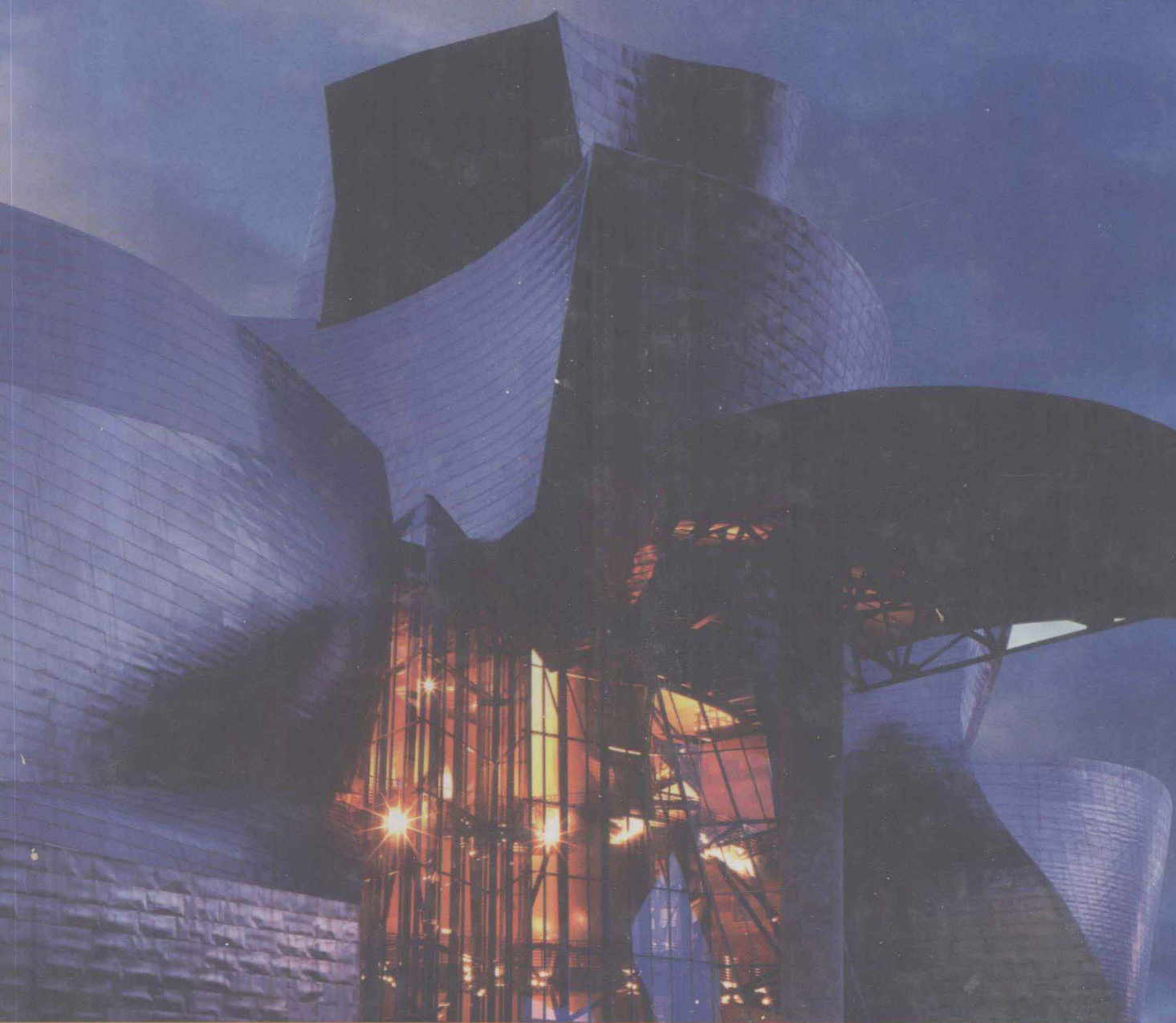


Finite Mathematics & Its Applications

Eighth Edition



Larry J. Goldstein David I. Schneider Martha J. Siegel

EIGHTH EDITION

Finite Mathematics & Its Applications

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Mathematics and Its Applications

This volume is one of a collection of texts for freshman and sophomore college mathematics courses. Included in this collection are the following.

Calculus and Its Applications, tenth edition, by L. Goldstein, D. Lay, and D. Schneider. A text designed for a two-semester course in calculus for students of business and the social and life sciences. Emphasizes an intuitive approach and integrates applications into the development.

