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Modeling and Simulation in Science, Engineering and Technology

Mathematical Modeling of Collective Behavior in Socio-Economic and Life Sciences

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Preface

The description of emerging collective behaviors and self–organization in a group of interacting individuals has gained increasing interest from various research communities in biology, engineering, physics, as well as sociology and economics. In the biological context, swarming behavior of bird flocks, fish schools, insects, bacteria, and people is a major research topic in behavioral ecology with applications to artificial intelligence. Likewise, emergent economic behaviors, such as distribution of wealth in a modern society and price formation dynamics, or challenging social phenomena such as the formation of choices and opinions are also problems in which the emergence of collective behaviors and universal equilibria has been shown.

These behaviors occur widely in nature and are part of our daily lives. It is quite surprising (ever astonishing) to learn that they can be fruitfully described and understood by means of suitable mathematical tools. Mathematical modeling using partial differential equations, which touches core areas of physics and engineering, is in fact playing an increasing role in emerging fields such as the social, economic and life sciences. Mathematical efforts are gradually gaining strength in this multidisciplinary area. Typically, the underlying equations are highly nonlinear; in many cases, they are also vectorial systems and represent a challenge even for the most modern and sophisticated mathematical-analytical and mathematical-numerical techniques. Among others, multidimensional computations of complex multi-scale phenomena are now within reach. Sophisticated nonlinear analysis deepens our understanding of increasingly complex models. Computational results feed back into the modeling process, and provide insight into detailed mechanisms that often cannot be studied by real-life experiments.

The novelty here is that important phenomena in seemingly different areas such as sociology, economics and biology can be described by closely related mathematical models. In this book we present selected research topics that can be regarded as new and challenging frontiers of applied mathematics. These topics have been chosen to elucidate the common methodological background underlining the main idea of this book: to identify similar modeling approaches, similar analytical and numerical techniques, for systems made out of a large number of "individuals" that show a "collective behavior," and obtain from them "average" information. The expertise obtained from dealing with physical situations is considered as the basis for the modeling and simulation of problems for applications in the socio-economic and life sciences, as a newly emerging research field.

In most of the selected contributions, the main idea is that the collective behaviors of a group composed of a sufficiently large number of individuals (agents) could be described using the laws of statistical mechanics as it happens in a physical system composed of many interacting particles. This opens a bridge between classical statistical physics and the socio-economic and life sciences. In particular, powerful methods borrowed from the kinetic theory of rarefied gases can be fruitfully employed to construct kinetic equations that describe the emergence of universal structures through their equilibria.

The book is subdivided into three parts, and each part contains several chapters listed alphabetically by author. Part I deals with the microscopic and kinetic modeling of simple economies and financial markets. Some of this kinetic modeling is clearly rather *ad hoc*, but if one is willing to accept the analogies between trading agents and colliding particles, then various well-established methods from statistical physics and applied mathematics seem ready for application to the field of economics. Most notably, the numerous tools originally devised for the study of the energy distribution in a rarefied gas can now be used to analyze wealth and price distributions.

Likewise, microscopic models of both social and political phenomena describing collective behaviors and self-organization in a society can be analyzed using these methods. These topics are the contents of Part II. Among others, the modeling of opinion formation and vote intention dynamics has attracted the interest of an increasing number of researchers in recent years.

The last part of the book deals mainly with the collective self-driven motion of self-propelled particles such as flocking of birds, schooling of fishes, swarming of insects and bacteria, traffic and crowd movements. These coherent and synchronized structures are apparently produced without the active role of a leader in the grouping, and can be described according to similar concepts of statistical physics and applied mathematics. General population dynamics and modern human warfare models are also presented here.

The idea of publishing a book to highlight these new emerging applications of mathematics occurred to us at the conclusion of a short series of lectures we organized in Vigevano in November 2008, with the support of the Municipality of Vigevano, the Center for Interuniversity Research in Land Economics founded by the Universities of Milano-Bicocca, Pavia and Ferrara (CRIET), the Advanced Applied Mathematical and Statistical Sciences Center of the University of Milan (ADAMSS) and the Center for Modeling Computing & Statistics of the University of Ferrara (CMCS). These lectures served as the springboard starting point for the present book, which includes several other contributions of distinguished researchers.

