

METHODS IN GEOCHEMISTRY  
AND GEOPHYSICS

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# ADVANCED THEORY OF DEEP GEOMAGNETIC SOUNDING

MARK N. BERDICHEVSKY  
and  
MICHAEL S. ZHDANOV



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*The road to  $\sigma$ ? Well, it's plain  
and simple to express:  
err and err and err again  
but less and less and less . . .*

## FOREWORD

*Deep Geomagnetic Sounding*, DGS, involves observation, analysis and interpretation of *anomalies* of the time-varying magnetic or electromagnetic field of the Earth. Many of these anomalies are caused by *deep, highly conducting bodies* originating from hydrothermal phenomena and partial melting processes. Thus, deep geomagnetic sounding provides information on the *phase* and *thermodynamic* states of the *Earth's crust* and *upper mantle*.

The basic ideas in the area of deep geomagnetic sounding have been put forward by H. Wiese, W.D. Parkinson, W. Kertz, M. Siebert, T. Rikitake, U. Schmucker. The seventies witnessed a rapid advance of this method of deep geophysics. The progress can be defined as follows:

First, the theory of electromagnetic induction in the Earth has been considerably advanced. Techniques of solving the direct problem have been developed; a library of models describing typical geological situations has been compiled.

Second, the efforts have shifted toward integration of deep geomagnetic sounding with Magneto-Telluric Sounding, MTS; the MT-sounding gives reference information on the geoelectrical cross-section of the Earth's crust and upper mantle.

Third, qualitative diagnostics of electromagnetic anomalies has been evolved and possibilities of studying *elongated, highly conducting bodies* in an *intermediate, poorly conducting layer*, say, in the Earth's crust, have been suggested.

Fourth, methods have been elaborated for solving the inverse problem, including those based on the optimization principle.

Fifth, deep geomagnetic sounding was exploited in the study of vast regions of the USSR, Central Europe, North America, Africa, and Australia. Deep geomagnetic sounding has become a powerful tool for exploration of the Earth's interior.

Currently, geophysicists investigating anomalies of the electromagnetic field of the Earth have at their disposal the fundamental theory including many thoroughly explored questions and a well-developed mathematics permitting the solution of both direct and inverse problems. Deep geomagnetic sounding helped in the discovery of deep conducting zones in the Carpathian Mountains, the Ukraine and Voronezh shields, the Asian sector of the Pacific island arc, the Canadian shield, the Rocky Mountains, and near the Baikal and African rifts. These results have brought geoelectrics to the forefront of up-to-date theoretical geology which is not short of ideas but which needs concrete geophysical information.

Geophysicists duly appreciate the successes of deep geomagnetic sounding; however, they are aware of its deficiencies and recognize that basically the method is not yet finalized and, hence, requires further improvements and modifications in many aspects.

The difficulties involved in interpretation of gravitational and magnetic anomalies are common knowledge. They are even more serious in the case of a time-varying electromagnetic field with its vortical nature and interaction of secondary sources.

The present book is intended to show some new approaches to the problem that would extend the potential of deep geomagnetic sounding. These methods make use of integral transforms of the electromagnetic field. What we want to stress is a new flexible approach based on the integrals of the Stratton-Chu type. The material presented in the book is largely original, generalizing the authors' papers published in geophysical journals over the period from 1970 to 1980. We have endeavored to make the book self-consistent. Therefore, it includes many previously known concepts underlying geoelectrics (for instance, division of the field into external and internal parts by the Gauss-Schmiedt method). While writing the book we thought that principles rather than technical details would be more interesting, hence, what has emerged is a rather theoretical book. No geological aspects of interpretation are treated.

Now we would like to say some words about organization of subject matter. We proceeded from the fact that the study of electromagnetic anomalies comprises the following three main stages:

(1) Determination of invariant *transfer functions* relating the components of the electromagnetic field at one or two points.

