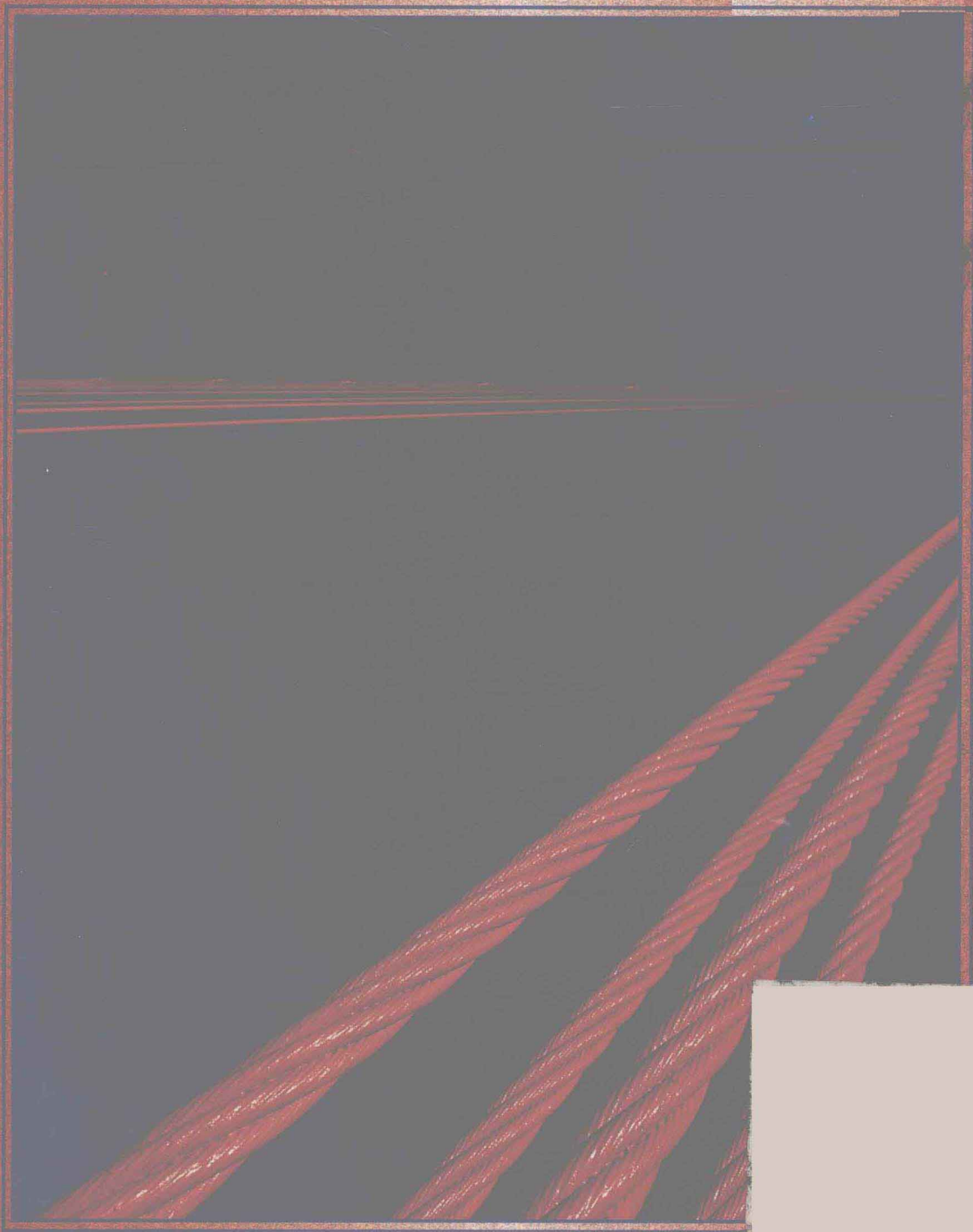
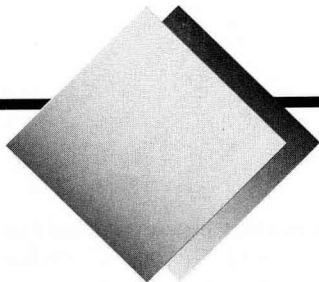


DAVID E. STEVENS

COLLEGE ALGEBRA
WITH GRAPHING TECHNOLOGY





College Algebra with Graphing Technology

David E. Stevens

Wentworth Institute of Technology



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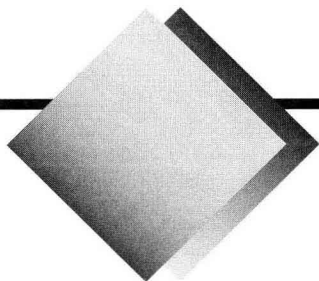
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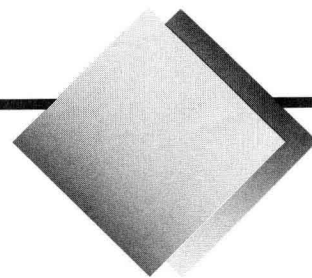
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College Algebra with Graphing Technology