Brief Calculus and Its Applications, tenth edition, by L. Goldstein, D. Lay, and D. Schneider. Consists of the first eight chapters of the above book with some material from later chapters.

Finite Mathematics & Its Applications, eighth edition, by L. Goldstein, D. Schneider, and M. Siegel. A traditional finite mathematics text for students of business and the social and life sciences. Allows courses to begin with either linear mathematics (linear programming, matrices) or probability and statistics. Includes topics in discrete mathematics.

Applied Calculus: A Graphing Approach by D. Schneider and D. Lay. A one-semester or two-quarter technology-required reform calculus text for students majoring in business, economics, life sciences, and social sciences.

Preface

This work is the eighth edition of our text for the traditional finite mathematics course taught to first- and second-year college students, especially those majoring in business and the social and biological sciences. Finite mathematics courses exhibit tremendous diversity with respect to both content and approach. Therefore, in revising this book, we incorporated a wide range of topics from which an instructor may design a curriculum, as well as a high degree of flexibility in the order in which the topics may be presented. For the mathematics of finance, we even allow for flexibility in the approach of the presentation.

In this edition, we attempt to maintain our popular student-oriented approach throughout and, in particular, through the use of the following features:

Applications

We provide realistic applications that illustrate the uses of finite mathematics in other disciplines. The reader may survey the variety of applications by referring to the Index of Applications located on the front endpapers. Wherever possible, we attempt to use applications to motivate the mathematics. For example, the concept of linear programming is introduced in Chapter 3 via a discussion of production options for a factory with a labor limitation.

Examples

We include many more worked examples than is customary in textbooks. Furthermore, we include computational details to enhance comprehension by students whose basic skills are weak.

Exercises

More than 2700 exercises comprise about one-quarter of the book, the most important part of the text in our opinion. The exercises at the ends of the sections are usually arranged in the order in which the text proceeds, so that homework assignments may be easily made after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Supplementary exercises at the end of each chapter amplify the other exercise sets and provide cumulative exercises that require skills acquired from earlier chapters. Answers to the odd-numbered exercises from the sections and answers to all the supplementary and chapter test exercises are included at the back of the book.

Practice Problems

The practice problems are a popular and useful feature of the book. They are carefully selected exercises located at the end of each section, just before the exercise set. Complete solutions follow the exercise set. The practice problems often focus on points that are potentially confusing or are likely to be overlooked. We recommend that the reader seriously attempt to do the practice problems and study their solutions before moving on to the exercises.

Use of Technology

Although the use of technology is optional for this text, many of the topics can be enhanced with graphing calculators, electronic spreadsheets, and mathematical software. Whenever relevant, we explicitly show the student how to use graphing calculators and electronic spreadsheets effectively to assist in understanding the fundamental concepts of the course. In addition, the text contains appendices on the fundamentals of using graphing calculators and electronic spreadsheets. The powerful mathematical software “Explorations in Finite Mathematics” is packaged with each *Student Solutions Manual*. Many sections of the book contain specially designed technology exercises intended to be solved with one of these technologies.

In our discussions of graphing calculators, we specifically refer to the TI-83 since this is the most popular graphing calculator. Therefore, most students will have a book customized to their calculator. Students with other graphing calculators can consult their guidebooks to learn how to make adjustments. Had the calculator material been written generically, every student would have to make adjustments. For the same reasons, the discussions of electronic spreadsheets refer to Microsoft Excel.

Examples from Professional Exams

We have included questions similar to those found on CPA, GMAT, and GRE Economics exams to illustrate further the relevance of the material in the course. These multiple-choice questions are identified with the notation PE (which stands for “professional exams”).

Review of Fundamental Concepts

Near the end of each chapter is a set of questions that help the student recall the key ideas of the chapter and focus on the relevance of these concepts.

Chapter Summaries

Each chapter contains a detailed summary of the important definitions and results from the chapter serving as a handy study tool for the student.

Chapter Tests

Each chapter has a sample test that can be used by the student to help determine if he or she has mastered the important concepts of the chapter. The answers to the chapter test are given at the back of the book.

Chapter Projects

Most chapters have extended projects that can be used as in-class or out-of-class group projects, or special assignments. These projects develop interesting applications or enhance key concepts of the chapters.

