

CONTEMPORARY

Introductory

ALGEBRA

IGNACIO BELLO

$$a + b = b + a$$

$$(a + b) + c = a + (b + c)$$

$$a + 0 = a$$

$$a + (-a) = 0$$

$$a + b = c ; a = c - b , b = c - a$$

CONTEMPORARY INTRODUCTORY ALGEBRA

IGNACIO BELLO

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1817



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In Memory of
My Father

PREFACE

This text is intended for students who either need help in elementary algebra or who have never taken a course in the subject. Each chapter is structured to maximize student understanding of basic algebraic concepts. The following are features designed to foster students' interest in these mathematical concepts and their application to real-world problems.

Pretests	Given with accompanying diagnostic answers at the beginning of each chapter to pinpoint students' areas of strength and weakness.
Objectives	Listed at the beginning of each section to indicate what students are expected to learn and coordinated with the Examples and Exercises by the use of the symbol ❶.
Motivation	An actual application or a cartoon designed to motivate students' interest in the topic being introduced.
Examples	Illustrate the concepts under discussion and coordinated with the Objectives and Problems by the use of the symbol ❶.
Problems	Correlated with corresponding examples and designed to check the students' facility in applying the concept developed.
Exercises	Provided for students to practice the principles being learned; coordinated with the Objectives and Examples by the use of the symbol ❶.
Using Your Knowledge	Follows Exercises and applies the ideas discussed in each section to different situations.
Self-Tests	Given with accompanying diagnostic answers at the end of each chapter to test students' understanding of the material covered.
Supplementary Exercises	Keyed to Self-Tests and included as alternate forms of these tests.
Calculator Corner	Calculators and their uses are discussed where appropriate.

This book can be used in various ways:

1. As a textbook in a traditional lecture course. Simply ignore the margin problems and lecture as usual.
2. As a textbook in an individualized study course. Assign the reading portion and the exercises one day; answer questions and collect the homework the next day.
3. As a lab text. Use as any other textbook in the mathematics lab.

We have tested the entire book using both standard and special classes at Hillsborough Community College with very satisfying results. Students responded well to the features

described above and used all or most of them to reinforce the principles discussed in each section.

The author would like to express his appreciation to the following people: reviewers Professors Wayne Andrepont and Charles Blanchard, University of Louisiana; Rosanne Donohoe, Northern Virginia Community College; Margaret M. Esser, Hillsborough Community College; Liana Fox, Hillsborough Community College; George Grisham, Illinois Central College; Paul N. Hutchens, Florissant Valley Community College; Donald R. Johnson, Scottsdale Community College; George Kosan, Hillsborough Community College; Robert Kosanovich, Ferris State College; Peter Lindstrom, Northlake College; Susan C. Meyers, Sinclair Community College; and Leonard M. Wapner, El Camino College; typist Cathy Sagendorf; *answer checkers* Robert Balman, Joseph Clemente, and John Hunt.

Special thanks go to Dr. Jack Britton, a gentleman and a scholar, who endured my insanity during this ordeal and came out of retirement to help me finish this project. And last but not least, thanks to my wife.