PREFACE

◆ Intent

A course in college algebra is a prerequisite for more advanced courses in mathematics as well as a prerequisite for other college courses in astronomy, biology, physics, chemistry, computer science, business, and engineering. The goal of this text is to effectively prepare students for these courses and to illustrate through real-life applied problems that knowledge of college algebra is fundamental to these disciplines. Unlike a traditional text, however, this text integrates *graphing technology* (graphing calculator or graphing software) into the college algebra course. In each chapter after Chapter 1, graphing technology is used as a true teaching aid, without sacrificing mathematical thought or rigor. Whenever a graphical interpretation of the mathematics is appropriate, graphing technology is included. See *Table of Contents: Graphing Technology Usage* on page ix.

◆ Approach

For their success in a college algebra course, it is essential that students be active rather than passive readers of the text. Therefore, I have written this text using an *interactive approach* with graphing technology. Each key mathematical concept is supported by a step-by-step text example, with marginal annotations and explanatory notes, and each text example is followed by a practice problem for the student to work. The practice problem may ask the reader to check the preceding text example by using graphing technology, to work the preceding text example using an alternative algebraic approach, to extend the preceding text example by asking for additional information, or to try an entirely different problem that has similar mathematical steps. In effect, the practice problems require the student to become involved with the mathematics and thus constitute a built-in workbook for the student. *A complete, detailed solution of each practice problem is given in the back of the text.*

◆ Features

Written in a warm and user-friendly style, this text addresses the concerns of writing across the curriculum, group learning, critical thinking, and the use


◆ Preface

of modern technology in the math classroom. The following features distinguish this text from other texts in the market.

◆ Applied Problems

Student interest in mathematics is undoubtedly greatest when applied problems are integrated directly within the curriculum. For this reason, applied problems from the fields of science, engineering, and business are introduced at every reasonable opportunity.

Chapter opening applications To arouse student interest, each chapter opens with an applied problem and a related photograph. The solution to the problem is presented within the chapter, after the necessary mathematics has been developed.


Application subsections Separate material has been included at the end of most sections to apply the mathematics to real-life situations. The logo  identifies these application subsections. See *Table of Contents: Applied Problems and Models* on page xii.

◆ Exercise Sets

The heart of any math textbook is its end-of-section exercise sets. It is here that students are given an opportunity to practice the mathematical concepts that have been developed. The exercise sets in this text are presented in two parts: *Basic Skills* and *Critical Thinking*. Applied problems appear in both parts.

Basic Skills These exercises are routine in nature, and most of them mimic the text examples that are worked out in each section. The reader is encouraged to use graphing technology to confirm results and to solve problems to which standard algebraic methods do not apply.

Critical Thinking These exercises require the student to think critically and to transcend routine application of the basic skills to the next level of difficulty. Exercises in this group may require the student to draw upon skills developed in earlier chapters or to use graphing technology to explore new ideas.

Some of the exercise sets also contain problems that are *calculus related*. Designed for students who are taking a college algebra course as a prerequisite to calculus, these exercises illustrate the algebraic support that is needed in calculus. The logo  identifies these problems.

◆ Chapter Reviews

To help students prepare for chapter exams, each chapter in this text concludes with an extensive chapter review, which is divided into two parts: *Questions for Writing and Group Discussion* and *Review Exercises*.

Questions for Writing and Group Discussion In keeping with the interactive approach, these questions allow students to state in their own words what they have learned in the chapter. Since many of these questions have open-ended answers, they are well suited for class or group discussions and are extremely valuable to those who believe in cooperative or collaborative learning.

Review Exercises These exercises reinforce the ideas discussed in the chapter and allow the instructor to indicate to the student the types of problems that may appear on a chapter test.

◆ Cumulative Reviews

To help students pull together ideas from several chapters, cumulative review exercises are placed strategically after Chapters 3, 6, and 8. The problems in these exercises are ungraded by difficulty and are presented in a random order. Some problems are basic and similar to those already studied; others are more challenging and require creative thinking.

◆ Pedagogy

Every effort has been made to make this a text from which students can learn and succeed. The following pedagogical features attest to this fact.

Caution notes flagged by the symbol **CAUTION**, help eliminate misconceptions and bad mathematical habits by pointing out the errors most commonly made by students.

Introductory comments at the start of each section introduce vocabulary and inform the reader of the purpose of the section.

