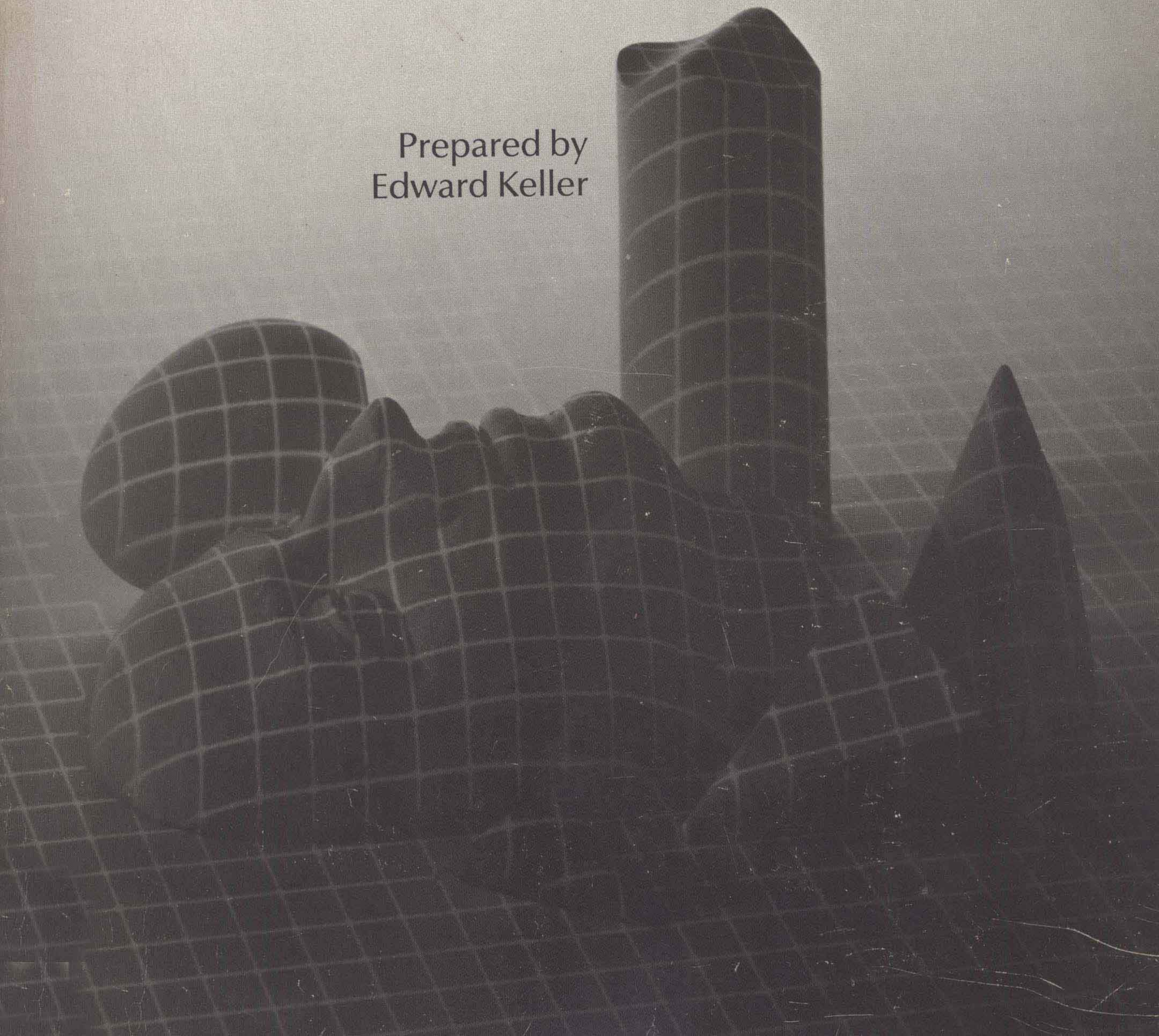
Solutions Manual to accompany

CALCULUS
with Analytic Geometry

Arthur B. Simon



Prepared by
Edward Keller

SOLUTIONS MANUAL
to accompany

CALCULUS WITH ANALYTIC GEOMETRY

by
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TO THE STUDENT

This book is intended for your use as you study Calculus with Analytic Geometry by Arthur B. Simon. Here you will find step-by-step solutions to over 800 exercises illustrating every major type of problem from the text. I have emphasized applied problems ("word" problems) because most students have difficulty with such problems. Many people find the most troublesome aspect of calculus to be the algebraic manipulation required; I have shown in detail the algebraic steps used in obtaining the answers. In non-routine problems I have tried to explain why I chose a particular approach to a problem. In some cases, alternative solution methods are indicated and common errors are pointed out.

When working on a problem I suggest that you *not* consult the solutions manual until you have attempted the problem on your own. If you are stuck on a problem, you will probably want to read only enough of the solution to get you started again. After you have finished the problems from a section of the text, you will find it useful to study the solutions manual so that you can see alternative methods of solution and can check your work.