We warmly hope this book will be of great interest to applied mathematicians, physicists, biologists, economists, and engineers involved in the modelling of complex socio-economic systems, and in aggregation and collective phenomena in general.

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Contents

Part I

Economic modelling and financial markets

Agent-based models of economic interactions

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Summary. The interdisciplinary field of econophysics has enjoyed recently a surge of activities especially with numerous agent-based models, which have led to a substantial development of this field. We review three main application areas of agent-based models in econophysics: order books, distributions of wealth in conservative economies, and minority games.[4]

1 Introduction

The term "complex system" was coined to cover a great variety of systems which include examples from natural sciences (physics, chemistry, biology, etc.) as well as social sciences (economics and sociology), especially those where the constituents are dissimilar (heterogeneous) and the interactions among them are not known particularly well. In this respect, a socio-economic system, as e.g. an economic market, is a perfect example of a complex system, since every constituent entity (agent) has different characteristics and behaves in a different way. The system is not just an assembly of many identical particles as we commonly encounter in physics, for example. Econophysicists have been particularly intrigued by a number of phenomena described by power laws in economic systems, for example the distribution of price changes and of individual income and wealth. All have a sort of an universal power-law tail. Fitting with power laws does not trigger any interest in most economists, but in does in physicists since many complex physical systems display a similar intermittent dynamics; for example, the avalanche dynamics in random magnets under a slowly varying external field and the progression of cracks in a slowly strained disordered material. The important thing about these physical examples is that the exogenous driving force is regular and steady, but

[4]Portions of this text have been taken from Sec. IV of the review by A. C. et al. "Econophysics: Empirical facts and agent-based models", arXiv:0909.1974.

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the resulting endogenous dynamics is complex and critical. Econophysicists have thus proposed simple multi-agent models with heterogeneities, where the nature of the dynamics comes from collective effects. In these models, the individual entities or components have a relatively simple behavior, but interactions lead to new, emergent and cooperative phenomena. The whole complex system is fundamentally different from its subparts or elementary constituents. This has been summarized in the famous paradigma "more is different" [103]. As this intermittent behavior appears to be generic for physical systems with both heterogeneities and interactions, it leads to think that the dynamics of socio-economic systems may also have the same underlying mechanisms.

In economic models there is usually a representative agent, who has unlimited foresight and capability of deliberation, "perfect rationality", and uses the "utility maximization" principle to act economically, taking into account all potential future events with the correct probabilities. The "rational-agent" paradigm is also coupled with another reductionism: As there is a single way to act perfectly rationally, all agents should display exactly the same behavior. So one representative agent would be sufficient. Thus the typical format of current economic models is that of a single agent or firm maximizing its utility or profit with perfect foresight over a finite or infinite period. Instead, the agent-based models that have originated from simple statistical physics considerations have allowed one, for example, to go beyond the prototype theories with a "representative" agent in traditional economics. The recent failure of economists to anticipate the collapse of markets worldwide since 2007, has led over a short period of time voices even from within the field of economics itself, suggesting that new foundations for the discipline are required [104,105]. Some scientists have suggested that econophysics and in particular agent-based models may provide such an alternative theoretical framework for rebuilding economics [106].

Statistical physics, a branch of physics that combines the principles and procedures of statistics with the laws of both classical and quantum mechanics, and explains the measurable properties of macroscopic systems on the basis of the properties and behavior of their microscopic constituents, has turned out to be useful in the study of these diverse complex systems. In the following, we review three main fields where econophysicists have applied this approach through agent-based models to the description of particular aspects of economics: order books, the distribution of wealth, and minority games. More recently these and other sectoral models have led to the development of comprehensive agent-based models of a whole economy including a spatial structure and the major markets considered in quantitative macroeconomic modelling (consumer goods, investment goods, labour, credit and finance) [107, 108]; however, due to a lack of time, we could not include the latter in our review.

