

Lial/Miller

Trigonometry / 3rd



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# Trigonometry

Third Edition

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American River College

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**Scott, Foresman and Company**

Glenview, Illinois

London, England

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## To the Student

A *Student Solutions Manual* to accompany this book is available from your local college bookstore. This book can help you study and review the course material.

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## Photo Acknowledgments

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## Library of Congress Cataloging in Publication Data

Lial, Margaret L.

Trigonometry.

Includes index.

I. Trigonometry. I. Miller, Charles David.

II. Title.

QA531.L5      1985      516.2'4      84-14179

ISBN 0-673-18016-6

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Printed in the United States of America.

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# Trigonometry

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# Preface

The Third Edition of *Trigonometry* is designed for a one-semester or one-quarter course that will prepare students either for calculus or for further work in electronics and other technical fields. Applications for both groups of students are given throughout the text.

We have written the book assuming a background in algebra. A course in geometry is a desirable prerequisite, but many students reach trigonometry with little or no background in geometry. For this reason, we have included a section on geometry in Chapter 1 and have explained the necessary ideas from geometry in the text as needed. In addition, the *Instructor's Guide* contains a geometry review unit, which can be reproduced and made available for students if desired.

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## KEY FEATURES

**Examples** More than 200 worked-out examples clearly illustrate concepts and techniques. Second color is used to identify pertinent steps within examples, as well as to highlight explanatory side comments.


**Applications** The number and variety of applications in examples and exercise sets have been increased in this edition. As before, these applications are spread throughout the text rather than being concentrated in one chapter. Applications from the fields of engineering, physics, biology, astronomy, navigation, and demography are included.

**Calculators** Without making the text a calculator instruction book, we do tell students when a calculator would be appropriate and how to use it. In addition, a calculator introduction gives a detailed discussion of calculator use, with references to appropriate chapters in the text. Flexibility has not been sacrificed; table use and evaluation of problems is taught as well as calculator use, and either approach can be used in the classroom.

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## EXERCISES

**Graded Exercises** The range of difficulty in the exercise sets affords students ample practice with drill problems. Then they are eased gradually through problems of increasing difficulty to problems that will challenge outstanding students. More than 3100 exercises, including approximately 2900 drill problems and 200 word problems, are provided in the text.

**Calculator Exercises** A large number of exercise sets feature calculator problems, and they are identified with the symbol . A scientific calculator is not essential for the course, however; most problems can be worked with a four-function calculator and the tables we supply.

**Chapter Review Exercises** A lengthy set of review exercises, about 525 problems in all, is given at the end of each chapter and reviews each section of the text thoroughly. This provides further opportunity for mastery of the material before students take an examination.

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## SECOND COLOR

Second color is used pedagogically in the following ways.

- Screens set off key definitions, formulas, and procedures, helping students review easily.
- Color side comments within examples explain the structure of the problem.
- For clarity, the end of each example is indicated with a color symbol, ●.

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## CONTENT FEATURES

The basic ideas of trigonometry are presented very early. **Angles** are discussed in Section 1.2, and the **trigonometric functions** are defined in Section 1.4. **Identities** are introduced briefly in Chapter 1, so that students see an early discussion of this key idea.

**Triangles** are presented early. In Chapter 2 we show how triangles are used to obtain the values of the trigonometric functions for acute angles and then show some applications of trigonometry. This is designed to give students a feel for the usefulness of trigonometry. All the right triangle applications are grouped together at the end of Chapter 2.

**Radian measure** is introduced in Chapter 3, and thereafter a balance between radian and degree measures is provided. Students going on to calculus will have received enough practice with radian measure to be comfortable using it in calculus.

The **identities** in Chapter 5 are proven in a format that shows the reasons for key steps at the side. This gives students a clear understanding of the structure of each proof.

**Inverse functions** are treated in detail in Chapter 6, which begins with a section on inverse functions in general for review before introducing the inverse trigonometric functions.

Although the **law of sines** and the **law of cosines** are presented in Chapter 7, this chapter could be taught after Chapter 2, except for some derivations.

The chapter on **logarithms** is written to emphasize the exponential and logarithmic functions and the solving of exponential and logarithmic equations.

## SUPPLEMENTS

The **Instructor's Guide** gives a geometry review unit, answers to the even-numbered exercises in the text, three alternative forms of chapter tests in a format that is ready for duplication, a test bank for each chapter, and answers to all test items.

A **Student Solutions Manual** has solutions to all the odd-numbered exercises in the text. Some students may want to use this book as an additional source of examples.

An 80-page supplement on **Analytic Geometry** covers all the standard topics for those who make this material part of their trigonometry course.

