.5

Biology 3.1

Business and finance

earnings 3.3
formulas 3.1
grocery store 4.2
income tax 4.2, 8.3
insurance 3.3
investment mixtures 3.2, 7.7
rental rates 8.2
shared expenses 6.4
social security tax 8.3

Calculators 1.7

Chemistry

isotopes 16.2
pH 3.4, 14.6

Computers

BASIC Appendix E
numeration bases 12.8

Distance and motion

distance 8.1
falling object 3.5
motion 3.1
path of particle 10.4
projectile 8.2, 15.5
stopping distance 14.7
stride 6.4
velocity 8.1

Electricity

AC circuit 9.3, 13.6
battery life 14.6
cardioid microphone 10.4
formulas 2.7, 3.1
Kirchoff laws 7.7
LC circuit 9.3
Lissajous figure 10.5
oscilloscope 10.5
power 3.5, 6.4
response curves 14.7
resistance 3.3, 3.5, 8.1
resistors 6.4, 7.7
signal strength 3.5
transmitting antenna 5.5

Energy 3.1

Fluids

fluid mechanics 2.7
formula 3.1

Geology 2.7

Illumination 14.7

Machine shop

machine parts 10.2
machine tool 5.5

Management 3.3, 16.3

Measurement 2.5

Mechanisms

gears 3.1, 3.5
levers 7.7
pendulum 9.3
pulleys 3.1
springs 2.7, 3.5, 8.1, 9.3
vibrating string 3.5

Medicine 3.1

Miscellaneous

age problems 3.2
angle between lines 11.2
archaeology 14.6
artillery observer 5.1
baseball diamond 10.2
fertilizer 3.5
industrial espionage 7.7
inequalities 12.7
music 14.7
robotics 15.2
telephone area codes 4.2
tolerances 3.4

Mixtures 3.2, 7.7

Navigation 5.5, 10.2, 10.3, 15.4

Optics and photography

f-stops 17.2
lenses 6.4
photography 3.1, 6.4
shadows 5.5

Physics

braking force 10.3
carbon dating 14.6
clock hands 9.2
gases 2.7, 3.5
gravitation, Newtonian 3.5
hydrostatics 2.7
mechanics 2.7
power 3.5
radiation 10.4
radioactivity 14.6
relativity 2.7
ship power 3.5
tension (rope) 10.3
vectors 10.3

Social sciences

death rates 16.3, 16.4
population density 1.5
population growth 14.6, 14.7
unemployment & crime rate 16.3

Sound 14.6**Statistics** Chapter 16**Strength of materials**

beams 2.7, 3.1, 3.3, 3.5
crushing load 14.7

Structures

bridge 5.5, 9.3
hanging cable 14.4
road bed 5.5
stovepipe 15.3
trough 6.4

Surveying 5.5, 8.2, 10.2**Thermal**

absolute zero 16.4
heat 3.1
insulation 3.5
Kelvin scale 16.4
Newton's law of cooling 14.6
temperature 8.1
thermodynamics 3.1

Contents

Preface v

List of Applications viii

1 Calculator Arithmetic

- 1.1 Keys and Features 1
- 1.2 Arithmetic of Signed Numbers 4
- 1.3 Squares and Square Roots 11
- 1.4 Powers and Roots 15
- 1.5 Scientific Notation 18
- 1.6 Store, Recall, and Sum 23
- 1.7 Number Sets and Properties 28
- Progress Check 31

2 Algebraic Manipulation

- 2.1 Literal Quantities 33
- 2.2 Operations and Simplification 34
- 2.3 Factoring 39
- 2.4 Algebraic Fractions 48
- 2.5 Units of Measure and Conversions 55
- 2.6 Rational Exponents 59
- 2.7 Operations with Radicals 65
- Progress Check 72

3 Basic Equations and Inequalities

- 3.1 Linear Equations 74
- 3.2 Problem-Solving Techniques 85
- 3.3 Linear Inequalities 93
- 3.4 Absolute Value 99
- 3.5 Variation 104
- Progress Check 110

4 Coordinates, Functions, and Graphs

- 4.1 Coordinates in the Plane 112
- 4.2 Functions and Function Notation 120
- 4.3 Graphs of Functions 129
- Progress Check 138

5 Right Triangle Trigonometry

- 5.1 Angles and Their Measurement 140
- 5.2 Trigonometric Functions of Acute Angles 145
- 5.3 Calculating Trigonometric Function Values 151
- 5.4 Solving Right Triangles 155
- 5.5 Applications of Solving Right Triangles 161
- Progress Check 168

6 Quadratic Equations and Related Topics

- 6.1 Quadratic Equations 170
- 6.2 Disguised Quadratic Equations 180
- 6.3 Fractional Equations 188
- 6.4 Applications 192
- 6.5 Finding Approximate Solutions (optional) 197
- Progress Check 201