Simply reading a solution will usually not be of much benefit. You should read with pencil in hand, verifying the steps as you read. You will also need a scientific calculator for many of the problems. The calculator need not be an expensive one, but it should have log and trig functions and some memory. I assume, of course, that you will have a copy of Calculus with Analytic Geometry as you use the solutions manual. I refer on occasion to theorems and formulas from the text and in some problems I use integration formulas from the list found in the textbook.

ACKNOWLEDGEMENTS

I would like to express my appreciation to Art Simon who discussed a number of the solutions with me. My special thanks go to Marguerite Hsu for cheerfully and accurately typing the manuscript. Also, I wish to thank Jan Muzzy, who with her usual skill assisted with the typing. Finally, I would like to thank my wife, Esther, and children, David and Beth, for their constant love and support.

Ed Keller
Hayward, California

February 1982

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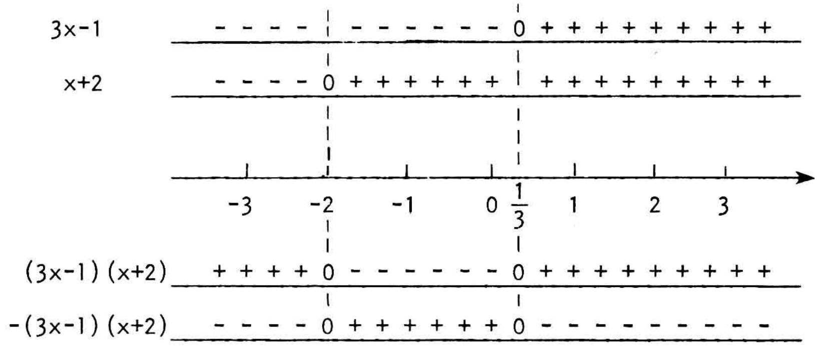
INTRODUCTION TO CALCULUS 1

SECTION 1-1 THE REAL LINE

19. We can write $-3x^2 - 5x + 2$ as $-(3x^2 + 5x - 2)$, which factors into $-(3x-1)(x+2)$. Examining the factors separately, we have

$$3x-1 \begin{cases} > 0 & \text{for } x \text{ in } (1/3,\infty) \\ = 0 & \text{for } x = 1/3 \\ < 0 & \text{for } x \text{ in } (-\infty,1/3) \end{cases} \qquad x+2 \begin{cases} > 0 & \text{for } x \text{ in } (-2,\infty) \\ = 0 & \text{for } x = -2 \\ < 0 & \text{for } x \text{ in } (-\infty,-2) \end{cases}$$

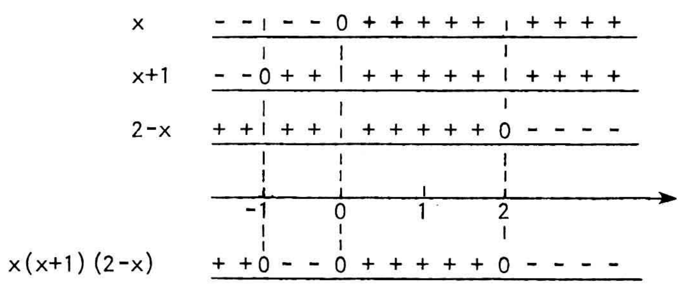
Next, we enter this information in a diagram.



