

WHY WE UNDERESTIMATE RISK IN  
THE FACE OF UNCERTAINTY

THE  
**F** OF **FLAW**  
AVERAGES

SAM L. SAVAGE

WITH ILLUSTRATIONS  
BY JEFF DANZIGER



# **The Flaw of Averages**

**WHY WE UNDERESTIMATE RISK IN  
THE FACE OF UNCERTAINTY**

**Sam L. Savage**



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## Preface

**T**he *Flaw of Averages* describes a set of common avoidable mistakes in assessing risk in the face of uncertainty. It helps explain why conventional methods of gauging the future are so *wrong so often*, and is an accessory to the recent economic catastrophe. Once grasped, these ideas can lead recent us to more effective forecasting and decision making. Traditionally, these topics have been the domain of probability and statistics. Although I will assume no prior knowledge of these subjects, for those who *have* had formal training in these subjects, it should take only a few chapters to repair the damage.

My perspective no doubt derives largely from my father, Leonard Jimmie Savage. Although well *below* average on academic scales during his early education, he emerged as a prominent mathematical statistician who collaborated closely with Milton Friedman, among others. One of their students was the founder of modern portfolio theory, Harry Markowitz, who claims that my father “indoctrinated him at point blank range with rational expectation theory.” Thus I am a child of the University of Chicago School of Economics.

Early on it was clear I possessed at least one of my father’s traits. I, too, was a below-average student, displaying neither athletic nor academic aptitude. The defining moment of my high school education came in an after-class conference with my English teacher in my junior year at the University of Chicago Laboratory School. She explained that I was failing the course, but with a monumental effort might achieve a D by the second semester. Then she helpfully explained the underlying problem: The Lab School was for students who would go on to college, which, quite clearly, I would not. Instead she suggested a technical school where I could get practical training as a mechanic or a plumber.

She therefore presented me with my first serious career decision: to work my butt off for a lousy D in English or play my guitar for

immediate gratification. I made the obvious choice, and music has brought me joy and solace ever since. Better yet, I was able to have my cake and eat it too, because I ended up with a D anyway. In retrospect I cannot question this individual teacher's judgment, because all told I received three D's in four years of high school English, each under a different instructor.

My father was in no position to complain, because when *he* graduated from high school in Detroit, he too "was classified by his teachers as 'not college material,' and consequently was refused admission to the University of Michigan."<sup>1</sup> My grandfather, in desperation, called on personal connections to get him admitted on probation to Wayne State University. Allen Wallis, with whom my father later cofounded the University of Chicago Statistics Department, reported what happened next: "In his year [at Wayne] he established a good enough record to be admitted on probation to the University of Michigan. However, he caused a fire in a chemistry laboratory and was expelled."<sup>2</sup>

Once again I followed in my father's footsteps, later flunking out of the University of Michigan myself, although based on academics rather than involuntary arson.

As adolescent misfits, then, neither of us was able to conform to the norms expected by our teachers. Thus nonaverageness itself became a family value, perhaps in some way inspiring this book. After being de-Michiganized, however, our careers diverged. My father fought his way back into Michigan, got his PhD in mathematics, and achieved great academic acclaim. I worked as a mechanic and briefly raced a sports car before ultimately getting a degree in computer science, which is, in deference to my English teacher, just plumbing with bits of information instead of water.

Although *The Flaw of Averages* will discuss concepts from both statistics and economics, I have little formal training in either of these subjects—just the basics picked up at an early age at the dinner table. Therefore, I have written not from the perspective of a statistician or an economist, but from the perspective of a former mechanic and current plumber of information who grew up surrounded by statisticians and economists.

I came up with some of the core ideas and title for this book in 1999, and I started writing. I knew the concept had potential, but somehow the book was not uplifting: The Flaw of Averages

asserts that everything is below projection, behind schedule, and beyond budget. Where was the happy ending?

In search of one, I continued to teach, consult, and write articles about various aspects of this problem. Feeling the need to stake out the real estate (in case I ever did finish the book), I wrote an article in October 2000 on the Flaw of Averages for the *San Jose Mercury News*.<sup>3</sup> When it was published, it was, to my surprise, adorned with a drawing by the renowned cartoonist Jeff Danziger depicting a statistician drowning in a river that is on average three feet deep. This is reproduced in Chapter 1 of this book.