7 Systems of Equations

- 7.1 Systems of Linear Equations 202
- 7.2 Matrix Solutions 209
- 7.3 Determinants 217
- 7.4 Cramer's Rule 223
- 7.5 Non-Singular Systems of Linear Equations 229
- 7.6 Systems of Non-Linear Equations 236
- 7.7 Applications 244
- Progress Check 254

8 More Functions and Graphs

- 8.1 Linear Functions and Lines 256
- 8.2 Quadratic Functions 266
- 8.3 Functions with "Split Definitions" 274
- 8.4 Transformations 281
- 8.5 Special Classes of Functions 288
- Progress Check 297

9 Trigonometric Functions

- 9.1 A Wrapping Function (optional) 299
- 9.2 Functions of Arbitrary Angles 303

- 9.3 Graphs of Sines and Cosines 310
- 9.4 Other Trigonometric Graphs 326
- 9.5 Sums of Functions 338
- 9.6 Inverse Trigonometric Functions 344
- Progress Check 352

10 Applications of Trigonometric Functions

- 10.1 Finding Unknown Angles 355
- 10.2 Solving General Triangles 363
- 10.3 Adding Plane Vectors 373
- 10.4 Polar Coordinates 384
- 10.5 Parametric Equations 393
- Progress Check 398

11 Trigonometric Equations and Identities

- 11.1 Simple Trigonometric Equations 400
- 11.2 Some Useful Identities 407
- 11.3 More Trigonometric Equations 417
- 11.4 Identities and Inconsistent Equations 423
- Progress Check 427

12 Polynomial and Rational Functions

- 12.1 Synthetic Division 428
- 12.2 The Remainder and Factor Theorems 433
- 12.3 Factors and Zeros 437
- 12.4 Real and Rational Roots 440
- 12.5 Graphs of Factored Polynomial Functions 449
- 12.6 Partial Fractions (optional) 455
- 12.7 Polynomial and Rational Inequalities (optional) 463
- 12.8 Changing Numeration Bases (optional) 467
- Progress Check 474

13 Complex Numbers

- 13.1 Complex Numbers and Their Arithmetic 477
- 13.2 Polynomials Using Complex Numbers 482
- 13.3 Geometry of Complex Numbers 488
- 13.4 Polar Form of Complex Numbers 490
- 13.5 Powers and Roots of Complex Numbers 495
- 13.6 Application to AC Circuits 502
- Progress Check 506

14 Exponential and Logarithmic Functions

- 14.1 Exponential Functions 507
- 14.2 Logarithmic Functions 513
- 14.3 Properties of Logarithms 518
- 14.4 Exponential Equations 525
- 14.5 Logarithmic Equations 531
- 14.6 Applications 535
- 14.7 Graphs on Semilog and Log Paper 543
- Progress Check 552

15 Conic Sections

- 15.1 The Conic Sections 553
- 15.2 Circles 568
- 15.3 Ellipses 574
- 15.4 Hyperbolas 582
- 15.5 Parabolas 592
- 15.6 Graphing Second-Degree Relations 598
- Progress Check 601

16 Descriptive Statistics

- 16.1 Measures of Central Tendency 604
- 16.2 Measures of Dispersion 612
- 16.3 Correlation 622
- 16.4 The Method of Least Squares 631
- Progress Check 639

17 Introduction to Limits

- 17.1 Limits of Functions 641
- 17.2 Sequences 649
- 17.3 Limits of Sequences 656
- 17.4 Solutions by Iteration 661
- 17.5 Speeding Up Slow Iterations 668
- Progress Check 672

Appendix A Reverse Polish Notation 673

Appendix B The Irrationality of $\sqrt{2}$ 676

Appendix C Formulas from Geometry 678

Appendix D The Pythagorean Theorem 684

Appendix E BASIC Programs 687

Answers to Odd-Numbered Exercises
and to Progress Checks 697

Index 739

1

Calculator Arithmetic

The hand-held scientific calculator can be a powerful tool in mathematics. In this chapter, we discuss many uses of the calculator in algebra and trigonometry. This chapter also includes a discussion of the types of numbers one encounters with calculators, and of some rather surprising properties of such numbers. In later chapters, we will discuss trigonometric and logarithmic functions in detail.

1.1

Keys and Features

Various brands and models of calculators differ widely in their features. However, there are many common characteristics. There are four types of keys on a calculator: data entry, function, operation, and special keys.

Data entry keys include:

$\boxed{0} \dots \boxed{9}$

Digits

$\boxed{\cdot}$

Decimal point

$\boxed{\pi}$

Pi key. Enters the important mathematical constant $\pi = 3.14159\dots$

$\boxed{+/-}$

Changes the sign of the number displayed from either positive to negative or negative to positive. Also serves as a function key (see below).

\boxed{EE}

Enter Exponent key. Used for scientific notation (see the discussion in section 1.5).