From the diagram we see that

$$-3x^2-5x + 2 \begin{cases} > 0 & \text{for } x \text{ in } (-2,1/3) \\ = 0 & \text{for } x = -2 \text{ or } 1/3 \\ < 0 & \text{for } x \text{ in } (-\infty,-2) \text{ or } (1/3,\infty). \end{cases}$$

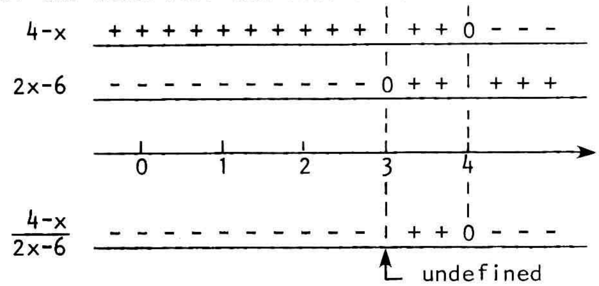
21. The diagram has a line for each of the three factors, x, $x+1$, and $2-x$.



The answer can be obtained directly from the diagram:

$$x(x+1)(2-x) \begin{cases} > 0 & \text{for } x \text{ in } (-\infty,-1) \text{ or } (0,2) \\ = 0 & \text{for } x = -1,\ 0 \text{ or } 2 \\ < 0 & \text{for } x \text{ in } (-1,0) \text{ or } (2,\infty). \end{cases}$$

29. The signs of the numerator and denominator are shown on separate lines.



NOTE: An expression of the form $\frac{0}{a}$ equals 0 provided $a \neq 0$, but any expression of the form $\frac{a}{0}$ is undefined.

Answer:

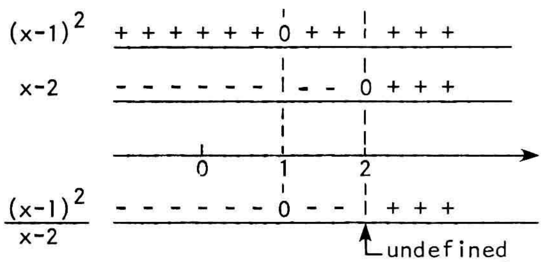
$$\frac{4-x}{2x-6} \begin{cases} > 0 & \text{for } x \text{ in } (3,4) \\ = 0 & \text{for } x = 4 \\ < 0 & \text{for } x \text{ in } (-\infty,3) \text{ or } (4,\infty) \end{cases}$$

The expression is undefined for $x=3$.

31. First add the fractions.

$$x + \frac{1}{x-2} = \frac{x(x-2)+1}{x-2} = \frac{x^2-2x+1}{x-2} = \frac{(x-1)^2}{x-2}.$$

Here is a sign diagram:



Answer:

$$x + \frac{1}{x-2} \begin{cases} > 0 & \text{for } x \text{ in } (2,\infty) \\ = 0 & \text{for } x = 1 \\ < 0 & \text{for } x \text{ in } (-\infty,1) \text{ or } (1,2) \end{cases}$$

The expression is undefined for $x=2$.

51. $|3x - 5| < 4$
$-4 < 3x - 5 < 4$ (since $|B| < a$ if and only if $-a < B < a$)
$-4 + 5 < 3x < 4 + 5$
$1 < 3x < 9$
$1/3 < x < 3$
Thus the original inequality holds if and only if x is in the interval $(\frac{1}{3}, 3)$.

55. $\left|\frac{x-3}{-2}\right| < .25$

$\frac{|x-3|}{2} < .25$ (since $\left|\frac{x-3}{-2}\right| = \frac{|x-3|}{|-2|} = \frac{|x-3|}{2}$)

$|x-3| < .5$

$-.5 < x-3 < .5$

$2.5 < x < 3.5$

SECTION 1-2 THE COORDINATE PLANE

1. The distance between the points $(x_1,y_1) = (1,3)$ and $(x_2,y_2) = (2,5)$ is

$$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(2-1)^2 + (5-3)^2} = \sqrt{1 + 4} = \sqrt{5} .$$

The midpoint of the line segment is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 2}{2}, \frac{3 + 5}{2}\right) = \left(\frac{3}{2}, 4\right).$$

19. In the equation $y = 2x^2 - 3x + 1$ we see that $a=2$, $b=-3$, and $c=1$. Since $a > 0$, the graph of the parabola opens upward.