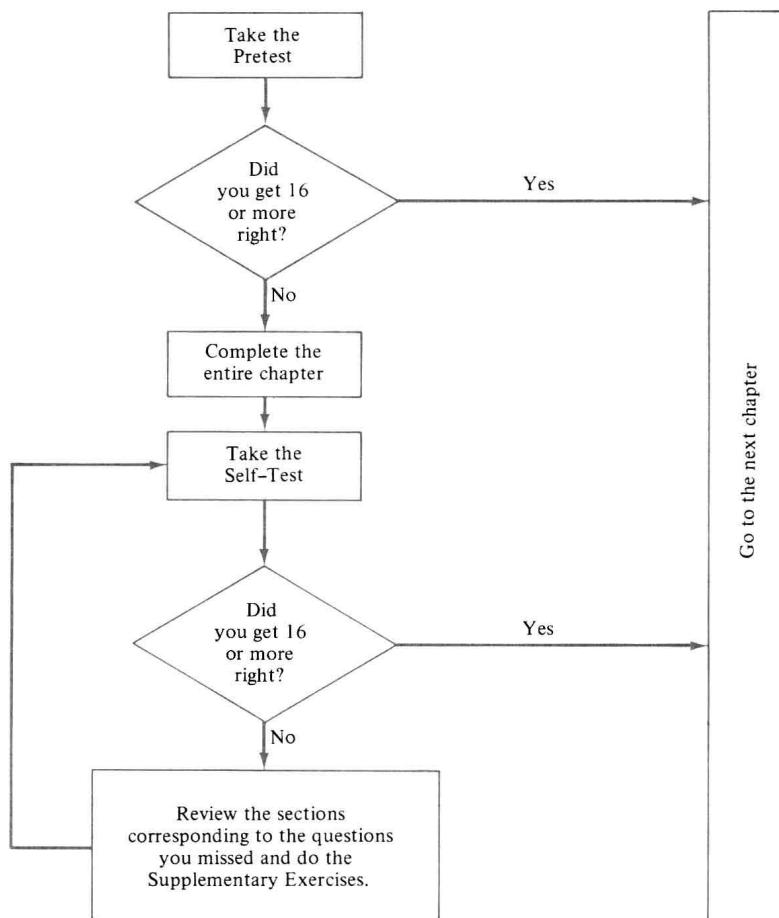
Ignacio Bello

TO THE STUDENT

HOW TO USE THIS BOOK

1. Take the Pretest. If you have 16 or more correct answers, go to the next chapter.
2. If you have fewer than 16 correct answers on the Pretest, perform the following steps:
 - a. Read the objectives in each section
 - b. Read and study the explanations and Examples
 - c. Do the marginal exercises (Problems)
 - d. Do the odd-numbered problems at the end of each section
3. Take the Self-Test at the end of each chapter. If you have 16 or more correct answers, go to the next chapter. Otherwise, review the Examples corresponding to the questions you missed, and then do part (a) of each problem in the Supplementary Exercises. If you have 16 or more correct answers, go to the next chapter.
4. If you have fewer than 16 correct answers, consult your instructor or a qualified tutor. You can then do part (b) of each problem in the Supplementary Exercises, and if you have 16 or more correct answers, go to the next chapter. If not, review the Examples corresponding to the questions you missed, and then do part (c) of each problem in the Supplementary Exercises. If you now have 16 or more correct answers, go to the next chapter. If you do not, take the Self-Test again until you obtain 16 or more correct answers.

The flow chart on the facing page may simplify things for you.



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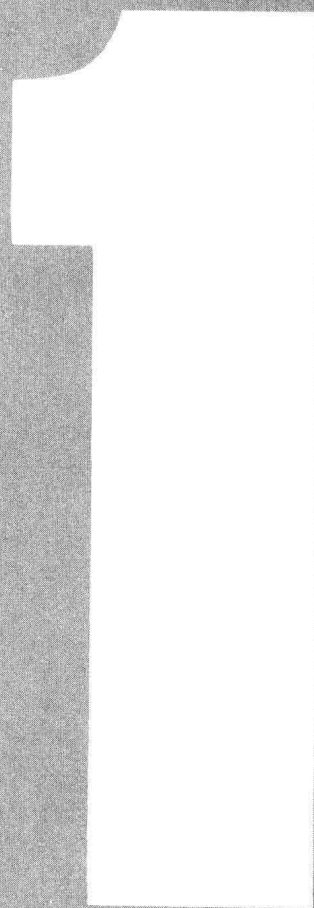
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CONTEMPORARY INTRODUCTORY ALGEBRA

NUMBERS AND THEIR PROPERTIES



PRETEST

CHAPTER 1

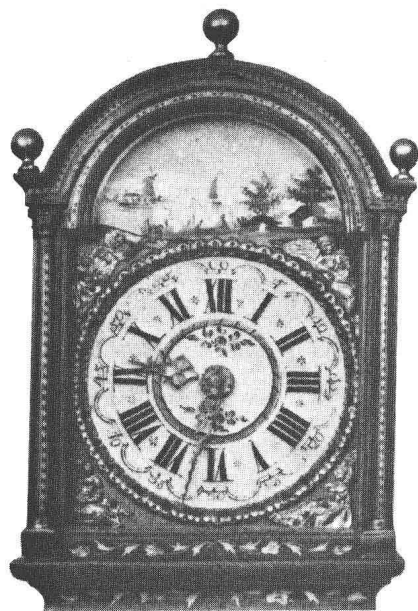
(Answers on next page)

