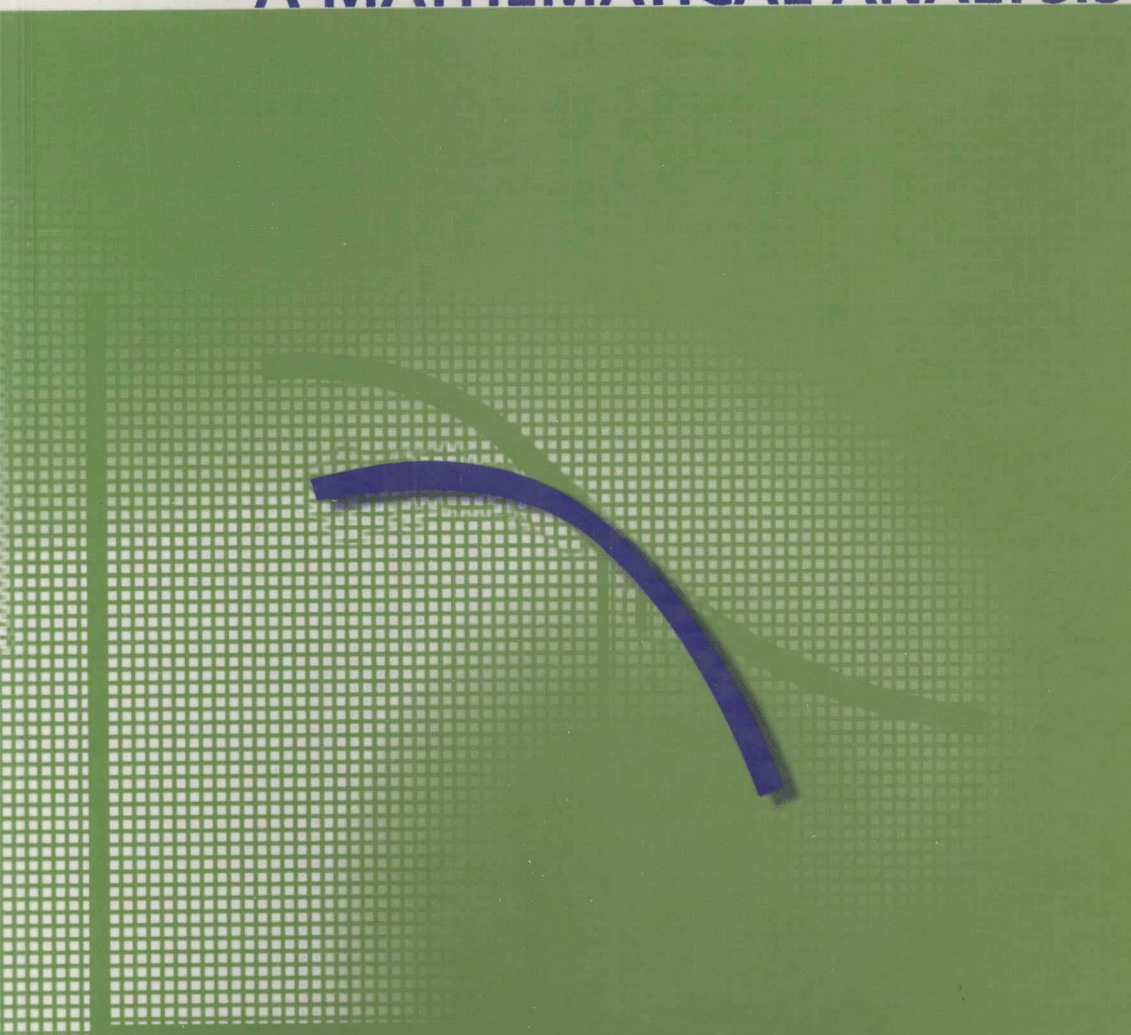
THIRD EDITION

THE STRUCTURE Of ECONOMICS

A MATHEMATICAL ANALYSIS



EUGENE SILBERBERG WING SUEN

McGRAW-HILL INTERNATIONAL EDITION
Economics Series

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THE STRUCTURE OF ECONOMICS

A MATHEMATICAL ANALYSIS

Eugene Silberberg
University of Washington

Wing Suen
University of Hong Kong

Irwin McGraw-Hill

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A Mathematical Analysis
International Edition 2001