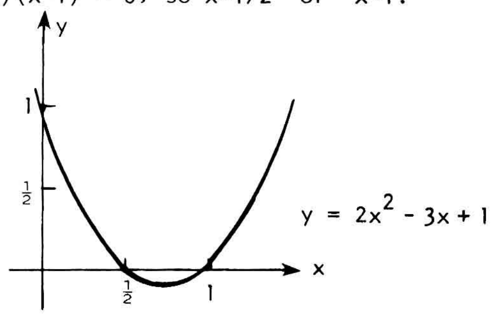
x coordinate of the vertex: $-\frac{b}{2a} = -(\frac{-3}{4}) = \frac{3}{4}$

y coordinate of the vertex: $2(3/4)^2 - 3(3/4) + 1 = -1/8$

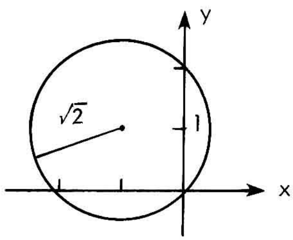
To find the y-intercept, set $x=0$ to obtain $y=1$. To find the x-intercepts we must find the values of x for which $y=0$; i.e., we must solve $2x^2 - 3x + 1 = 0$. Factoring gives $(2x-1)(x-1) = 0$, so $x=1/2$ or $x=1$.

In summary, we have found these points:

x	0	1/2	3/4	1
y	1	0	-1/8	0



25. To find the center and radius, complete the square:



$$x^2 + y^2 + 2x - 2y = 0$$
$$(x^2 + 2x + \_) + (y^2 - 2y + \_) = 0 + \_ + \_$$
$$(x^2 + 2x + 1) + (y^2 - 2y + 1) = 0 + 1 + 1$$
$$(x + 1)^2 + (y - 1)^2 = 2$$

Comparing with the standard equation (1.22) we see that $h=-1$, $k=1$ and $r=\sqrt{2}$. Hence, the graph is a circle with center $(-1,1)$ and radius $\sqrt{2}$.

49. Since the center is $(1,-2)$, the equation is of the form

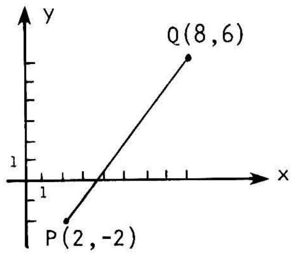
$$(x-1)^2 + (y + 2)^2 = r^2.$$

The point $(2,4)$ must satisfy the equation, so

$$(2-1)^2 + (4 + 2)^2 = r^2; \text{ that is, } 37 = r^2$$

The equation is $(x-1)^2 + (y + 2)^2 = 37$.

51. To find the equation of the circle, we need to determine the center and radius. The solution is easier to visualize if we first draw a picture. The center of the circle must be the midpoint of PQ, which is



$$\left(\frac{2+8}{2}, \frac{-2+6}{2}\right) = (5,2)$$

by the midpoint formula. The radius is half the distance between P and Q. Thus

$$r = \frac{1}{2}\sqrt{(8-2)^2 + (6-(-2))^2}$$
$$= \frac{1}{2}\sqrt{36 + 64}$$
$$= \frac{1}{2}\sqrt{100} = 5$$

Hence, the equation is $(x-5)^2 + (y-2)^2 = 25$.