(2) *Analysis* of electromagnetic anomalies, i.e. division of the electromagnetic field into normal and anomalous parts, separation of surface and deep anomalies, their qualitative diagnostics.

(3) *Interpretation* of electromagnetic anomalies, i.e. determination of the geoelectrical structure of the Earth's interior.

The problems of analysis and interpretation are resolved by the methods requiring a priori information on the conductance of the sedimentary section as well as on the normal geoelectrical profile through the Earth's crust and upper mantle. Such information can be provided by geomagnetic profiling or sounding and by magnetotelluric profiling and sounding. Thus, one can construct a closed system of analysis and interpretation of electromagnetic anomalies, which would include:

(1) Determination of conductance of the sedimentary section by the generalized Schmucker method or by MT-profiling.

(2) Determination of a normal geoelectrical profile through the Earth's crust and upper mantle from the normalized values of spectral impedance or by MT-sounding.

(3) Division of the electromagnetic field into external and internal parts



by the classical Gauss-Schmiedt method or by the generalized Kertz-Siebert method.

(4) Division of the electromagnetic field into normal and anomalous parts for a specified normal profile of the Earth's crust and upper mantle and a specified conductance of the sedimentary section.

(5) Integral transformation of the electromagnetic field and its analytic continuation into the Earth, in order to localize highly conducting bodies (the method of singular points, the flux-function method, the multipole expansion method).

(6) Determination of the form of conducting bodies and their geoelectrical cross-section (the method of tightening contours, the finite-function method).

The above specific features of deep geomagnetic sounding dictated the content and breakdown of the book. It is arranged in five parts dealing, respectively, with: (1) equations of the electromagnetic field and boundary conditions on thin layers; (2) the fundamental models of electromagnetic induction in the Earth; (3) the theory of transfer functions; (4) methods of analysis of the electromagnetic field and its analytic continuation; and (5) methods of interpretation of electromagnetic anomalies.

In the book a description is given of the computing system TEMF, Transformation of Electro-Magnetic Field. The TEMF system implements the principal algorithms of analysis for a two-dimensional  $E$ -polarized field.

All the formulas are written in the International System of units (SI).

The book is intended for readers familiar with electrodynamics, geomagnetism, and geoelectrics.

The Russian version of the book was published in 1980 by the "Nedra" publishers under the title "Interpretatsiya anomalii elektromagnitnogo pola Zemli" (Interpretation of Anomalies of the Electromagnetic Field of the Earth). While preparing the English version we wanted to make use of the opportunity to fill some gaps and present the latest developments. Thus, we have expanded the book and changed its title. The readers will find here some new sections devoted to magnetovariation and magnetotelluric soundings and to processing of data. Besides the present book includes novel approaches emerging in deep geomagnetic sounding, say, the multipole expansion method and finite-function method.

Many people helped us.

When working at the book we often met V.I. Dmitriev and discussed the most complicated questions. Under his influence we wrote some sections of the book. It is with direct cooperation of Dmitriev that Sections 3, 4, 9, 12 were prepared.

A participant of our work was E.B. Fainberg. He contributed much to the development of the theory of transfer functions and is a coauthor of the method for dividing the electromagnetic field into inductive and galvanic modes (Chapters III, VI, VIII, IX).

No less significant was the cooperation of I.M. Varentsov. Owing to his

efforts, the TEMF computing system has been constructed and the method of tightening contours developed; he implemented the algorithm for finite-difference simulation (Sections 13, 51 and Chapter XIV).

Among other colleagues assisting in the preparation of the book we should like to express our acknowledgement to O.N. Zhdanova (experiments on interpretation of surface anomalies and development of a program for the TEMF computing system, Sections 47, 48), N.G. Golubev (finite-difference simulation and finite-function method, Sections 13, 52), S.V. Plotnikov (calculation of filters, construction of nomograms for analytic continuation of the field, Sections 42, 44), A.A. Avaghimov, A.I. Bilinsky and A.M. Shilova (interpretation of the Kopetdag and Carpathian anomalies, Sections 48, 53), and N.V. Baglaenko (method of tightening contours, Section 51).