*Trigonometry*, Third Edition, is part of a series of texts, including *Beginning Algebra*, Fourth Edition; *Intermediate Algebra*, Fourth Edition; *College Algebra*, Fourth Edition; *Fundamentals of College Algebra*; and *Algebra and Trigonometry*, Third Edition. The use of a series of texts increases student understanding by offering continuity of notation, definitions, and format.

We thank the many users of the previous editions of this book who were kind enough to share their experiences with us. This revision has benefited from their comments and suggestions.

We also thank the people who reviewed all or part of the revised manuscript and gave us many helpful suggestions: Elizabeth Cauley, Pensacola Junior College; Virginia Deus, Diablo Valley College; Louis M. Edwards, Valencia Community College; Allen E. Hansen, Riverside City College; Earl E. Hasz, Metropolitan State College; Joannah Hill, Montana State University; Louis F. Hoelzle, Bucks County Community College; Virgil C. Kowalik, Texas A & I University; Robert G. Kuller, Northern Illinois University; Harry Smith, Iowa State University; George T. Wales, Ferris State College; Dennis Weltman, North Harris County Junior College; and August J. Zarcone, College of DuPage.

Our appreciation also goes to the staff at Scott, Foresman and Company, who did an excellent job in working with us toward publication.

Margaret L. Lial  
Charles D. Miller

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## Using a Calculator

This discussion of calculators is broken into two parts. First we discuss things to keep in mind if you are buying a calculator for use in this course. Then we discuss some of the things to remember as you actually use the machine to work trigonometry problems.

**Buying a Calculator** A scientific calculator is very helpful for trigonometry. These calculators once cost hundreds of dollars, but now can be purchased for less than the cost of a pizza. Scientific calculators can be recognized by the following keys (among others):



The first of these keys is used in trigonometry, the second to find the logarithm of a number, and the third to raise a number to a power. A scientific calculator has the advantage of doing away with the need for most tables; with a scientific calculator you would seldom if ever use a table in a trigonometry course. (The general skill of table reading is still useful, however, so we include it in the book and many instructors discuss it in class.)

Some advanced calculators are *programmable*: instead of starting each new problem of a given type from scratch, only the necessary keystrokes are entered. Then the data for a new problem can be entered, with only one or two keys needed to get the result. A programmable calculator is not really necessary for this course. However, such calculators do offer two advantages: first, since you cannot program a calculator for a group of problems unless you completely understand the basic ideas of the problems, your understanding would be enhanced; and second, the programming skills taught in using such calculators are useful in further course work in science or mathematics.





There are two types of logic in common use in calculators today. Both algebraic and Reverse Polish Notation (RPN) have advantages and disadvantages. Algebraic logic is the easiest to learn. For example, the problem  $8 + 17$  is entered into an algebraic machine by pressing

$$8 + 17 = .$$

On a machine with Reverse Polish Notation (named for the eminent Polish mathematicians who developed the system), this same problem would be entered as

$$8 \text{ ENTER } 17 + .$$

Some people claim that Reverse Polish machines work advanced problems more easily than algebraic machines. Others claim that algebraic machines are easier to use for the great bulk of ordinary, common problems. It is up to you to decide which to buy. One of us has a Reverse Polish machine, and the other has an algebraic machine.

**Calculator Errors** A calculator can store only so many digits in its memory. Because of this, numbers that have more digits than can be stored must be rounded. For example,  $1/3$  is not stored as the exact fraction  $1/3$ , but rather as a decimal, perhaps .333333333333. Since this rounded form of  $1/3$  is used, errors can occur in calculations. To see how this happens, use a calculator to divide 1 by 3, and then multiply the result by 3. You should get 1 (exactly), but many machines produce

$$(1 \div 3) \times 3 = \left(\frac{1}{3}\right) \times 3 = .999999999.$$

Some machines round this result to 1; however, the machine does not treat the number internally as 1. To see this, subtract 1 from the result above; you should get 0 but probably will not.

Another calculator error results when numbers of greatly different size are used in addition. For example,

$$10^9 + 10^{-5} - 10^9 = 10^{-5}.$$

However, most calculators would give

$$10^9 + 10^{-5} - 10^9 = 0.$$

These calculator errors seldom occur in realistic problems, but if they do occur you should know what is happening.

**Using a Calculator** While this introduction is not designed to replace your calculator instruction manual, we do list a few things to keep in mind as you use your calculator.