55. The lengths of the sides can be computed using the distance formula:

$$d(A,B)=\sqrt{(-2-1)^2 + (0 - 1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$d(B,C) = \sqrt{\left[\frac{-1+\sqrt{3}}{2} - (-2)\right]^2 + \left[\frac{1-3\sqrt{3}}{2} - 0\right]^2} = \sqrt{\left(\frac{3+\sqrt{3}}{2}\right)^2 + \left(\frac{1-3\sqrt{3}}{2}\right)^2}$$
$$= \tfrac{1}{2}\sqrt{9+6\sqrt{3}+3+1-6\sqrt{3} + 27} = \tfrac{1}{2}\sqrt{40} = \tfrac{1}{2}\sqrt{4\cdot 10} = \sqrt{10}$$

$$d(A,C) = \sqrt{\left[\frac{-1+\sqrt{3}}{2} - 1\right]^2 + \left[\frac{1-3\sqrt{3}}{2} - 1\right]^2}$$

$$= \sqrt{\left(\frac{-3+\sqrt{3}}{2}\right)^2 + \left(\frac{-1-3\sqrt{3}}{2}\right)^2}$$

$$= \tfrac{1}{2}\sqrt{9 - 6\sqrt{3} + 3 + 1 + 6\sqrt{3} + 27} = \tfrac{1}{2}\sqrt{40} = \sqrt{10}$$

Since d(A,B)=d(B,C)=d(A,C), the triangle is an equilateral triangle.

SECTION 1-3 FUNCTIONS

5. $g(\sqrt{2}) = (\sqrt{2})^2 - 3\sqrt{2} + 1 = 2 - 3\sqrt{2} + 1 = 3 - 3\sqrt{2}$

$g(-3.7) = (-3.7)^2 - 3(-3.7) + 1 = 13.69 + 11.1 + 1 = 25.79$

$g(-1 + h) = (-1 + h)^2 - 3(-1 + h) + 1 = 1 - 2h + h^2 + 3 - 3h + 1$

$= 5 - 5h + h^2$

11. You can compute the function values using the formula in its original form. The computation will be easier, however, if you first simplify the expression:

$$F(x) = \frac{x^2 - 4x + 3}{x - 3} = \frac{(x-3)(x-1)}{x-3} = x - 1, \text{ provided } x \neq 3.$$

Now, $F(3.1) = 3.1 - 1 = 2.1$
$F(3.01) = 3.01 - 1 = 2.01$
$F(3.001) = 3.001 - 1 = 2.001$

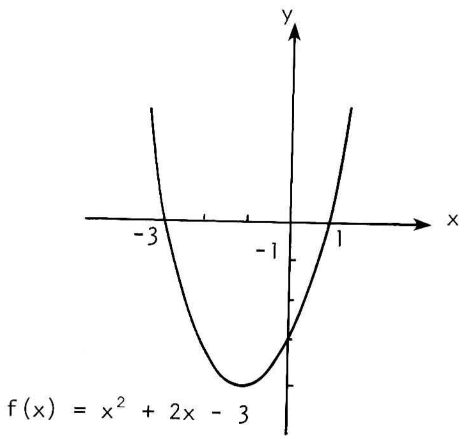
19. To find the domain we must determine the largest set of numbers for which $\sqrt{4 - x^2}\,/\,x^2$ is defined. First, we see that x cannot be zero, since division by zero is not defined. Also, since the square root of a negative number is not a real number, we must have $4-x^2 \geq 0$. This inequality is equivalent to $x^2 \leq 4$, that is, $-2 \leq x \leq 2$. Putting the two parts together we see that the domain is the set of all numbers in the interval $[-2,2]$ except 0. This set can also be expressed as the set of all numbers which are either in $[-2,0)$ or in $(0,2]$.

In determining the range, notice that both x^2 and $\sqrt{4 - x^2}$ are nonnegative. (Remember that "$\sqrt{\ }$" denotes the nonnegative square root.) Since $\sqrt{4 - x^2}\,/\,x^2$ can be any nonnegative number, the range of the function is $[0,\infty)$.

$$f(\sqrt{2}) = \sqrt{4 - (\sqrt{2})^2}\,/\,(\sqrt{2})^2 = \sqrt{4-2}\,/2 = \sqrt{2}/2$$