Boxed definitions, formulas, laws, and properties state key mathematical ideas and provide the reader with quick, easy access to this information.

Step-by-step procedural boxes indicate the sequence of steps that a student can follow for tasks such as simplifying algebraic expressions, solving certain types of equations and inequalities, or finding the inverse of a function or a matrix.

◆ Development

The prerequisite for a course in college algebra is two years of high school algebra or a semester of college intermediate algebra. Chapter 1, *Prerequisites of College Algebra*, provides a quick review of the topics in intermediate algebra, such as exponents, radicals, factoring, algebraic fractions, equations, inequalities, and formulas. Complex numbers are introduced in this chapter in preparation for their use with quadratic functions in Chapter 3 and polynomial functions in Chapter 4.

Chapter 2, *Functions and Graphs*, introduces the coordinate plane and the language of functions and graphs. Graphing technology is introduced in Section 2.2 and is used extensively throughout the remainder of the text to verify results, solve equations, and explore new ideas. Section 2.5 lists eight basic functions and their graphs (constant, identity, absolute value, squaring, cubing, reciprocal, square root, and cube root), and then applies the vertical and horizontal shift rules, the x - and y -axes reflection rules, and the vertical stretch and compress rules to sketch the graphs of several related functions. These eight basic functions and their graphs are then used to discuss composition of functions, inverse functions, applied functions, and variation.

Chapter 3, *Linear and Quadratic Functions*, uses the ideas of shifting, reflecting, stretching, and compressing to develop the linear and quadratic functions from the identity and squaring functions. The vertex formula for a parabola is developed and used to solve some max–min applied problems.

Section 3.4 discusses the algebraic and graphical methods of solving quadratic equations and inequalities, and Section 3.5 gives an introduction to solving a system of two equations in two unknowns by looking at the intersection points of their graphs.

Chapter 4, *Polynomial and Rational Functions*, focuses on the relationship between factors, roots, zeros, and x -intercepts. Graphing technology and the rational zero theorem are used to find the real zeros of a polynomial function. This chapter also discusses algebraic and graphical methods for solving polynomial equations and inequalities and rational equations and inequalities. With the aid of graphing technology, it is possible to solve max–min applied problems that deal with polynomial and rational functions.

Chapter 5, *Exponential and Logarithmic Functions*, discusses the properties of real exponents, defines the exponential function, and develops the logarithmic function as the inverse of the exponential function. By letting the number of compounding periods in the compound interest formula increase without bound, the reader is shown how the number e develops in a real-life situation. The properties of logarithms are used in graphing functions that contain logarithmic expressions (Section 5.3) and also in solving exponential and logarithmic equations (Section 5.4). Graphing technology is used to support the work and to help solve equations that are not solvable by ordinary algebraic methods.

Chapter 6, *Conic Sections*, discusses the conics and their related quadratic equations in two unknowns. This chapter states the geometric properties of the circle, parabola, ellipse, and hyperbola, as well as the distinguishing characteristics of their equations. The reflection properties of the conics are applied to various technical applications. Throughout the chapter, graphing technology is used to support algebraic manipulations.

Chapters 7 and 8 include other topics of interest in a college algebra course. Chapter 7, *Linear Systems and Matrices*, extends the discussion of 2×2 systems (from Section 3.5) to a procedure for solving $n \times n$ linear systems by the elimination method. Matrices are first introduced as an aid for solving a system of linear equations and are then applied to problems that require using the matrix operations of addition and multiplication. Determinants and inverses of matrices are evaluated algebraically and by using the matrix features of a graphing calculator. Section 7.4 introduces linear systems of inequalities and applies the results to linear programming problems in two unknowns.

Chapter 8, *Sequences and Series*, offers an introduction to these topics for the precalculus student. Section 8.2 introduces the idea of proof by mathematical induction. In this section, the reader is encouraged to use pattern recognition to guess a formula for the sum of a series and then to prove this guess by mathematical induction. Similarly, pattern recognition and mathematical induction are used to develop a formula for the general element of an arithmetic sequence, for the general element of a geometric sequence, and for the expansion of $(A + B)^n$. The sequence and series features of a graphing calculator are used to support the work.

◆ Supplements

The following supplements are available for users of this text.

Instructor's Solution Manual, by Eleanor Canter of Wentworth Institute of Technology, includes detailed solutions to all the even-numbered exercises.

Student's Solution Manual, by Eleanor Canter of Wentworth Institute of Technology, provides detailed solutions for the odd-numbered exercises from the text.

Instructor's Manual with Test Bank, by Cheryl Roberts of Northern Virginia Community College, includes sample syllabi, suggested course schedules, chapter outlines with references to videos, homework assignments, chapter tests, and a test bank of multiple-choice questions and open-ended problems.