**Parentheses** Many calculators have parentheses keys,  $\boxed{[}$  and  $\boxed{]}$ . These are used as in algebra. For example,  $(3 \cdot 5 + 8 \cdot 2) \cdot 4$  could be found as follows.

$$\boxed{[} \ 3 \ \boxed{\times} \ 5 \ \boxed{+} \ 8 \ \boxed{\times} \ 2 \ \boxed{]} \ \boxed{\times} \ 4 \ \boxed{=} \ 124$$

**Memory** A memory key is like an electronic piece of scratch paper. Pressing **M** or **STO** will cause the number in the display to be stored, and pressing **MR** or **RCL** will cause it to be recalled.

**Scientific Notation** A key labeled **EE** permits numbers to be entered in scientific notation. For example, entering 9.68, pressing **EE**, and then entering 5, followed by **+/-**, results in the display

$$9.68 - 05,$$

which represents  $9.68 \times 10^{-5}$ . Pressing **INV** and then **EE** causes the number in display to be written in regular notation.

The next few paragraphs discuss some of the special keys on the calculator that are used in trigonometry. Refer to these when you reach the appropriate portion of the book.

**Degrees, Minutes, Seconds** [Chapter 1] Many scientific calculators will convert automatically between angle measures in degrees, minutes, seconds and angle measures in decimal degrees. A circle is divided into 360 degrees, written  $360^\circ$ . Each degree is divided into 60 minutes, written  $60'$ , and each minute into 60 seconds, written  $60''$ . The notation  $12^\circ 51'$  represents

$$12^\circ 51' = 12 + \frac{51}{60} = 12.85^\circ,$$

and  $58^\circ 30' 15''$  is

$$58 + \frac{30}{60} + \frac{15}{3600} = 58.504167^\circ.$$

Some calculators will make these conversions automatically, often with a key labeled **DMS**. A calculator does the two examples above as follows.

Enter	Press	Display
12.51	<b>DMS</b>	12.85
58.3015	<b>DMS</b>	58.504167

To convert  $106.5^\circ$  and  $249.89^\circ$  to degrees, minutes, seconds, use a calculator as follows. (The column headed "Result" shows how to interpret the display.)

Enter	Press	Display	Result
106.5	<b>INV</b> <b>DMS</b>	106.30	$106^\circ 30'$
249.89	<b>INV</b> <b>DMS</b>	249.5324	$249^\circ 53' 24''$

**Values of Trigonometric Functions** [Chapter 2] Values of the sine, cosine, and tangent functions can be found directly with a scientific calculator. First make sure that the angle is expressed in decimal degrees, and that the calculator is set for degree measure.

Angle	Decimal degrees	Press	Display
84°	84.	<b>SIN</b>	.99452190
92° 50'	92.833333	<b>COS</b>	-.04943084
52° 12' 47"	52.213056	<b>TAN</b>	1.2897990

Values of the cotangent, secant, and cosecant functions can be found from the identities

$$\cot \theta = \frac{1}{\tan \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \text{and} \quad \csc \theta = \frac{1}{\sin \theta},$$

which are developed later in this book. Use the reciprocal key, **1/x**, on your calculator. For example, to find  $\csc 58.5^\circ$ ,  $\sec 119.7213^\circ$ , and  $\cot 1000^\circ$ , use the following keystrokes.

Enter	Press	Display	Result (rounded)
58.5	<b>SIN</b> <b>1/x</b>	1.1728277	$\csc 58.5^\circ = 1.17283$
119.7213	<b>COS</b> <b>1/x</b>	-2.0170174	$\sec 119.7213^\circ = -2.01702$
1000	<b>TAN</b> <b>1/x</b>	-.17632698	$\cot 1000^\circ = -.17633$

**Radian-Degree Conversion** [Chapter 3] Some calculators will convert directly between the degree measure and the radian measure of an angle, often with a **DR** key. For example, to find the radian measure of  $86^\circ$  and the degree measure of 2 radians, work as shown below.

Enter	Press	Display	Result (rounded)
86	<b>DR</b>	1.5009832	$86^\circ = 1.50098$ radians
2	<b>INV</b> <b>DR</b>	114.59156	2 radians = $114.5916^\circ$

**Inverse Trigonometric Functions** [Chapter 6] There is a great variation in the keys used to find the values of the inverse trigonometric functions. To find  $\theta$  in the statement

$$\sin \theta = .5,$$

some calculators require that you press the combination **INV** **SIN**, while others use **SIN<sup>-1</sup>**, and still others use **ARCSIN**. To find out how your calculator works, make sure that it is set for degree measure, enter .5, and find the appropriate keys to give a result of 30 (since  $\sin 30^\circ = .5$ .) Then, for example, to find values of  $\theta$  such that  $\sin \theta = .85546$  and  $\cos \theta = .115892$ , work as follows.