We are greatly indebted to S.M. Sheinmann, who has reviewed the Russian version of the book. We did our best to take due account of his comments. Our deepest gratitude is to G.V. Keller for encouragement and support in preparing the English version. We are also grateful to L.M. Al'pin, B.S. Svetov, and V.N. Strakhov for helpful discussions. Special appreciation is expressed to U. Schmucker and P. Weidelt with whom we have had many inspiring talks concerning the problems of deep geomagnetic sounding. We pay tribute to O.N. Zhdanova for her assistance in the preparation of the typescript for publication and to T.M. Pyankova who has translated our book into English dealing with such cantankerous authors as

M.N. BERDICHEVSKY  
M.S. ZHDANOV

## NOTATIONS

$E, e$	— electric field and its spectrum
$H, h$	— magnetic field and its spectrum
$E^n, H^n, e^n, h^n$	— normal electromagnetic field and its spectra
$E^a, H^a, e^a, h^a$	— anomalous electromagnetic field and its spectra
$H^e, h^e$	— external magnetic field and its spectrum
$H^i, h^i$	— internal magnetic field and its spectrum
$E^A, H^A$	— main part of an anomalous electromagnetic field
$E^s, H^s, e^s, h^s$	— surface electromagnetic anomaly and its spectra
$E^d, H^d, e^d, h^d$	— deep electromagnetic anomaly and its spectra
$E^D, H^D$	— main part of a deep electromagnetic anomaly
$H^T$	— transformed magnetic field
$U^H, U^E, u^H, u^E$	— Debye potentials and their spectra
$U, u$	— scalar potential of the magnetic field and its spectrum
$U^e, U^i, u^e, u^i$	— scalar potentials of the external and internal magnetic fields and their spectra
$\Psi, \psi$	— flux function and its spectrum
$J, j$	— density of an electric current and its spectrum
$J^Q$	— density of an extrinsic electric current distributed in the domain $Q$ of the atmosphere
$J^s, j^s$	— density of an excess electric current distributed in an inhomogeneous near-surface layer
$J^d$	— density of an excess electric current distributed in a deep inhomogeneity $D$
$I, i$	— density of a surface electric current and its spectrum
$I^s, i^s$	— density of an excess surface current distributed in the S-shell and its spectrum
$\mu_0$	— magnetic permeability in vacuum
$\sigma$	— electrical conductivity of a medium
$\sigma_n$	— normal electrical conductivity of a medium
$\sigma_s$	— electrical conductivity of the near-surface layer
$\sigma_d$	— electrical conductivity of a deep inhomogeneity
$S$	— total conductance
$\rho$	— electrical resistivity of a medium
$d$	— thickness of a layer
$k$	— wave number of a medium
$\kappa = -ik$	— modified wave number of a medium
$\lambda$	— electromagnetic wave length
$l$	— field penetration depth