ANSWERS

- Write -13 as a fraction.
- Find a fraction equivalent to $\frac{2}{7}$ with a denominator of 21.
- Find a fraction equivalent to $\frac{6}{15}$ with a denominator of 5.
- Reduce $\frac{27}{36}$ to lowest terms.
- Multiply $5\frac{1}{4} \cdot \frac{20}{21}$.
- Divide $2\frac{1}{4} \div \frac{5}{8}$.
- Add $\frac{3}{10} + \frac{5}{12}$.
- Subtract $4\frac{1}{6} - 1\frac{7}{10}$.
- Write 48.123 in expanded form.
- Write as a decimal. a. $\frac{19}{1000}$ b. $\frac{3}{11}$
- Write as a reduced fraction.
a. 0.025 b. 2.12
- Write 43.5% as a fraction.
- Write 34.8% as a decimal.
- Write as a percent. a. 0.49 b. $\frac{3}{8}$
- Which law is illustrated in each of the following statements?
a. $b \cdot (c \cdot a) = b \cdot (a \cdot c)$
b. $(5 + 9) + 2 = 5 + (9 + 2)$
- Find.
a. $(9 \times 8) + 4$ b. $9 \times (8 + 4)$ c. $4(x + 7)$
- Remove parentheses (simplify).
a. $(8 + 2a) + 4$ b. $(8a) \cdot 9$
- Evaluate.
a. $^{\circ}\text{C} = \frac{5}{9}(^{\circ}\text{F} - 32)$ when $^{\circ}\text{F} = 50$
b. $y \div 3x - z + 5$ for $x = 2$, $y = 12$, $z = 1$
- A rectangle is 8.1 in. long and 1.9 in. wide. Find a. its area and b. its perimeter.
- A circle has a 9-in. radius. Find a. its area and b. its circumference. (Use 3.14 for the value of π .)

- _____
- _____
- _____
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- _____
- a. _____ b. _____
- a. _____ b. _____
- _____
- _____
- a. _____ b. _____
- a. _____ b. _____
- a. _____ b. _____
c. _____
- a. _____ b. _____
- a. _____ b. _____
- a. _____ b. _____
- a. _____ b. _____

ANSWERS	IF YOU MISSED	REVIEW		
	QUESTION	SECTION	EXAMPLES	PAGE
1. $\frac{-13}{1}$	1	1.1	1	6
2. $\frac{6}{21}$	2	1.1	2	6
3. $\frac{2}{5}$	3	1.1	3	7
4. $\frac{3}{4}$	4	1.1	4	8
5. $\frac{5}{18}$	5	1.2	1, 2, 3	13, 14
6. $\frac{5}{43}$	6	1.2	4, 5	15, 16
7. $\frac{60}{37}$	7	1.2	6	17
8. $\frac{15}{40 + 8 + \frac{1}{10}}$	8	1.2	8	19
9. $\frac{+ \frac{2}{100} + \frac{3}{1000}}$	9	1.3	1	26
10. a. $\frac{0.019}{0.27}$	10a	1.3	2	26
b. $\frac{1}{40}$	b	1.3	3	27
11. a. $\frac{1}{40}$ b. $\frac{53}{25}$	11a, b	1.3	4, 5	28
12. $\frac{87}{200}$	12	1.4	1	33
13. $\frac{0.348}{0.348}$	13	1.4	2	34
14. a. $\frac{49\%}{37.5\%}$	14a	1.4	3	34
b. $\frac{37.5\%}{49\%}$	b	1.4	4	35
15. a. $\frac{\text{Comm. } \times}{\text{Assoc. } +}$	15a	1.5	2	42
b. $\frac{\text{Assoc. } +}{\text{Comm. } \times}$	b	1.5	2	42
16. a. $\frac{76}{108}$	16a	1.5	1	41
b. $\frac{108}{4x + 28}$	b	1.5	1	41
c. $\frac{4x + 28}{2a + 12}$	c	1.5	3	43
17. a. $\frac{2a + 12}{72a}$	17a	1.5	4	43
b. $\frac{72a}{2a + 12}$	b	1.5	4	43
18. a. $\frac{10^\circ}{12}$	18a	1.6	1, 2	49, 50
b. $\frac{12}{10^\circ}$	b	1.6	7	54
19. a. $\frac{15.39 \text{ in.}^2}{20 \text{ in.}}$	19a	1.6	4	52
b. $\frac{20 \text{ in.}}{15.39 \text{ in.}^2}$	b	1.6	4	52
20. a. $\frac{254.34 \text{ in.}^2}{56.52 \text{ in.}}$	20a	1.6	6	53
b. $\frac{56.52 \text{ in.}}{254.34 \text{ in.}^2}$	b	1.6	6	53



OBJECTIVES

You should be able to:

- ① Write an integer as a fraction.
- ② Find a fraction equivalent to a given one with a specified denominator.
- ③ Reduce a fraction to lowest terms.

