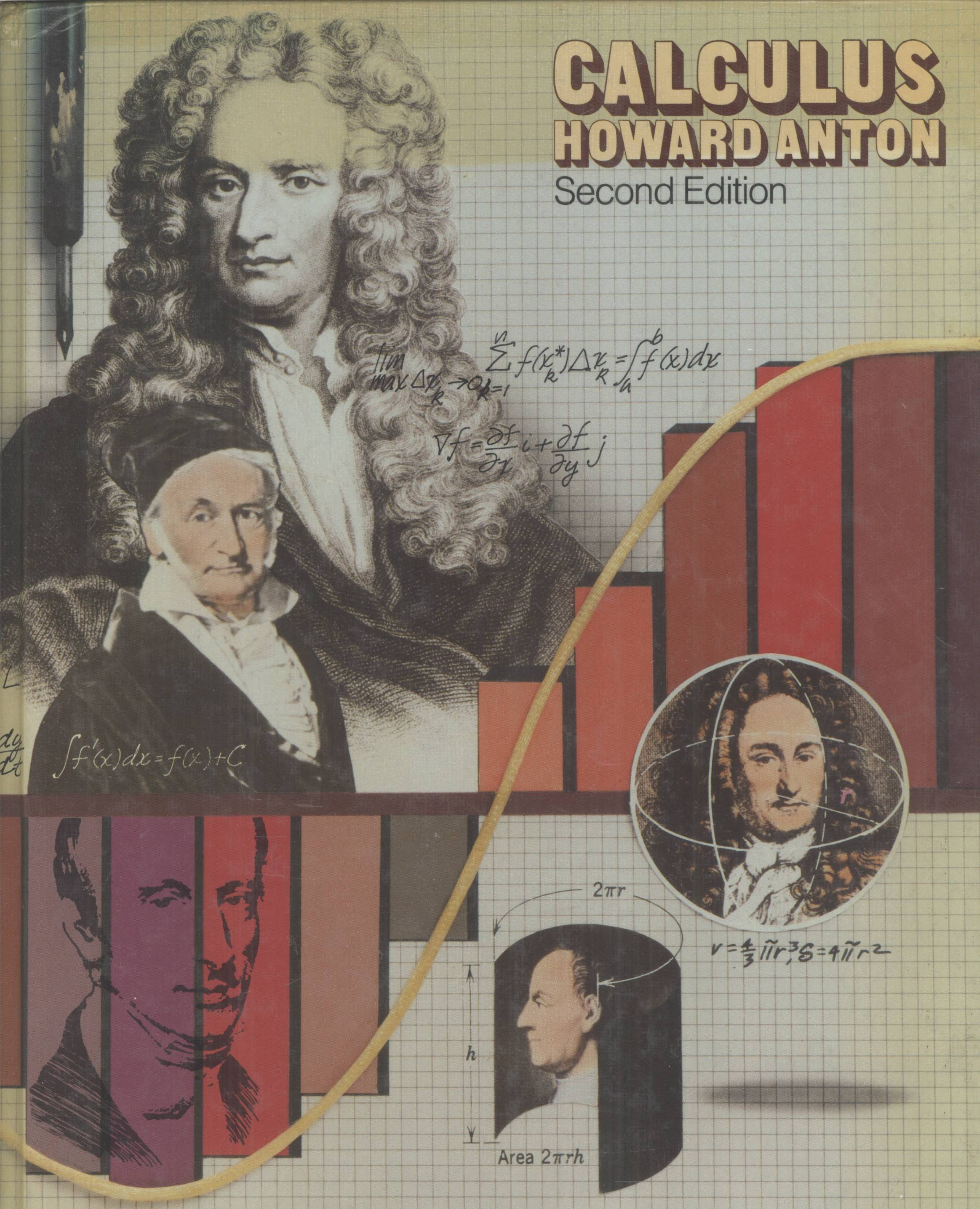
CALCULUS

HOWARD ANTON

Second Edition



Calculus

with analytic geometry

HOWARD ANTON
Drexel University

SECOND EDITION

JOHN WILEY & SONS
New York Chichester Brisbane Toronto Singapore

To the Student Study guides and a solutions manual for this textbook are available through your college bookstore under the titles:

The Calculus Companion, volumes 1 and 2, to Accompany Calculus with Analytic Geometry, Second Edition by Howard Anton, prepared by William H. Barker and James Ward

Student's Solutions Manual to Accompany Calculus with Analytic Geometry, Second Edition by Howard Anton, prepared by Albert Herr

The study guide can help you with course material by acting as a tutorial, review, and study aid. The solutions manual contains detailed solutions to all odd-numbered exercises. If either the study guide or solutions manual is not in stock, ask the bookstore manager to order a copy for you.

Library of Congress Cataloging in Publication Data:

Anton, Howard.
Calculus, with analytic geometry.

Includes index.
1. Calculus. I. Title.
QA303.A53 1984 516'.04 83-19778
ISBN 0-471-08271-6

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Calculus

with analytic geometry

To
My wife Pat
My children Brian, David, and Lauren
My mother Shirley

In memory of
Stephen Girard (1750-1831) benefactor

preface

This text is designed for a standard three-semester or four-quarter calculus course. My intent is to present the material in the clearest possible way. The primary effort has been devoted to *teaching* the subject matter with a level of rigor suitable for the mainstream calculus audience.

Anyone who has taught calculus knows that the course contains so much material that it is impossible for an instructor to spend an adequate amount of classroom time on every topic. For this reason a calculus textbook bears a major portion of the burden in the teaching process. I am hopeful that this book will be one that can be relied on for sound, clear, and complete explanations, thereby freeing valuable classroom time for the instructor.

FEATURES

Precalculus Material Because of the vast amount of material to be covered, it is desirable to spend as little time as possible on precalculus topics. However, it is a fact that freshmen have a wide variety of educational backgrounds and different levels of preparedness. Therefore, I have included an optional first chapter devoted to precalculus material. It is written in enough detail to enable the instructor to feel confident in moving quickly through these preliminaries.

Trigonometry Deficiencies in trigonometry plague many students throughout the entire calculus sequence. Therefore, I have included a substantial trigonometry review in the appendix. It is more detailed than most such reviews because I feel that students will appreciate having this material readily available. The review is broken into two units: the first to be mastered before reading Section 1.4 and the second before Section 3.3

Illustrations This calculus text is more heavily illustrated than most. It is my experience that beginners in mathematics frequently have difficulty extracting concepts from mathematical formulas, yet when the right picture is presented the concept becomes clear immediately. For this reason, I have chosen to take full advantage of modern two-color typography and the most up-to-date illustrative techniques. There are over 1200 illustrations designed to clarify and enhance the exposition. (Most calculus books have half this number.)

Pedagogy I have devoted special effort to the explanations of more difficult concepts. In places where students traditionally have trouble, I have tried to give the reader some foothold on the problem. At times I have simply used artwork designed to focus the student's thinking process in the right direction; at other times I have paraphrased ideas informally in a way that cuts through the technical roadblocks; and at still other times I have simply broken the discussion into smaller, more understandable pieces than generally found in most calculus texts.

Rigor Where possible, theory is presented precisely in a style tailored for freshmen. However, in those places where precision conflicts with clarity, I have presented informal intuitive discussions. Whenever this occurs, there is a clear indication that the arguments given are not intended as formal proof. I have tried to make a clear distinction between rigorous mathematics and informal developments. Theory involving $\delta\epsilon$ arguments has been placed in optional sections so it can be avoided if desired.

Flexibility This book is designed to ensure maximum flexibility. There will be no difficulty in permuting chapters in any reasonable way.

Order of Presentation The order in which topics are presented is fairly standard, with two exceptions: derivatives of trigonometric functions are introduced early (Chapter 3) and a discussion of first order separable and linear differential equations is given in the chapter on logarithmic and exponential functions (Chapter 7). This placement of differential equations material allows us to give some nice applications of logarithms and exponentials immediately, and also helps meet the needs of those engineering and science students who will encounter this material in other courses taken concurrently with calculus.