2 Order books

Much has been done in the past fifteen years by providing new models that can reproduce the "stylized facts," that have often been left aside when modeling financial markets referring to them as "anomalous" characteristics, as if observations failed to comply with theory. These recent developments have been built on top of early attempts at modeling mechanisms of financial markets with agents. Stigler [1], who investigated some rules of the Security Exchange Commission (SEC), and Garman [2], who investigated double-auction microstructure, belong to those historical works. The first modern attempts at this type of models were perhaps made in the field of behavioral finance. Agent-based models in financial economics were built with numerous agents who can exchange shares of stocks according to exogenously defined utility functions reflecting their preferences and risk aversions. LeBaron [3] provides a recent review of that type of models. Although achieving some of their goals, these models suffer from many drawbacks: (a) they were complicated, and it was a difficult task to identify the role of their numerous parameters and their dependencies; (b) the chosen utility functions did not reflect what was observed on the mechanisms of financial markets. Simpler models were introduced in the last decade, implementing only well-identified and presumably more realistic behavior: Cont and Bouchaud [4] used noise traders that are subject to "herding," i.e., form random clusters of traders sharing the same view on the market. The idea was used by Roberto et al. [5] too. A complementary approach was to characterize traders as fundamentalists, chartists, or noise traders. Lux and Marchesi [6] proposed an agent-based model in which these types of traders interact. In all these models, the price variation directly results from the excess demand: at each time step, all agents submit orders and the resulting price is computed. Therefore, everything is cleared at each time step and there is no order book structure to keep track of orders.

One big step was made with models taking into account limit orders and keeping them in an order book once submitted and not yet executed. Chiarella and Iori [7] built an agent-based model where all traders submit orders depending on the three groups identified in the Lux and Marchesi model [6]: chartists, fundamentalists and noise traders. Submitted orders are then stored in a persistent order book. One of the first simple models with this feature was proposed by Bak, Paczuski and Shubik [8]. In this model, orders are particles moving along a price line, and each collision is a transaction. Due to numerous caveats in this model, the authors proposed in the same paper an extension with fundamentalist and noise traders in the spirit of the models mentioned previously. Maslov [9] went further in the modeling of trading mechanisms by taking into account fixed limit orders and market orders that trigger transactions, and the order book was here really simulated. This model was solved analytically by Slanina using a mean-field approximation [10].

Following this trend of modeling, the rational agent models in economics started to be replaced by models with a notion of flows: orders are not submitted any more by an agent following a strategic behavior, but are viewed as arriving in flows whose properties are to be determined by empirical observations on market mechanisms. Biais, Hilllion and Spatt [11] made a thorough empirical study of the order flows in the Paris Bourse a few years after its complete computerization. Market orders, limit orders, time of arrivals, and placement were studied. Challet and Stinchcombe [12] proposed a simple model of order flows: limit orders are deposited in the order book and can be removed if not executed, in a simple deposition–evaporation process. Bouchaud and Potters [13] used this type of model with empirical distribution as inputs. Preis et al. [14] attempted to add perturbations to the order flows in order to reproduce stylized facts. An empirical model was proposed by Mike and Farmer [15], where order placement and cancellation models are proposed and fitted on empirical data. Here, we review some of these models that we feel are representative of a specific trend of modeling; certainly our review is not exhaustive.

2.1 Order-driven market models

Starting from the mid-1990s, physicists have propose order book models directly inspired from physics, where the analogy "order = particle" is emphasized.