1.1 NUMBERS

The symbols on the sundial and the Roman clock have something in common. They both use numerals to name the numbers from 1 to 12. Algebra and arithmetic also have something in common. They both use the same numbers.

① Numbers of Arithmetic

In arithmetic you learned about the counting numbers. The numbers used for counting are the **natural numbers**

1, 2, 3, 4, 5,

and so on. These numbers are used in algebra. We also use the **whole numbers** 0, 1, 2, 3, 4, and so on. Later on, you probably learned about the **integers**. The integers include the **positive integers**,

+1, +2, +3, +4, +5,

and so on (read “positive one, positive two,” and so on). The negative integers,

-1, -2, -3, -4, -5,

and so on, and the number 0, which is neither positive nor negative. Thus the integers are

..., -2, -1, 0, 1, 2, ...

where the dots (...) indicate that the enumeration continues without end.

All of the preceding numbers can be written as **common fractions** of the form $\frac{a}{b}$ (or a/b) in which the numerator a and the denominator b are both integers and the denominator is not 0. If a number can be written in this form, it is called a **rational number**. Thus, $\frac{1}{3}$, $\frac{-5}{2}$, and $\frac{0}{7}$ are rational numbers. Of course, any integer can be written as a fraction by writing it with a denominator of 1. For example,

$$4 = \frac{4}{1}, \quad 8 = \frac{8}{1}, \quad 0 = \frac{0}{1}, \quad \text{and} \quad -3 = \frac{-3}{1}$$

Note that

$$+1 = 1$$

$$+2 = 2$$

$$+3 = 3$$

and so on. Thus, the positive integers are the natural numbers.

Rational numbers can be written as fractions of the form

$\frac{a}{b}$ ← Numerator

$\frac{a}{b}$ ← Denominator

Problem 1. Write as fractions:

- a. 47
b. -13

Example 1. Write the following numbers as fractions.

- a. 10
b. -15

Solution:

$$\text{a. } 10 = \frac{10}{1}$$

$$\text{b. } -15 = \frac{-15}{1}$$

The rational numbers we have discussed are part of a larger set of numbers, the set of *real numbers*. The **real numbers** include the *rational numbers* and the *irrational numbers*. Thus each real number is either rational or irrational.

The **irrational numbers** are numbers that *cannot* be written as the ratio of two integers. For example, $\sqrt{2}$, π , $-\sqrt[3]{10}$, and $\frac{\sqrt{3}}{2}$ are irrational numbers.

We will say more about the irrational numbers later in the book.

② Equivalent Fractions

Can you find other ways of writing 10 as a fraction? Here are some

$$10 = \frac{10}{1} = \frac{10 \times 2}{1 \times 2} = \frac{20}{2}$$

$$10 = \frac{10}{1} = \frac{10 \times 3}{1 \times 3} = \frac{30}{3}$$

and
$$10 = \frac{10}{1} = \frac{10 \times 4}{1 \times 4} = \frac{40}{4}$$

As you can see, the fraction $\frac{10}{1}$ is **equivalent** to many other fractions. We can always obtain other fractions equivalent to any given fraction by **multiplying** the numerator and denominator of the original fraction by the **same** nonzero number. In fact, this is the same as multiplying the fraction by 1. For example,

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

$$\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$

and
$$\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

Example 2. Find a fraction equivalent to $\frac{3}{5}$ with a denominator of 20.

Solution: We must solve the problem

$$\frac{3}{5} = \frac{?}{20}$$

Note that $\frac{2}{2} = 1$, $\frac{3}{3} = 1$, and $\frac{4}{4} = 1$.

Problem 2. Find a fraction equivalent to $\frac{7}{6}$ with a denominator of 18.