10 09 08 07 06 05 04 03
20 09 08 07 06 05 04 03 02 01
CTP SLP

Library of Congress Cataloging-in-Publication Data

Silberberg, Eugene.
The structure of economics: a mathematical analysis / Eugene Silberberg, Wing Suen.—3rd ed.
p. cm.
Includes bibliographical references and indexes.
ISBN 0-07-234352-4
1. Economics, Mathematical. I. Suen, Wing Chuen. II. Title
HB135.S54 2000
330'-01'51—dc21 00-037220

www.mhhe.com

When ordering this title, use ISBN 0-07-118136-9

Printed in Singapore

PREFACE

It's safe to say that the most interesting and important developments in microeconomic theory since the publication of the second edition of this work in 1990 are in the area of choice under imperfect information. With uncertainty, the choices individuals make may reflect the problems of moral hazard and adverse selection, and the operation of the market changes as well to reflect these actions. In the third edition, therefore, we expand the scope of the text to include these new developments in economic theory. In particular, the new Chapter 15, "Contracts and Incentives," covers the recent developments in contract theory, and the new Chapter 16, "Markets with Imperfect Information," covers recent developments in information economics. Wing Suen, of the University of Hong Kong, penned these chapters. Wing was also the secret author in the second edition of Chapter 13, "Behavior Under Uncertainty," to which we have added a few examples.

To accommodate this new material, we discarded the old Chapter 19 on stability of equilibrium. We feel that this material is now less relevant to today's economics courses, both absolutely and relative to the new material. Also, since today's students are much better prepared mathematically than students were when the first edition was first published, we discarded most of the material in Chapter 2, "Review of Calculus (One Variable)," assuming that students have rudimentary knowledge of the calculus of one variable. We maintained the discussion of calculus of several variables but deleted some of the formalisms, in order to make the material accessible to students whose knowledge of that material is less than in working order. Various other changes in the traditional parts of the book include a discussion of discriminating monopoly in Chapter 4, "Profit Maximization"; a theorem and application related to complementary factors of production in Chapter 6, "Comparative Statics: The Traditional Methodology"; an extended but easier discussion of

TO TWO MAGNIFICENT STRUCTURES:

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$I \times 4$ $IV_7 \times 2$ $I \times 2$ $V_7 \times 2$ $I \times 2$

AND TO THE FOLKS WHO DELIVERED THEM SO WELL:
JOE, LOUIS, BIX, JELLY, GERTRUDE, BESSIE AND THEIR FRIENDS.

the LeChâtelier effects in Chapter 7, "The Envelope Theorem and Duality"; and a variety of extensions and emendations throughout the text.

Although all the analysis contained herein derives from topics in microeconomics, the real subject of this book is *meta*economics rather than economics itself. That is, we concern ourselves principally with the methodology of positive economics, in particular, the way meaningful theorems are derived in economics. Paul Samuelson explained in his monumental *Foundations of Economic Analysis* (Harvard University Press, 1947) that the meaningful theorems in economics consist not in laying out various equilibrium conditions, which are rarely observable and therefore empirically sterile, but in deriving predictions that the direction of change of some decision variable in response to a change in some observable parameter must be in some particular direction. The statement that consumers equate their marginal rates of substitution to relative prices is not testable unless we can measure indifference curves. By contrast, the law of demand, which merely requires us to be able to measure the direction of change of an observable price and quantity, is a meaningful, i.e., refutable theorem. Thus in this book, in both the new chapters as well as the old, we devote ourselves almost exclusively to exploring the conditions under which models with a maximization hypothesis generate propositions that are at least in principle refutable.

