PERSPECTIVES IN COMPUTING

W. RHEINBOLDT, D. SIEWIOREK, EDITORS

Discrete Algorithms and Complexity

Proceedings of the Japan-US Joint Seminar
June 4–6, 1986, Kyoto, Japan

Edited by
David S. Johnson, Takao Nishizeki,
Akihiro Nozaki, Herbert S. Wilf

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Discrete Algorithms and Complexity

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Foreword

The Japan-US Joint Seminar on Discrete Algorithms and Complexity Theory was held 4 – 6 June, 1986, in Kyoto, Japan. It was jointly organized by A. Nozaki and H.S. Wilf and generously sponsored by the Japan Society for Promotion of Sciences and the U.S. National Science Foundation. Thirty-three of the participants were invited to give talks. This volume contains most of the papers presented there.

Some papers describe the state of the art in specific fields, some explore new problems, and others present new results that will soon appear in more detailed form in scientific journals. We expect that the reader will come away from this volume with a better understanding of and insight into Discrete Algorithms and Complexity.

The editors wish to thank all delegates; their efforts made the seminar stimulating and fruitful. Especially we would like to express our gratitude to the executive committee of the Seminar: Takao Asano, Tetsuo Asano, Kazuo Iwama, Shigeki Iwata, Takumi Kasai and Hajime Machida. Finally, we would like to thank the staff of Academic Press Boston for their cooperation in producing this volume.

David S. Johnson
Takao Nishizeki
Akihiro Nozaki
Herbert S. Wilf

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* Speakers

An Upper Bound on the Expected Cost of an Optimal Assignment

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Introduction

An instance of the $n \times n$ assignment problem (AP) is specified by a $n \times n$ matrix (c_{ij}) of real numbers. The problem is to find a permutation that minimizes $A^* = \sum_{i=1}^{n} c_{i,\sigma(i)}$.

When n is fixed and the c_{ij} are drawn independently from the uniform distribution over $[0,1]$, A^* becomes a random variable. Computational experiments indicate that, when $n > 100$, $E[A^*]$ is close to 1.6. Lazarus [1979] shows that

$$E[A^*] \geq 1 + \frac{1}{e} + O\left(\frac{1}{n}\right) \approx 1.37,$$

and Walkup [1979] shows that $E[A^*] < 3$ for all n. Our main result is

Theorem 1. For all n, $E[A^*] < 2$.

A Regularity Condition

Call the matrix (c_{ij}) <u>regular</u> if no two distinct subsets of its elements have the same sum. This implies in particular that the optimal assignment σ is unique. Under the stated assumptions about the probability distribution of the c_{ij} the matrix (c_{ij}) is regular with probability 1. Throughout the paper we restrict attention to regular instances of the AP.

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The Transportation Problem and its Dual

The proof of Theorem 1 is based on well-known properties of the following dual pair of linear programming problems ([1]).

PRIMAL	**DUAL**
$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$	$\max \sum_{i=1}^{m} a_i u_i + \sum_{j=1}^{n} b_j v_j$
subject to	subject to
$x_{ij} \geq 0$	$c_{ij} - u_i - v_j \geq 0$
$1,2,\ldots,m$	$i = 1,2,\ldots,m$
$j = 1,2,\ldots,n$	$j = 1,2,\ldots,n.$
$\sum_j x_{ij} = a_i \quad i = 1,2,\ldots,m$	
$\sum_i x_{ij} = b_j \quad j = 1,2,\ldots,n$	

Here the a_i and b_j are nonnegative real numbers satisfying $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. The *AP* is the special case in which $m = n$ and all a_i and b_j are equal to 1. In this special case the primal and dual have A^* as their common optimal value. Also, every basic feasible solution to the primal has all x_{ij} equal to 0 or 1; for every such basic feasible solution there is a permutation σ of $\{1,2,\ldots,n\}$ such that $x_{ij} = 1$ and only if $j = \sigma(i)$.

In general, the basic solutions of the primal and dual can be characterized in graph-theoretic terms. Let G be the complete bipartite graph with vertex set $\{s_1,s_2,\ldots,s_m\} \cup \{t_1,t_2,\ldots,t_n\}$ and edge set $\{\{s_i, t_j\}, i = 1,2,\ldots,m, j = 1,2,\ldots,n\}$. Let T be the edge set of a spanning tree of G. There is a unique solution of the primal satisfying: if $\{s_i, t_j\} \notin T$ then $x_{ij} = 0$. There is a unique solution of the dual satisfying: $u_1 = 0$ and if $\{s_i, t_j\} \in T$ then $c_{ij} - u_i - v_j = 0$. The solutions obtained in this way from spanning trees of G are the basic solutions of the primal and dual. Call T *feasible* if the associated basic solution of the primal satisfies $x_{ij} \geq 0$ for all i and j; call T *dual feasible* if the associated basic solution of the dual satisfies $c_{ij} - u_i - v_j \geq 0$ for all i and j. If T is both feasible and dual feasible we say that T is *optimal;* in this case the basic solutions associated with T are optimal for the primal and dual respectively.