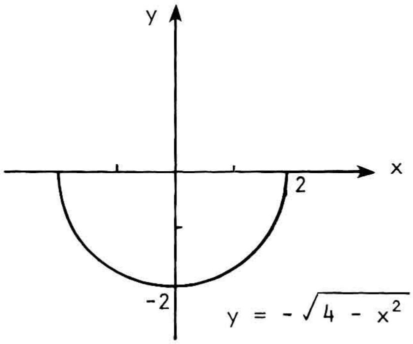
25. For the function f defined by $f(x) = x^2 + 2x - 3$, the domain is the set of all real numbers. To find the range we will need to know the y-coordinate of the vertex of the parabola, since that value is the smallest number in the range. Now, the x-coordinate of the vertex is $-b/2a = -2/2(1) = -1$, and therefore the y-coordinate is $f(-1) = (-1)^2 + 2(-1) - 3 = -4$. We see that -4 is the smallest possible y value. Thus the range of f is $[-4,\infty)$. Here is the graph:

x	-3	-2	-1	0	1
f(x)	0	-3	-4	-3	0



33. For $y = -\sqrt{4 - x^2}$, the domain is the set of all x for which $4 - x^2 \geq 0$; that is, the set of all x in the interval $[-2,2]$ (see the solution to Exercise 19 above). To determine the range, observe that $-\sqrt{4 - x^2}$ is smallest when $4 - x^2$ is largest. This occurs when $x = 0$, which gives $y = -2$. The largest value of $-\sqrt{4 - x^2}$ is zero. We conclude that the range is $[-2,0]$.

To identify the graph of $y = -\sqrt{4 - x^2}$, square both sides to obtain $y^2 = 4 - x^2$. This gives $x^2 + y^2 = 4$, which we recognize as the equation of a circle with center (0,0) and radius 2. The graph of $y = -\sqrt{4 - x^2}$ is the lower half of this circle.



SECTION 1-4 APPLICATIONS OF FUNCTIONS

7. If $f(x) = 2 + x - 4x^2$,

$$\begin{aligned} f(x + \Delta x) - f(x) &= 2 + (x + \Delta x) - 4(x + \Delta x)^2 - (2 + x - 4x^2) \\ &= 2 + x + \Delta x - 4(x^2 + 2x\Delta x + (\Delta x)^2) - 2 - x + 4x^2 \\ &= \Delta x - 4x^2 - 8x\Delta x - 4(\Delta x)^2 + 4x^2 \\ &= \Delta x - 8x\Delta x - 4(\Delta x)^2 \end{aligned}$$

9. If $f(x) = x^3$,

$$\begin{aligned} f(x + \Delta x) - f(x) &= (x + \Delta x)^3 - x^3 \\ &= x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3 = 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 \end{aligned}$$

29. The volume when $r=3$ is $\frac{4}{3}\pi(3)^3 = 36\pi$.

$$\Delta V = \frac{4}{3}\pi(3+\Delta r)^3 - \frac{4}{3}\pi 3^3$$
$$= \frac{4}{3}\pi\left[(3+\Delta r)^3 - 3^3\right]$$
$$= \frac{4}{3}\pi\left[3^3 + 3\cdot 3^2\Delta r + 3\cdot 3(\Delta r)^2 + (\Delta r)^3 - 3^3\right]$$
$$= \frac{4\pi\Delta r}{3}\left[27 + 9\Delta r + (\Delta r)^2\right]$$

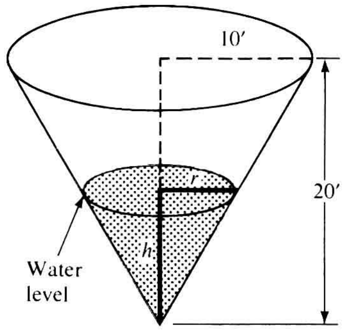
33. The area of a triangle is given by $A = \frac{1}{2}bh$, where b is the base and h is the height. In this problem $h = 3b$, so $A = \frac{1}{2}b(3b) = \frac{3}{2}b^2$. The domain of this function is $[0,\infty)$, since the base cannot be negative.

41. By similar triangles, $\frac{r}{h} = \frac{10}{20}$,

so, $r = \frac{1}{2}h$.