Applications An abundance of applications to physics, engineering, biology, population growth, chemistry, and economics appear throughout the text.

Exercises Each exercise set begins with routine drill problems and progresses gradually toward problems of greater difficulty. I have tried to construct well-balanced exercise sets with more variety than is available in most calculus texts. Answers, including art, are given for the odd-numbered exercises, and each chapter contains a set of supplementary exercises to help the student consolidate his or her mastery of the chapter.

Supplements There are two supplements available to students: a solutions manual containing worked-out solutions to odd-numbered exercises and a study guide for students who need additional help with text material.

Computer Graphics The use of computers to generate mathematical surfaces is fascinating and growing in importance. To help make the student aware of this tool, I have included a brief (optional) discussion of this topic at the end of Section 16.1.

Reviewing and Class Testing All of the material in this text has been refined and polished as a result of our extensive experience with the first edition. In addition to the hundreds of instructors who wrote to me with comments and constructive suggestions, *twenty-two* reviewers worked with me on the second edition to ensure mathematical accuracy and quality exposition.

CHANGES FOR THE SECOND EDITION

The second edition is a refinement of the first. Although some new material has been added, the major effort has been devoted to improving the exposition and the exercise sets.

A team of reviewers examined each exercise set to determine which ones needed more drill problems, more challenging problems, or more problems of specific types. The exercise sets were expanded accordingly. In addition, many exercise sets were modified to create a better balance between the even and odd exercises in order to give instructors more flexibility in assigning exercises with answers or without.

Throughout the text, material has been rewritten or reorganized to improve the exposition, and some new material has been added based on reviewer and user suggestions. The main changes are as follows:

- New material on graphing $y = ax^2 + bx + c$ was added to Chapter 1 to better prepare students for exercises in which such curves occur.
- The section on differentials was extensively rewritten and a discussion of linear approximation was added to it.
- The section on the chain rule was revised to give equal emphasis to the functional and differential versions of the formula. A rigorous proof of the chain rule was added to the appendix.
- Chapter 4 was totally reorganized as requested by many users.
- A new section on Newton's method was added.
- A review of logarithms was added to the chapter on the natural logarithm, the integral definition of $\ln x$ is now better motivated, and it is shown that $\ln x$ and $\log_e x$ are equivalent.
- The sections on inverse functions and inverse trigonometric functions were rewritten for greater clarity.
- New sections on surface integrals, Stokes' Theorem, and the Divergence Theorem in three dimensions were added.

HOWARD ANTON

acknowledgments

Reviewers It was my good fortune to have the advice and guidance of many fine reviewers. The knowledge and skills that they shared with me have greatly enhanced this book. For their contributions, I thank:

Dennis DeTurck, *University of Pennsylvania*
Garret J. Etgen, *University of Houston*
William R. Fuller, *Purdue University*
Douglas W. Hall, *Michigan State University*
Albert Herr, *Drexel University*
Robert Higgins, *Quantics Corporation*
Harvey B. Keynes, *University of Minnesota*
Leo Lampone, *Quantics Corporation*
Stanley M. Lukawecki, *Clemson University*
Melvin J. Maron, *University of Louisville*
Mark A. Pinsky, *Northwestern University*
Donald R. Sherbert, *University of Illinois*
William F. Trench, *Drexel University*
Richard C. Vile, *Eastern Michigan University*
Irving Drooyan, *Los Angeles Pierce College*
Charles Denlinger, *Millersville State College*
Edith Ainsworth, *University of Alabama*
Norton Starr, *Amherst College*
Katherine Franklin, *Los Angeles Pierce College*
Barbara Flajnik, *Virginia Military Institute*
Jacqueline Dewar, *Loyola Marymount University*
Hugh B. Easler, *College of William and Mary*
Paul Kumpel, *SUNY Stony Brook*
A.L. Deal, *Virginia Military Institute*
Michael Grossman, *University of Lowell*
David Bolen, *Virginia Military Institute*
David Armacost, *Amherst College*
Gary Grimes, *Mt. Hood Community College*
Joseph Meier, *Millersville State College*
Joseph M. Egar, *Cleveland State University*