In the special case of the *AP* there are in general many optimal spanning trees of G. If the *AP* satisfies our regularity hypothesis there is a unique optimal tree which remains feasible when the a_i and b_j are perturbed by setting

$a_i = 1 + n\varepsilon \quad i = 1, \quad i = 2,\ldots,n$

and $b_j = 1 + \varepsilon$, $j = 1,2,\ldots,n$. Let us call this unique tree the canonical optimal tree.

Proof of Theorem 1: Each regular instance (c_{ij}) of the *AP* determines a 3-tuple $< T, u, v >$. Here T is the canonical optimal tree and the n-vectors $u = (u_1, u_2, \ldots, u_m)$ and $v = (v_1, v_2, \ldots, v_n)$ are the associated optimal solution of the dual; call this 3-tuple the *optimality data* for (c_{ij}).

Now assume that the c_{ij} are drawn independently from the uniform distribution over [0,1]. Define the following two random variables is therefore $\frac{1}{2} + \frac{1}{2}$ $\max(0, u_i + v_j)$. Hence,

over the sample space of instances (c_{ij}):

$$X = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}$$

and $Y = < T, u, v >$, the optimality data for (c_{ij}). The proof of Theorem 1 will emerge from consideration of the identity

$$E[X] = E[E[X \mid Y]].$$

Clearly $E[X] = \frac{n^2}{2}$, since the expected value of each c_{ij} is $\frac{1}{2}$. Let us compute $E[X \mid Y]$, where $Y = < T, u, v >$ is fixed. The matrices from our sample space having $< T, u, v >$ as optimality data are precisely those satisfying

$$\begin{aligned} c_{ij} - u_i - v_j &= 0, \quad \{s_i, t_j\} \in T \\ c_{ij} - u_i - v_j &\geq 0 \quad \{s_i, t_j\} \in T \end{aligned}$$

The *a priori* distribution of each c_{ij} is uniform over [0,1]. The *a posteriori* distribution of c_{ij} given $< T, u, v >$ is uniform over $[\max(0, u_i + v_j), 1]$, and its conditional expectation given $< T, u, v >$

□

Dyer, Frieze and McDiarmid (1984) have recently used the proof

$$E[\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} | <T, u, v>]$$

$$= \sum_{\{s_i,t_j\}\in T} (u_i + v_j)$$

$$+ \sum_{\{s_i,t_j\}\notin T} (\frac{1}{2} + \frac{1}{2}\max(0, u_i + v_j)).$$

Noting that $u_i + v_j = c_{ij} \geq 0$ when $\{s_i, t_j\} \in T$, that $\max(0, u_i + v_j) \geq u_i + v_j$ and that $\sum_{i=1}^{n} u_i + \sum_{j=1}^{n} v_j = A^*$, we obtain

$$E[\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} | <T, u, v>] \geq$$

$$\frac{1}{2}\sum_{\{s_i,t_j\}\in T} (u_i + v_j) + \sum_{\{s_i,t_j\}\notin T} (\frac{1}{2} + \frac{1}{2}(u_i + v_j))$$

$$= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} (u_i + v_j) + \frac{n^2 - 2n + 1}{2}$$

$$= \frac{n}{2} A^* + \frac{n^2 - 2n + 1}{2}.$$

Applying $E[X] = E[E[X|Y]]$ we obtain

$$\frac{n^2}{2} \geq \frac{n}{2} E[A^*] + \frac{n^2 - 2n + 1}{2},$$

giving $E[A^*] \leq \frac{2n-1}{n} < 2$

technique introduced in this paper to obtain a broad generalization of Theorem 1.

Acknowledgement

Thanks to Colin McDiarmid for suggesting a major simplification in the proof.

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The Principal Partition of Vertex-Weighted Graphs and Its Applications

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Abstract The principal partition of vertex-weighted graphs is utilized to solve certain assignment problems or flow problems which are formulated using such graphs. The well-known labeling algorithm or labyrinth algorithm for augmenting flows is used to find the principal partition and to solve the first three problems. The fourth problem which is originated from a routing problem in three-dimensional integrated circuits, requires, in addition to the flow augmentation step, a step of finding an optimal flow assignment for a part of the graph.

1. Introduction.

The principal partition introduced by Kishi and Kajitani is a partition of a graph into three parts satisfying certain minimality

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