Therefore, $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$
$$= \frac{1}{12}\pi h^3$$



43. If h is as shown, the area of the window is

= (area of semicircle)+(area of rectangle)

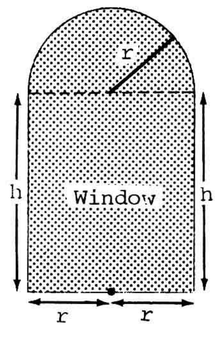
$= \frac{1}{2}\pi r^2 + 2rh$

But we are given that the perimeter is 12 feet.
Therefore,

$12 =$ (circumference of the semicircle) + h + 2r + h

$= \frac{1}{2}(2\pi r) + 2r + 2h$

$2h = 12 - \pi r - 2r$



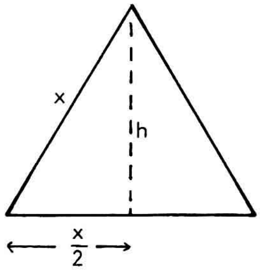
Substituting in the expression for the area:

$$A = \frac{1}{2}\pi r^2 + 2hr = \frac{1}{2}\pi r^2 + (12 - \pi r - 2r)r$$
$$= 12r - \frac{1}{2}\pi r^2 - 2r^2$$
$$= 12r - \left(\frac{\pi}{2} + 2\right) r^2$$

45. If x is the length of a side and h is the height, the area of the triangle is $\frac{1}{2}$(base)(height) $= \frac{xh}{2}$.

By the Pythagorean theorem,

$$\frac{x^2}{4} + h^2 = x^2, \text{ so } h^2 = \frac{3x^2}{4}, \text{ or } h = \frac{\sqrt{3}x}{2}$$



Therefore, we have

$$A = \frac{xh}{2} = \frac{x}{2}\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}x^2}{4}$$

The domain of this function is $(0,\infty)$

SECTION 1-5 LINES

17. The slope of the line through $(-1,2)$ and $(4,1)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{1-2}{4-(-1)} = \frac{-1}{5}$$

Using the point $(-1,2)$ in the point-slope formula gives the equation

$$y - 2 = \frac{-1}{5}(x - (-1))$$

$$y = \frac{-1}{5}x + \frac{9}{5}$$

We could have used the point $(4,1)$ in the point-slope formula instead.

22. The slope of $x - 2y + 1 = 0$ can be found by solving for y:

$$2y = x + 1 \quad \text{or} \quad y = \frac{1}{2}x + \frac{1}{2}.$$

We see that the slope of $x - 2y + 1 = 0$ is $1/2$, and so the slope of the desired line is -2. Using the given point $(6,1)$ in the point-slope formula gives

$$y - 1 = -2(x - 6) \quad \text{or} \quad y = -2x + 13$$

27. $\Delta x = x_2 - x_1 = -.9 - (-1) = .1.$

$$\frac{\Delta f}{\Delta x} = \frac{f(-.9) - f(-1)}{.1} = \frac{[(-.9)^3 + 1] - [(-1)^3 + 1]}{.1}$$

$$= \frac{-.729 + 1}{.1} = \frac{.271}{.1} = 2.71$$

35. $\frac{\Delta f}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^2 - 2(x + \Delta x) + 3 - (x^2 - 2x + 3)}{\Delta x}$

$= \frac{\not{x}^2 + 2x\Delta x + (\Delta x)^2 - 2\not{x} - 2\Delta x + \not{3} - \not{x}^2 + 2\not{x} - \not{3}}{\Delta x}$

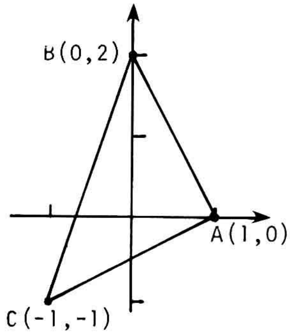
$= \frac{\Delta x\ (2x + \Delta x - 2)}{\Delta x} = 2x + \Delta x - 2$

41. $m_{BA} = \frac{0 - 2}{1 - 0} = -2$

$m_{CA} = \frac{0-(-1)}{1-(-1)} = \frac{1}{2}$

Since $m_{CA} = -1/m_{BA}$,

side CA is perpendicular to side BA, and therefore, the triangle is a right triangle.