New in This Edition

Among the changes in this edition, the following are the most significant:

1. *Personal financial decisions section.* A section has been added to the end of Chapter 10 that considers financial decisions that students often have to

make shortly after graduating from college. The section discusses IRAs, consumer loans, and mortgages. The differences between traditional and Roth IRAs are discussed along with the advantages of starting them early. The common add-on method for computing the finance charges for a consumer loan is presented and analyzed. The section shows how to compare mortgages having different up-front fees by calculating their APRs, which must be provided by the lender, and their effective mortgage rates, which are used by mortgage analysts to compare mortgages.

2. *Electronic spreadsheets.* An appendix titled “Spreadsheet Fundamentals” has been added. In addition, relevant material on the use of spreadsheets is presented alongside appropriate topics in the book.
3. A new chapter project for the Mathematics of Finance chapter provides useful insights into the effects of interest.
4. The discussion of sensitivity analysis of linear programming problems has been expanded.
5. *Additional exercises and updated data.* We have added many new exercises (including about two-dozen similar to those appearing in professional exams, such as the GMAT) and have updated the real-world data appearing in exercises and examples.

Minimal Prerequisites

Because of great variation in student preparation, we keep formal prerequisites to a minimum. We assume only one year of high school algebra. Furthermore, we review, as needed, those topics that are typically weak spots for students.

Topics Included

This edition has more material than can be covered in most one-semester courses. Therefore, the instructor can structure the course to the students’ needs and interests. The book divides naturally into four parts. The first part consists of linear mathematics: linear equations, matrices, and linear programming (Chapters 1–4); the second part is devoted to probability and statistics (Chapters 5–7); the third part covers topics utilizing the ideas of the other parts (Chapters 8–10); and the fourth part explores key topics from discrete mathematics that are sometimes included in the modern finite mathematics curriculum (Chapters 11–13). We prefer to begin with linear mathematics since it makes for a smooth transition from high school mathematics and leads quickly to interesting applications, especially linear programming. Our preference notwithstanding, the instructor may begin this book with Chapter 5 (“Sets and Counting”) and then do either the linear mathematics or the probability and statistics.

Supplements

1. *Instructor’s Resource Manual* 013-046620-4. Contains the answers to all the exercises in the sections, and contains all the answers to the supplementary exercises, chapter tests, and chapter projects. In addition, the manual contains an extensive test item file with questions (both multiple-choice and standard) for each section of the book.
2. *Student Solutions Manual and Explorations in Finite Mathematics Software* 013-046633-6. Includes the solution to every odd-numbered exercise from

the sections, solutions to all the supplementary and chapter test exercises in the text, as well as a copy of the premier software package for finite mathematics, developed by David I. Schneider. The software, “Explorations in Finite Mathematics”, consists of 28 routines that include an animated solution of geometric linear programming problems, student-directed solutions to Gaussian elimination and simplex method problems, interactive shading of Venn diagrams, and detailed analyses of loans and annuities. Matrix operations use rational arithmetic, and matrices are displayed on the screen with typeset quality. An animated Galton board routine shows in a dynamic way how the binomial distribution eventually approaches the normal distribution as n increases.

3. *TestGen*. Provides nearly 1000 suggested test questions, keyed to chapter and section. *TestGen* is a text-specific testing program networkable for administering tests and capturing grades online. Edit and add your own questions, or use the new “Function Plotter” to create a nearly unlimited number of tests and drill worksheets. (Available via your Prentice Hall representative.)
4. *Prentice Hall Companion Website* (<http://www.prenhall.com/goldstein>) Created as an extra resource for both students and professors, the site includes the following features:
 - (a) *Excel Tutorials and Projects written by Revathi Narasimhan* at St. Peter’s College. Uses Excel to enhance the understanding of many of the topics in the course. Using a combination of specially designed projects and tutorials, students are able to analyze data, draw conclusions, and present their analysis in a professional format.
 - (b) *Graphing Calculator Help*. Offers support to students on how to use all the major brands of graphing calculators, including some program downloads.
5. *Prentice Hall Grade Assist*. Helps you create assignments and tests quickly and easily—you can use problems from your text, problems you create, and algorithmic problems. Choose from nearly thirty question types including multiple choice, short answer, and gradable free response mathematics.
6. *Finite Math for Windows*. As the first software package of its kind, *Finite Math for Windows* transitions students from using calculators to using Windows to solve finite math problems. This user-friendly software enables students to easily check computations and work step-by-step through problems.
7. *Course Management*. Available on demand through your Prentice Hall Sales Representative, WebCT, Blackboard & Course Compass.
 - (a) *Net Tutor*. Real time, online tutoring allows students to ask questions and get help on the text material from mathematics instructors.
 - (b) *Online Calculator Manuals* for the TI-82, TI-83, and TI-83 Plus, TI-85, TI-86, TI-89, TI-92, HP, Sharp and Casio graphing calculators.