Although the mathematics we use is elementary, it is extremely useful. The late G. H. Hardy wrote in his delightful essay *A Mathematician's Apology* (Cambridge University Press, 1940) that

> It is the dull and elementary parts of applied mathematics, as it is the dull and elementary parts of pure mathematics, that work for good or ill. Time may change all this. No one foresaw the applications of matrices and groups and other purely mathematical theories to modern physics, and it may be that some of the "highbrow" applied mathematics will become useful in as unexpected a way; but the evidence so far points to the conclusion that, in one subject as in the other, it is what is commonplace and dull that counts for practical life.

Moreover,

> The general conclusion, surely, stands out plainly enough. If useful knowledge is, as we agreed provisionally to say, knowledge which is likely now or in the comparatively near future, to contribute to the material comfort of mankind, so that mere intellectual satisfaction is irrelevant, then the great bulk of mathematics is useless.

But this is precisely what an economist would expect! Hardy was observing the law of diminishing marginal product in the application of mathematical tools to science. A large gain in clarity and economy of exposition can be had from the incorporation of elementary algebra and calculus. The gain from adding real analysis and topology, however, is apt to be less. And perhaps, when such arcane fields as complex analysis and algebraic topology are brought to bear on scientific analysis, their marginal product will be found to be approximately zero, fitting Hardy's definition of "useless." (It is amusing to note, though, that number theory,

long considered one of the most useless of all mathematical inquiries, has recently found important application in modern cryptography.)

In this book we explore the insights that elementary mathematics affords the study of positive economics. We do not explore these issues to their fullest generality or mathematical rigor. Although generality and rigor are important economic goods, their production, because of the above-mentioned law of diminishing returns, entails increasing marginal costs. Thus we are usually content with intuitive, heuristic proofs of many mathematical propositions. We refer students to standard mathematics texts for rigorous discussions of various theorems we use in this book. We aimed for that unobservable margin where for the bulk of our readers, the marginal benefits of greater rigor and generality equal their respective marginal costs. By example after example we hope to convince the reader that these elementary tools yield interesting and sometimes profound insights into modern economics.

A note to students and instructors: Long experience teaching this material, and the authors' own experiences in learning it, have made it abundantly clear that mastering this material is impossible without doing the problems. So do the problems! The only true indicator of understanding is that you can explain the solution to someone else. An *Instructor's Manual* is available from McGraw-Hill.

Eugene Silberberg
Wing Suen

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CHAPTER 1

COMPARATIVE STATICS AND THE PARADIGM OF ECONOMICS

1.1 INTRODUCTION

Suppose we are in a conversation about social changes that have taken place in the past generation. We might discuss, for example, the substantial increase in the rate of participation of women in the competitive labor market, especially in "nontraditional" occupations such as engineering, law, and medicine, the increasing prominence of the "two-earner" family, the increase in the age of first marriage, the rise of "women's liberation," and the like. Suppose now that someone says, "Let me give you an 'economic explanation' of these events." What do you expect to hear? What is meant by the phrase "economic explanation," and what would distinguish it from, say, a sociological or political explanation? For that matter, what do we mean by the term "explanation"?

A list of facts, for example, is not an explanation. Compilations of changes in the weather as seasons pass, or changes in various stock market indices, are not explanations of those events. The stylized data presented in the preceding paragraph are not an explanation of anything; they are only a collection of economic (and sociological) facts, which we typically call "data." The data may be interesting, but they are not "explanations." The term *explanation* means that there is some more general proposition than the observed data for which these facts are special cases. We interpret or understand these facts by applying some general laws or rules by which these events are supposedly guided. For example, physicists "explain" the

motion of ordinary objects on the basis of Newton's classical laws of mechanics. An explanation of the previous socioeconomic data would mean an interpretation of these events in terms of a framework of systematic human behavior, not merely a documentation that these events happened to occur at a particular time. Moreover, we would want to apply that same framework to different sets of facts, allowing the investigator to interpret these other data sets using the same guiding principles. The development of the framework and the specific models employed by economists to explain social phenomena is the subject of this book.