$k_x, k_y$	— spatial frequency along the $x$ and $y$ -axes, $\eta_0 = \sqrt{k_x^2 + k_y^2}$ , $\eta = \sqrt{k_x^2 + k_y^2 - k^2}$
$\omega$	— cyclic frequency of oscillations
$T$	— period of oscillations
$i$	— imaginary unit
$\mathbf{n}$	— unit normal vector
$\mathbf{d}_x, \mathbf{d}_y, \mathbf{d}_z$	— unit vectors of Cartesian basis
$E_n, H_n$	— normal components of the electromagnetic field
$E_\tau, H_\tau$	— tangential vector components of the electromagnetic field
$\text{grad}, \text{grad}^\Gamma$	— three- and two-dimensional gradient
$\text{div}, \text{div}^\Gamma$	— three- and two-dimensional divergence
$\text{curl}, \text{curl}^\Gamma$	— three- and two-dimensional curl
$\nabla, \nabla_\tau$	— three- and two-dimensional Hamilton's operator
$\Delta, \Delta_\tau$	— three- and two-dimensional Laplace's operator
$W_{,\alpha}$	— derivative of the scalar field $W$ with respect to $\alpha$
$A_{\alpha,\beta}$	— derivative of the $\alpha$ -component of the vector field $A$ with respect to $\beta$
$A \cdot B$	— scalar product of the vectors $A$ and $B$
$A \times B$	— vector product of the vectors $A$ and $B$
$f(x_1) * \phi(x_1, x_2)$	— convolution of the functions $f, \phi$ with respect to the variable $x_1$
$M$	— observation point
$M \in V$	— the point $M$ belongs to the domain $V$
$M \notin V$	— the point $M$ does not belong to the domain $V$
$V_1 \subset V_2$	— the domain $V_1$ is enclosed into the domain $V_2$
$V_1 \cup V_2$	— the domain $V_1$ is united with the domain $V_2$
$Z_m$	— spectral impedance of a normal spherical model, $m$ being the degree of a spherical harmonic
$Z_\eta$	— spectral impedance of a normal plane model, $\eta_0 = \sqrt{k_x^2 + k_y^2}$
$Z$	— Tikhonov-Cagniard impedance
$\rho_a$	— apparent resistivity
$G$	— Green function
$\mathbf{G}$	— Hilbert transform operator
$\delta$	— Dirac function; symmetric Kronecker symbol
$\epsilon$	— skew-symmetric Kronecker symbol
$J_n$	— Bessel function of the $n$ -th order
$H_n^{(1)}$	— Hankel function of the first kind, $n$ -th order
$A$	— characteristic vector of a field
$\mathcal{A}^N$	— linear space whose elements are characteristic vectors; $N$ being space dimensionality
$\mathbf{a}, \mathbf{b}^E, \mathbf{b}^H$	— excitation operators
$\mathbf{e}, \mathbf{h}$	— characteristic operators
$\mathbf{e}^{-1}, \mathbf{h}^{-1}$	— inverse characteristic operators

$\mathcal{E}_K, \mathcal{H}_L$	— linear spaces whose elements are the vectors $E, H$ ; $K, L$ - being space dimensionalities
$\mathbf{Z}$	— impedance operator
$Z_{\alpha\beta}$	— impedance tensor matrix
$\mathbf{Y}$	— admittance operator
$Y_{\alpha\beta}$	— admittance tensor matrix
$\mathbf{t}$	— telluric operator
$t_{\alpha\beta}$	— telluric tensor matrix
$t_{\alpha\beta}^F$	— Fourier transform of the telluric matrix
$\mathbf{m}$	— magnetic operator
$m_{\alpha\beta}$	— magnetic tensor matrix
$m_{\alpha\beta}^F$	— Fourier transform of the magnetic matrix
$\mathbf{J}^{uv}$	— inductive operator
$J_{\alpha\beta}^{uv}$	— inductive tensor matrix
$\mathbf{W}^v$	— Wiese-Parkinson matrix
$W^v$	— complex Wiese-Parkinson vector
$\mathbf{W}$	— Wiese vector
$S^{uv}$	— Schmucker complex vector
$K$	— Cauchy type integral
$K^E, K^H$	— Stratton-Chu type integrals
$M^E, M^H$	— modified Stratton-Chu type integrals
$I$	— misfit functional, S-stabilizing functional
$M_\alpha$	— Parametric functional, $\alpha$ being regularization parameter
$\tilde{\alpha}$	— optimal value of the regularization parameter
$\ H\ $	— norm of the vector $H$
$\ B\ $	— norm of the operator $B$
$P_m^n(x)$	— associated Legendre polynomial of degree $m$ , order $n$
$\mathbf{I}$	— unit operator (identity operator)

$$\text{sign } x = \begin{cases} +1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

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