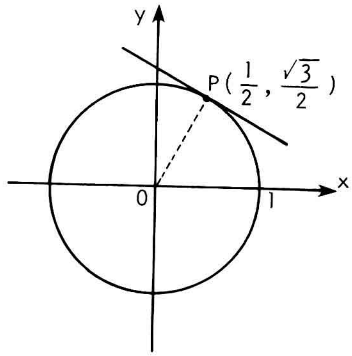
46. The tangent line at P is perpendicular to the line OP. Now, the slope of OP is

$\frac{\sqrt{3}}{2} / \frac{1}{2} = \sqrt{3}$, and thus the slope of the tangent line is

$\frac{-1}{\sqrt{3}} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$. The equation

of the tangent line is $y - \frac{\sqrt{3}}{2} = \frac{-\sqrt{3}}{3}(x - \frac{1}{2})$ or

$y = \frac{-\sqrt{3}}{3}x + \frac{2\sqrt{3}}{3}$.



48. The problem is to find the equation of the line through (0,32) and (100,212). This line has slope (212 - 32)/(100 - 0) = 180/100 = 9/5. By the slope-intercept form, the equation is $F = \frac{9}{5}C + 32$.

If F = 98.6, we need to solve

$$98.6 = \frac{9}{5}C + 32$$

$$66.6 = \frac{9}{5}C$$

$$C = \frac{5}{9}(66.6) = 5(7.4) = 37.$$

50. First, using the formula from exercise 49 with $t=0$ and $\Delta t=3$, we see that the average speed during the first three seconds is

$$\frac{\Delta s}{\Delta t} = \frac{s(0+3) - s(0)}{3} = \frac{-16(3^2) + 192(3) - 0}{3}$$

$$= -16(3) + 192 = 144 \text{ ft/sec.}$$

The average speed during the next three seconds is found by setting $t=3$ and $\Delta t=3$. We have

$$\frac{\Delta s}{\Delta t} = \frac{s(3+3) - s(3)}{3} = \frac{-16(6^2) + 192(6) - [-16(3^2) + 192(3)]}{3}$$

$$= 48 \text{ ft/sec.}$$

SECTION 1-6 EXPONENTS AND LOGARITHMS

3. $$32^{-2/5} = \frac{1}{32^{2/5}} = \frac{1}{(\sqrt[5]{32})^2} = \frac{1}{2^2} = \frac{1}{4}$$

11. Let $\log_{\frac{1}{3}} 81 = x$. Then $(1/3)^x = 81 = 3^4$, and therefore, $3^{-x} = 3^4$.

It follows that $x = -4$, so $\log_{\frac{1}{3}} 81 = -4$.

20. Substituting $y = 9 \cdot 3^h$ in the equation $y = 3^x$ gives $9 \cdot 3^h = 3^x$. But $9 \cdot 3^h = 3^2 \cdot 3^h = 3^{2+h}$. Thus, $3^{2+h} = 3^x$, and we have $x=2+h$.

25. Letting $y = 1/2$ in the equation $y = \log_4 x$ yields $1/2 = \log_4 x$, which is equivalent to $x = 4^{1/2}$ by the definition of logarithm. Therefore, $x=2$.

33. If the graph of $y = \log_a x$ contains the point $(81,-2)$, we must have $-2 = \log_a 81$ or $a^{-2} = 81$. We can write $81 = 9^2 = (1/9)^{-2}$, so $a=1/9$.

43. The amount, Q, present after t years is given by $Q = 10(1/2)^{t/2}$. (To check this, notice that when $t=2$, $Q = 10(1/2)$, half the original amount.)

After 8 years, the amount present will be $10(1/2)^{8/2} = 10(1/2^4)$
$= 10/16 = 5/8$ gr.

To compute the average rate of change as t changes from 2 to 4, we use $\Delta t = 4-2 = 2$. Then