Graphing Technology Laboratory Manual, by David Lawrence of Southwestern Oklahoma State University, includes keystroke instructions for various types of graphing calculators—Texas Instruments, Casio, Sharp, and Hewlett-Packard, as well as Derive software.

DERIVE[®] Laboratory Manual, by Lloyd R. Jaisingh of Morehead State University contains sixteen laboratory experiments using the computer algebra system DERIVE[®].

WESTEST, a computer-generated testing program, includes algorithmically generated questions and is available in both Macintosh and PC versions.

West Math Tutor Software by Mathens, an algorithmically based tutorial, is available for Macintosh and PC platforms. The instructor's version contains the complete bank of algorithms so that an instructor can create assignments, save them to disk for student use, and grade automatically when assignments are turned in. The student version is customized for each chapter of the appropriate text. The package is available to instructors, and a disk is available to students. It is also available as a site license for a math laboratory.

Videos, which are produced specifically for this text.

Fifty Transparency Masters illustrate the important figures, rules, and procedures in the text.

Please ask your West representative about qualifications for these supplements.

◆ Acknowledgments

Most of the material in this text has been class-tested with several hundred students at Wentworth Institute of Technology. I thank these students for their helpful comments and critiques. Special thanks go to my friend and colleague Eleanor Canter for her work in checking the answers and writing complete solutions to the more than 4500 exercises in this text. I also express my sincere thanks to the following reviewers—their ideas were extremely helpful in shaping this text into its present form.

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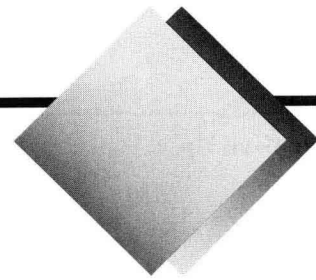
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The production of a textbook is a team effort between the editorial staff and the author. My editor, Nancy Hill-Whilton, offered the support and guidance that I needed to complete this project. Denise Bayko organized our reviewers' comments into a format that revealed where extra work was needed. Kathi Townes copyedited the manuscript and encouraged me to provide additional information that would benefit the reader, and Sandy Gangelhoff kept the project moving through the various stages of production. I thank each of you for the encouragement and enthusiasm that you provided in the preparation of this book.

D. E. Stevens
Boston, Massachusetts
1995



TO THE STUDENT

In order to use this text effectively, you should purchase a graphing calculator. As an alternative to a graphing calculator, you can use a computer equipped with graphing software. Your instructor has information on the type of calculator that you should purchase and the type of software that is available from West.

◆◆ Get Involved

The key to success in a college algebra course is to *get involved* with the mathematics. Do the homework exercises the day that your instructor assigns them. Before beginning the homework exercises, read the appropriate section in the text. Read carefully each text example in the section and try the practice problem that follows the text example. (A complete, detailed solution of each practice problem is given in the back of the text.) Form a study group with other members of your class and discuss the homework exercises with the group. You'll be surprised how much you can learn from each other.

◆◆ Ask Questions

You are in college to learn. You have an inquiring mind, and inquiring minds want to know. So, be sure to *ask questions*. Make note of the homework exercises that you did not understand, and come to class prepared to ask specific questions. Remember, there is no such thing as a “dumb” question. In fact, many other members of your class probably want to ask the same question that is causing you difficulty. Your instructor will welcome your participation in the discussion.

◆◆ Have Confidence

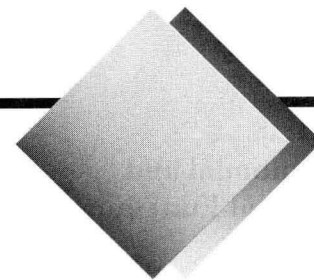
In order to do well in a mathematics test, you must *have confidence* in your ability to do the work. Building confidence requires study, intense work, and perseverance. If you have put in an honest effort, then you will never feel intimidated, defensive, or flustered during a test situation. You can enter the classroom knowing that you have given your very best.

◆ Never Give Up

Life is exciting when you're in college. You are full of optimism and enthusiasm for the opportunities that lie ahead. In the future, you will look back at your college experience as one of the best times of your life. So laugh, enjoy yourself, and *never give up!*

Achievement consists of never giving up . . . If there is no dark and dogged will, there will be no shining accomplishment; if there is no dull and determined effort, there will be no brilliant achievement.

HSUN TZU
Chinese philosopher



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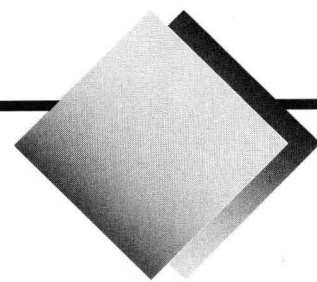
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