Over the years, I have had the good fortune to interact with some exceptional people in academia and industry who were grappling with the Flaw of Averages themselves. As a result of this interaction, an approach that we call Probability Management has recently emerged, offering a potential cure for many variants of this problem. And so at last with a happy ending in hand, I renewed my writing efforts in earnest in 2006. All told, on *average*, I have written 21 words per day since 1999.

When my stepbrother, John Pearce, first heard of this writing project, he assumed that I was working through some sort of psychodrama involving my late father. Wrong. This work has been fueled by a psychodrama involving my high school English teachers.

SAM L. SAVAGE

*Palo Alto, California*  
*April 2009*



## Acknowledgments

I must start by chronologically acknowledging those who were directly involved in the evolution of Probability Management. I am indebted to Ben Ball of MIT, first for infecting me with his interest in portfolios of petroleum exploration projects in the late 1980s, and second for the collaboration that laid the foundations for much that lay ahead. In 1992 Mark Broadie of Columbia University gave me a key (a simple spreadsheet model) that unlocked a world of stochastic modeling. In 2003 I had the pleasure of working with Andy Parker of Bessemer Trust on a retirement planning model that pioneered some important ideas in interactive simulation. In 2004, I began an exciting three-way collaboration with Stefan Scholtes of Cambridge University and Daniel Zweidler, then at Shell. This truly put Probability Management on the map with a large interactive simulation application at Shell and a coauthored article in *ORMS Today*. During this time, Dan Fylstra of Frontline Systems made a breakthrough in interactive simulation, turning my dream of interactive simulation in spreadsheets into reality.

The following group also played critical roles in the development of this book. My father, Leonard Jimmie Savage, and his colleagues Milton Friedman and Allen Wallis served as towering intellectual role models from my earliest memories. Next, I must thank Linus Schrage of the University of Chicago for his collaboration on *What's Best!*, without which I would not have been reborn as a management scientist. By supporting my seminar series on management science in spreadsheets, Jack Gould, then dean of the University of Chicago Graduate School of Business, helped launch the odyssey during which I discovered the Flaw of Averages. Stanford's Department of Management Science and Engineering, with which I have been affiliated since 1990, has been the ideal environment in which to experiment with and teach the ideas underlying the book. I owe special thanks to Peter Bernstein, whose own book, *Capital*

*Ideas*, assisted me in my own work and who personally helped get this book off the ground. In 1999, Mina Samuels, who was then an editor for John Wiley & Sons, was inspirational in helping me conceive the book and, when I tracked her down in 2007, was even more supportive as a midwife. In the meantime, Bill Falloon, who inherited my nine-year project at John Wiley, deserves the Most Patient Editor of the Century Award: Thanks. Bill Perry of Stanford University has served as both an inspiration and a foundation of support. Marc Van Allen, of the law firm Jenner and Block, realized that the Flaw of Averages underlies the nation's accounting standards and collaborated in researching and publicizing the issue. Several chapters were inspired by discussions with Howard Wainer, and by a prepublication draft of his book, *Picturing the Uncertain World: How to Understand, Communicate and Control Uncertainty Through Graphical Display*, which I highly recommend. Finally, I owe special thanks to David Empey and Ronald Roth for their programming support over the years and in particular for the implementation of the application at Shell and subsequent development of the DIST (Distribution String) data type.

When it takes you nine years to write a book, there is plenty of time to pick up useful ideas from others. So many people provided assistance, contributions, or comments over the years that they won't fit into a paragraph. Therefore I have used the following table. The laws of probability ensure that I have missed a few people who belong here, for which I apologize in advance.

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Mike Campbell

David Cawfield

Kevin Chang

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Mike Dubis

Ken Dueker

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Roland Frenk

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Bob Glick

Peter Glynn

Joe Grundfest

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In the end, I could not possibly have written this without the guiding light of my wife Daryl, who helped extensively with the editing and who continues to make life so much fun.

S. L. S.

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## PROBABILITY MANAGEMENT

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