John Brothers, *Indiana University*
Phil Locke, *University of Maine, Orono*
Richard Thompson, *University of Arizona*
David Cohen, *University of California, Los Angeles*
Marilyn Blockus, *San Jose State University*
Thomas McElligott, *University of Lowell*
Mark Bridger, *Northeastern University*
David Sandell, *U.S. Coast Guard Acdemy*

Exercises and Proofreading I gratefully acknowledge the fine work of Melvin J. Maron who helped prepare the end-of-chapter supplementary exercises. Thanks are also due for the contribution of those who helped with the text exercises and proofreading: Albert Herr, Robert Higgins, Leo Lampone, Hal Schwalm, Andrew Galardi, Steven Fratini, Douglas McCloud, and Evelyn Weinstock.

Assistants For their help with duplicating, paste-up, and collating, I thank my children: Brian Anton, David Anton, and Lauren Anton.

Computer Graphics The excellent computer graphics were prepared by Robert Conley, James Warner, and Richard Yuskaitis of Precision Visuals, P.O. Box 1185, Boulder, Colorado, 80306.

Typing The fine appearance of this text is due in part to the careful typing and formatting of the manuscript by Technitype, 304 Fries Lane, Cherry Hill, New Jersey, 08003. I am especially grateful to Kathleen R. McCabe, who personally handled the project and magically transformed my rough work into polished copy. A word of thanks is also due to Susan Gershuni who typed preliminary versions of the manuscript.

Wiley Staff I give special thanks to the entire Wiley staff for the care and cooperation they have shown in the production and marketing of this edition of the text. In particular, I thank the following people with whom I had the good fortune to work personally:

Robert Pirtle, *Mathematics Editor*
Elaine Rauschal, *Production Manager*
Alejandra Longarini, *Production Supervisor*
Eugene T. Patti, *Supervising Editor*
Trumbull Rogers, *Manuscript Editor*
John Balbalis, *Consulting Artist*
Rafael H. Hernandez, *Assistant Design Director*
Carolyn Moore, *Marketing Manager*

Finally, I wish to acknowledge the major role of my former editor, Gary W. Ostedt, whose dedication to excellence contributed in large measure to the success of the first edition. I feel fortunate to have been the beneficiary of his talent for the past eleven years.

HOWARD ANTON

introduction

Calculus is the mathematical tool used to analyze changes in physical quantities. It was developed in the seventeenth century to study four major classes of scientific and mathematical problems of the time:

1. Find the tangent to a curve at a point.
2. Find the length of a curve, the area of a region, and the volume of a solid.
3. Find the maximum or minimum value of a quantity—for example, the maximum and minimum distances of a planet from the sun, or the maximum range attainable for a projectile by varying its angle of fire.
4. Given a formula for the distance traveled by a body in any specified amount of time, find the velocity and acceleration of the body at any instant. Conversely, given a formula that specifies the acceleration or velocity at any instant, find the distance traveled by the body in a specified period of time.

These problems were attacked by the greatest minds of the seventeenth century, culminating in the crowning achievements of Gottfried Wilhelm Leibniz and Isaac Newton—the creation of calculus.

Gottfried Wilhelm Leibniz (1646–1716)
This gifted genius was one of the last people to have mastered most major fields of knowledge—an impossible accomplishment in our own era of specialization. He was an expert in law, religion, philosophy, literature, politics, geology, metaphysics, alchemy, history, and mathematics.

Leibniz was born in Leipzig, Germany. His father, a professor of moral philosophy at the University of Leipzig, died when Leibniz was six years old. The precocious boy then gained access to his father's library and began reading voraciously on a wide range of subjects, a habit that he maintained throughout his life. At age 15 he entered the University of Leipzig as a law student and by the age of 20 received a doctorate from the University of Altdorf. Subsequently, Leibniz followed a career in law and international politics, serving as counsel to kings and princes.