The Bak, Paczuski and Shubik model

This is a simple model [8] where the authors consider a market with N noise traders able to exchange one share of stock at a time. Price $p(t)$ at time t is constrained to be an integer (i.e., price is quoted in number of ticks) with an upper bound $\bar{p}$: $\forall t, p(t) \in \{0, \dots, \bar{p}\}$. Simulation is initiated at time 0 with half of the agents asking for one share of stock (buy orders, bid) with price

$$p_{\mathrm{b}}^{j}(0) \in \{0, \bar{p}/2\}, \qquad j = 1, \dots, N/2, \tag{1}$$

and the other half offering one share of stock (sell orders, ask) with price

$$p_{\mathrm{s}}^{j}(0) \in \{\bar{p}/2, \bar{p}\}, \qquad j = 1, \dots, N/2. \tag{2}$$

At each time step t, agents revise their offer by exactly one tick, with equal probability to go up or down. Therefore, at time t, each seller or buyer chooses his new price as

$$p_{\mathrm{s}}^{j}(t+1) = p_{\mathrm{s}}^{j}(t) \pm 1 \qquad \text{or } p_{\mathrm{b}}^{j}(t+1) = p_{\mathrm{b}}^{j}(t) \pm 1. \tag{3}$$

A transaction occurs when there exists $(i,j) \in \{1, \dots, N/2\}^2$ such that $p_{\mathrm{b}}^{i}(t+1) = p_{\mathrm{s}}^{j}(t+1)$. In such a case the orders are removed and the transaction price is recorded as the new price $p(t)$. Once a transaction has been recorded, two orders are placed at the extreme positions on the grid:

$p_b^i(t+1) = 0$ and $p_s^j(t+1) = \bar{p}$. As a consequence, the number of orders in the order book remains constant and equal to the number of agents.

As pointed out by the authors, this process of simulation is similar to the reaction–diffusion model $A + B \rightarrow \emptyset$ in Physics. In such a model, two types of particles are inserted at each side of a pipe of length $\bar{p}$ and move randomly with steps of size 1. Each time two particles collide, they are annihilated and two new particles are inserted. Following this analogy, it thus can be shown that the variation $\Delta p(t)$ of the price $p(t)$ obeys

$$\Delta p(t) \sim t^{1/4} \left(\log \left(\frac{t}{t_0} \right) \right)^{1/2} . \tag{4}$$

Thus, at long time scales, the series of price increments simulated in this model exhibit a Hurst exponent $H = 1/4$. As for the stylized fact $H \approx 0.7$, this subdiffusive behavior appears to be a step in the wrong direction compared to the random walk $H = 1/2$. Moreover, Slanina [16] pointed out that no fat tails are observed in the distribution of the returns of the model, but rather fits the empirical distribution with an exponential decay. Many more drawbacks of the model could be mentioned, the main one being that "moving" orders is highly unrealistic as for modeling an order book. However, we feel that such a model is interesting because of its simplicity and its representation of an order-driven market.

The Maslov model

Maslov [9] keeps the zero-intelligence structure of the earlier model [8] but adds more realistic features in the order placement and evolution of the market: (1) limit orders are submitted and stored in the model, without moving, (2) limit orders are submitted around the best quotes, and (3) market orders are submitted to trigger transactions. More precisely, at each time step, a trader is chosen to perform an action. In contrast to previous models, the number of traders is not fixed, but one trader enters the market at each time step. This trader can either submit a limit order with probability q_l or submit a market order with probability $1 - q_l$. Once this choice is made, the order is a buy or sell order with equal probability. All orders have a one unit volume.

As usual, we denote $p(t)$ as the current price. In case the submitted order at time step $t+1$ is a limit ask (respectively bid) order, it is placed in the book at price $p(t) + \Delta$ (respectively $p(t) - \Delta$), Δ being a random variable uniformly distributed in $]0; \Delta^M = 4]$. In case the submitted order at time step $t+1$ is a market order, one order at the opposite best quote is removed and the price $p(t+1)$ is recorded. In order to prevent the number of orders in the order book from large increase, two mechanisms are proposed by the author: either keeping a fixed maximum number of orders (new limit orders are the discarded) or removing them after a fixed lifetime if they haven't been executed. As for numerical simulations, results show that this model exhibits