Students who have come this far in economics will undoubtedly have encountered the standard textbook definition of economics that goes something like, "Economics is the science that studies human behavior as a relationship between ends and scarce means which have alternative uses."† This is indeed the substantive content of economics in terms of the class of phenomena generally studied. To many economists (including the authors), however, the most striking aspect of economics is not the subject matter itself, but rather the conceptual framework within which the previously mentioned phenomena are analyzed. After all, sociologists and political scientists are also interested in how scarce resources are allocated and how the decisions of individuals are related to that process. What economists have in common with each other is a methodology, or paradigm, in which *all* problems are analyzed. In fact, what most economists would classify as *noneconomic* problems are precisely those problems that are incapable of being analyzed with what has come to be called the *neoclassical* or *marginalist* paradigm.

The history of science includes many paradigms or schools of thought. The Ptolemaic explanation for planetary motion, in which the earth was placed at the center of the coordinate system (perhaps for theological reasons), was replaced by the Copernican paradigm which moved the origin to the sun. When this was done, the equations of planetary motion were so vastly simplified that the older school was soon replaced (though the Ptolemaic paradigm is essentially maintained in problems of navigation). The Newtonian paradigm of classical mechanics served admirably well in physics, and still does, in fact, in most everyday problems. For study of fundamental processes of nature, however, it has been found to be inadequate and has been replaced by the Einsteinian paradigm of relativity theory.

In economics, the classical school of Smith, Ricardo, and Marx provided explanations of the growth of productive capacity, the gains from specialization and trade (comparative advantage), and the like. One outstanding puzzle persisted: the diamond-water paradox. The classical paradigm, dependent largely on a theory of value based on inputs, was incapable of explaining why water, which is essential to life, is generally available at modest cost, while diamonds, an obvious frivolity, are expensive, even if dug up accidentally in one's backyard (considering the

†Taken from Lionel Robbins' classic monograph, *An Essay on the Nature and Significance of Economic Science*, Macmillan & Co., Ltd., London, 1932, p. 15.

opportunity cost of withholding one from sale).[†] With the advent of marginal analysis, beginning in the 1870s and continuing in later decades by Jevons, Walras, Marshall, Pareto, and others, the older paradigm was supplanted. Economic problems came to be analyzed more explicitly in terms of individual choice. Values were perceived to be determined by consumers' tastes as well as production costs, and the value placed on goods by consumers was not considered to be "intrinsic," but rather depended on the quantities of that good and other goods available.

The structure of this new paradigm was explored further by Hicks, Allen, Samuelson, and others. As this was done, the usefulness and limitations of the new paradigm became more apparent. It is with these properties that this book is concerned.

1.2 THE MARGINALIST PARADIGM

Let us consider the definition of economics in more depth. Economics, first and foremost, is an *empirical science. Positive* economics is concerned with questions of *fact,* which are in principle either true or false. What *ought* to be, as opposed to what *is,* is a normative study, based on the observer's value judgments. In this text, we shall be concerned only with positive economics, the determination of what *is.* (For expositional ease the term *positive* will generally be dropped.) Two economists, one favoring, say, more transfers of income to the poor, and the other favoring less, should still come to the same conclusions regarding the effects of such transfers. Positive economics consists of propositions that are to be tested against facts, and either confirmed or refuted.

But what *is* economics, and what distinguishes it from other aspects of social science? For that matter, what is social science? *Social science is the study of human behavior.* One particular paradigm of social science, i.e., the conceptual framework under which human behavior is studied, is known as the *theory of choice.* This is the framework that will be adopted throughout this book. Its basic postulate is that individual behavior is fundamentally characterized by individual choices, or decisions.[‡]

This fundamental attribute distinguishes social science from the physical sciences. The atoms and molecular structures of physics, chemistry, biology, etc., are not perceived to possess conscious thought. They are, rather, passive adherents to the laws of nature. The choices humans make may be pleasant (e.g., whether to buy a Porsche or a Jaguar) or dismal (e.g., whether to eat navy beans or potatoes for subsistence), but the aspect of choice is asserted to be pervasive.

[†]Of course, being different commodities with different "quantity" measurements, it is not possible to say that diamonds are *more* expensive than water.

[‡]A complicating feature, not relevant to the present discussion but also peculiar to the social sciences, is that the participants often have a vested interest in the results of the analysis.