During his numerous foreign missions, Leibniz came in contact with outstanding mathematicians and scientists who stimulated his interest in mathematics—most notably, the physicist Christian Huygens. In mathematics Leibniz was self-taught, learning the subject by reading papers and journals. As a result of this fragmented mathematical education, Leibniz often duplicated the results of others, and this ultimately led to a raging conflict over the inventor of calculus—Leibniz or Newton? The argument over this question engulfed the scientific circles of England and Europe with most scientists on the continent supporting Leibniz and those in England supporting Newton. The conflict was unfortunate, and both sides suffered in the end. The continent lost the benefit of Newton's discoveries in astronomy and physics for more than 50 years, and for a long period England became a second-rate country mathematically because its mathematicians were hampered by Newton's inferior calculus notation. It is of interest to note that Newton and Leibniz never went to the lengths of vituperation of their advocates—both were sincere admirers of each other's work. The fact is that both men invented calculus independently. Leibniz invented it 10 years after Newton, in 1685, but he published his results 20 years before Newton published his own work on the subject.

Leibniz never married. He was moderate in his habits, quick tempered, but easily appeased, and charitable in his judgment of other people's work. In spite of his great achievements, Leibniz never received the honors show-

Gottfried Leibniz
(Culver Pictures)

Isaac Newton
(Culver Pictures)

ered on Newton, and he spent his final years as a lonely embittered man. At his funeral there was one mourner, his secretary. An eyewitness stated, "He was buried more like a robber than what he really was—an ornament of his country."

Isaac Newton (1642–1727)
Newton was born in the village of Woolsthorpe, England. His father died before he was born and his mother raised him on the family farm. As a youth he showed little evidence of his later brilliance, except for an unusual talent with mechanical devices—he apparently built a working water clock and a toy flour mill powered by a mouse. In 1661 he entered Trinity College in Cambridge with a deficiency in geometry. Fortunately, Newton caught the eye of Isaac Barrow, a gifted mathematician and teacher. Under Barrow's guidance Newton immersed himself in mathematics and science, but he graduated without any special distinction. Because the plague was spreading rapidly through London, Newton returned to his home in Woolsthorpe and stayed there during the years of 1665 and 1666. In those two momentous years the entire framework of modern science was miraculously created in Newton's mind—he discovered calculus, recognized the underlying principles of planetary motion and gravity, and determined that "white" sunlight was composed of all colors, red to violet. For some reason he kept his discoveries to himself. In 1667 he returned to Cambridge to obtain his Master's degree and upon graduation became a teacher at Trinity. Then in 1669 Newton succeeded his teacher, Isaac Barrow, to the Lucasian chair of mathematics at Trinity, one of the most honored chairs of mathematics in the world. Thereafter, brilliant discoveries flowed from Newton steadily. He formulated the law of gravitation and used it to explain the motion of the moon, the planets, and the tides; he formulated basic theories of light, thermodynamics, and hydrodynamics; and he devised and constructed the first modern reflecting telescope.

Throughout his life Newton was hesitant to publish his major discoveries, revealing them only to a select circle of friends, perhaps because of a fear of criticism or controversy. In 1687, only after intense coaxing by the astronomer, Edmond Halley (Halley's comet), did Newton publish his masterpiece, *Philosophae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy). This work is generally considered to be the most important and influential scientific book ever written. In it Newton explained the workings of the solar system and formulated the basic laws of motion which to this day are fundamental in engineering and physics. However, not even the pleas of his friends could convince Newton to publish his discovery of calculus. Only after Leibniz published his results did Newton relent and publish his own work on calculus.

After 35 years as a professor, Newton suffered depression and a nervous breakdown. He gave up research in 1695 to accept a position as warden and later master of the London mint. During the 25 years that he worked at the mint, he did virtually no scientific or mathematical work. He was knighted in 1705 and on his death was buried in Westminster Abbey with all the honors his country could bestow. It is interesting to note that Newton was a learned theologian who viewed the primary value of his work to be its support of the existence of God. Throughout his life he worked passionately to date biblical events by relating them to astronomical phenomena. He was so consumed with this passion that he wasted years searching the Book of Daniel for clues to the end of the world and the geography of hell.

Newton described his brilliant accomplishments as follows, "I seem to have been only like a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

contents