Enter	Press	Display	Result (rounded)
.85546	<b>appropriate</b>	58.810591	$\sin 58.811^\circ = .85546$
.115892	<b>keys</b>	83.344923	$\cos 83.3449^\circ = .115892$

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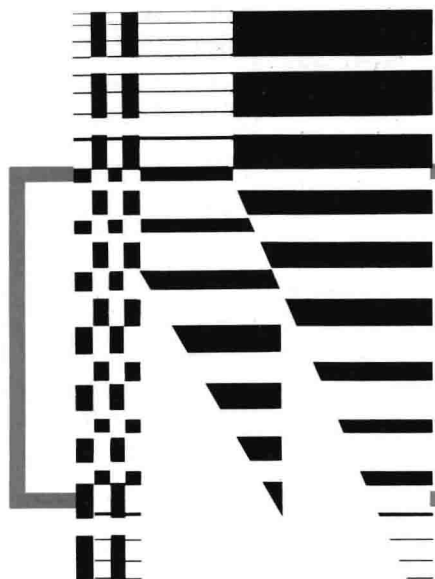
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# The Trigonometric Functions

The foundations of trigonometry go back at least three thousand years. The ancient Egyptians, Babylonians, and Greeks developed trigonometry to find the lengths of the sides of triangles and the measures of their angles. In Egypt trigonometry was used to reestablish land boundaries after the annual flood of the Nile River. In Babylonia it was used in astronomy. The very word *trigonometry* comes from Greek words for triangle (*trigon*) and measurement (*metry*). Today trigonometry is used in electronics, surveying, and other engineering areas, and is necessary for further courses in mathematics, such as calculus.

Perhaps the most exciting thing to happen to trigonometry in the last few years is the widespread availability of inexpensive calculators. With these calculators, arithmetic calculations that formerly required many hours, or were not done at all, can now be done in a few moments. In this book, while a calculator is not required, it would be a big help. Exercises where a calculator would be especially helpful are identified by the symbol .

## 1.1 Basic Terms

Many ideas in trigonometry are best explained with a graph. Graphs in the plane are drawn with the aid of **ordered pairs**, two numbers written inside parentheses, such as  $(-2, 4)$ . Graphs are set up with two axes, one for each number in an



ordered pair. (See Figure 1.1.) The horizontal axis is called the **x-axis**, and the vertical axis is the **y-axis**. The two axes cross at a point called the **origin**. To locate the point that corresponds to the ordered pair  $(-2, 4)$ , start at the origin, and go 2 units left and 4 units up. The point  $(-2, 4)$  and other sample points are shown in Figure 1.1.

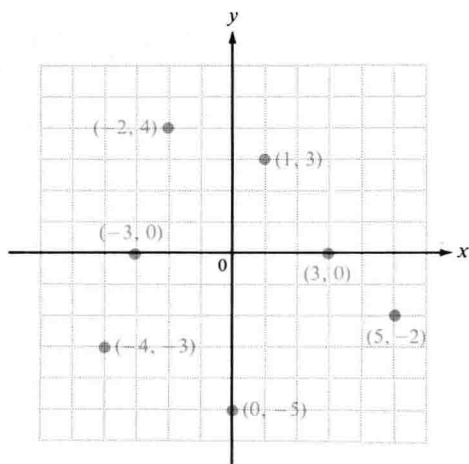


Figure 1.1

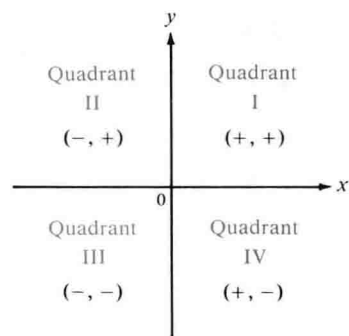


Figure 1.2

The axes divide the plane into four regions called **quadrants**. The quadrants are numbered in a counterclockwise direction, as shown in Figure 1.2. The points on the axes themselves belong to none of the quadrants. Figure 1.2 also shows that in quadrant I both the  $x$ -coordinate and the  $y$ -coordinate are positive; in quadrant II the value of  $x$  is negative while  $y$  is positive, and so on.

The distance between any two points on a plane can be found by using a formula derived from the **Pythagorean theorem**.

### Pythagorean Theorem

If the two shorter sides of a right triangle have lengths  $a$  and  $b$ , respectively, and if the length of the hypotenuse (the longest side) is  $c$ , then

$$a^2 + b^2 = c^2.$$