Acknowledgments

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If you have comments or suggestions, we would like to hear from you. We hope that you enjoy using this book as much as we have enjoyed writing it.

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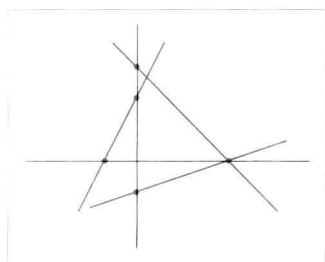
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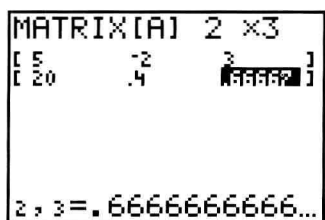
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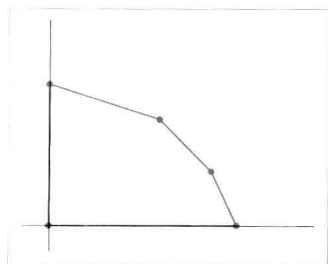
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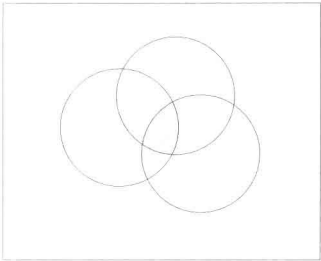
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	x	y	u	v	M
y	0	1	$\frac{3}{16}$	$-\frac{1}{4}$	0
x	1	0	$-\frac{1}{8}$	$\frac{1}{2}$	0
M	0	0	$\frac{3}{16}$	$\frac{7}{4}$	1

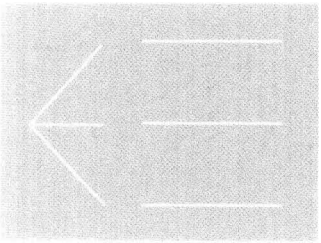
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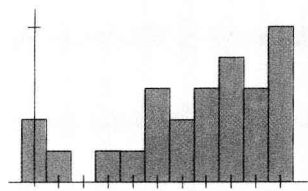
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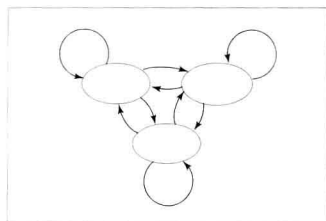
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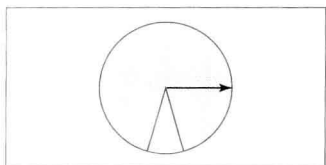
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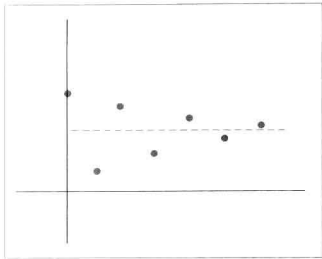
	A	B	C
1	Principal	\$100.00	
2			
3	Compound Amount		
4	Interest Rate	5 Years	10 Years
5	3.00%	\$116.12	\$134.83
6	3.50%	\$119.03	\$141.69
7	4.00%	\$122.02	\$148.89
8	4.50%	\$125.08	\$156.44
9	5.00%	\$128.20	\$164.36
10	5.50%	\$131.41	\$172.68
11	6.00%	\$134.69	\$181.40
12	6.50%	\$138.04	\$190.56
13	7.00%	\$141.48	\$200.16

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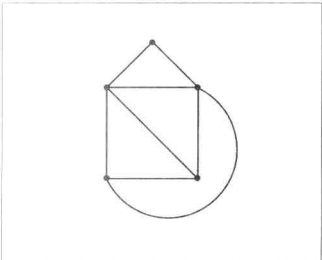


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Linear Equations and Straight Lines

- 1.1 Coordinate Systems and Graphs
- 1.2 Linear Inequalities
- 1.3 The Intersection Point of a Pair of Lines
- 1.4 The Slope of a Straight Line
- 1.5 The Method of Least Squares

Many applications considered later in this text involve linear equations and their geometric counterparts—straight lines. So let us begin by studying the basic facts about these two important notions.

1.1 Coordinate Systems and Graphs

Often we can display numerical data by using a *Cartesian coordinate system* on either a line or a plane. We construct a Cartesian coordinate system on a line by choosing an arbitrary point O (the *origin*) on the line and a unit of distance along the line. We then assign to each point on the line a number that reflects its directed distance from the origin. Positive numbers refer to points on the right of the origin, negative numbers to points on the left. In Fig. 1 we have drawn a Cartesian coordinate system on the line and have labeled a number of points with their corresponding numbers. Each point on the line corresponds to a number (positive, negative, or zero). Conversely, every number corresponds to a point on the line.

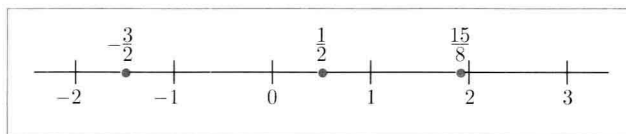


Figure 1

A Cartesian coordinate system may be used to numerically describe points on a line. In a similar fashion, we can construct a Cartesian coordinate system to numerically locate points on a plane. Such a system consists of two perpendicular lines called the *coordinate axes*. These lines are usually drawn so that one is horizontal and one is vertical. The horizontal line is called the *x-axis*, the vertical line the *y-axis*. Their point of intersection is called the *origin* (Fig. 2). Each point of the plane is identified by a pair of numbers (a, b) . The first number, a , tells the number of units from the point to the *y-axis* (Fig. 3). When a is positive, the point is to the right of the *y-axis*; when a is negative, the point is to the left

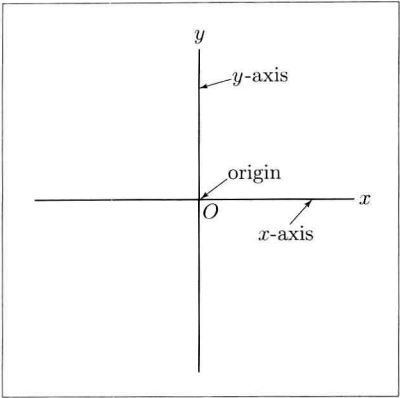


Figure 2

of the y -axis. The second number, b , gives the number of units from the point to the x -axis (Fig. 4). When b is positive, the point is above the x -axis; when b is negative, the point is below. The numbers a and b are called, respectively, the x - and y -coordinates of the point. In order to plot the point (a, b) , begin at the origin and move a units in the x -direction and then b units in the y -direction.

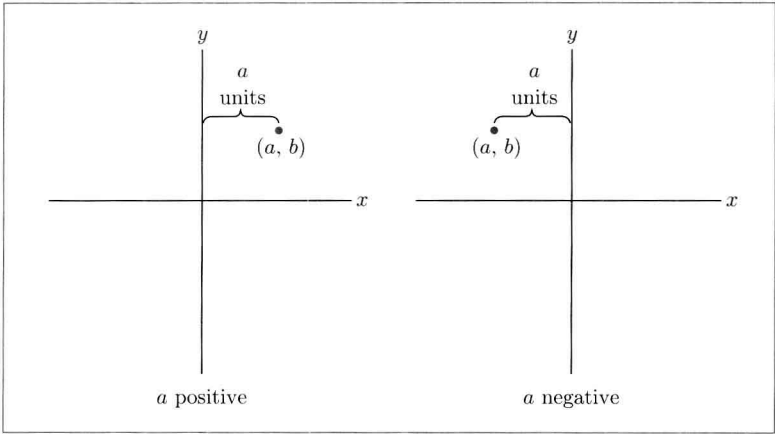


Figure 3

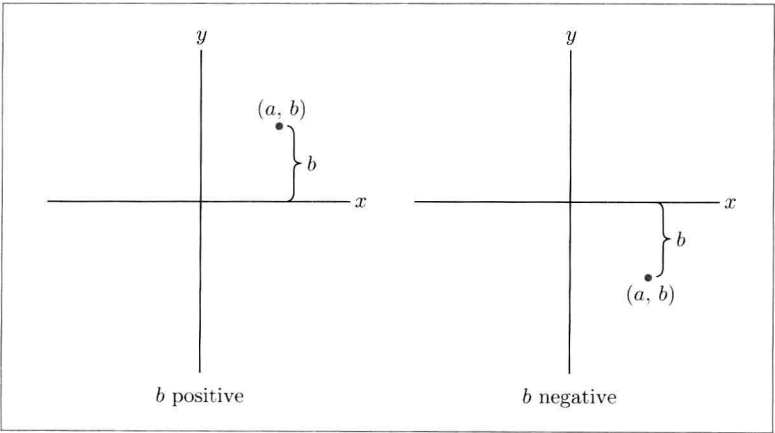


Figure 4