WILEY FINANCE

WITH DVD

Credit risk modeling using Excel and VBA

Second Edition

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WILEY
A John Wiley and Sons, Ltd., Publication

This edition first published 2011

Registered office
John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

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ISBN 978-0-470-66092-8

A catalogue record for this book is available from the British Library.

Typeset in 10/12pt Times by Aptara Inc., New Delhi, India
Printed in Great Britain by CPI Antony Rowe, Chippenham, Wiltshire

Preface to the 2nd Edition

It is common to blame the inadequacy of risk models for the fact that the 2007–2008 financial crisis caught many market participants by surprise. On closer inspection, though, it often appears that it was not the models that failed. A good example is the risk contained in structured finance securities such as collateralized debt obligations (CDOs). In the first edition of this book, which was published before the crisis, we already pointed out that the rating of such products is not meant to communicate their systematic risk even though this risk component can be extremely large. This is easy to illustrate with simple, standard credit risk models, and surely we were not the first to point this out. Hence, in terms of risk, an AAA-rated bond is definitely not the same as an AAA-rated CDO. Many institutions, however, appear to have built their investment strategy on the presumption that AAA is AAA regardless of the product.

Recent events therefore do not invalidate traditional credit risk modeling as described in the first edition of the book. A second edition is timely, however, because the first edition dealt relatively briefly with the pricing of instruments that featured prominently in the crisis (CDSs and CDOs). In addition to expanding the coverage of these instruments, we devote more time to modeling aspects that were of particular relevance in the financial crisis (e.g., estimation error). We also examine the usefulness and limitations of credit risk modeling through case studies. For example, we discuss the role of scoring models in the subprime market, or show that a structural default prediction model would have assigned relatively high default probabilities to Lehman Brothers in the months before its collapse. Furthermore, we added a new chapter in which we show how to predict borrower-specific loss given default.

For university teachers, we now offer a set of powerpoint slides as well as problem sets with solutions. The material can be accessed via our homepage `www.loeffler-posch.com`. The hybrid character of the book – introduction to credit risk modeling as well as cookbook for model implementation – makes it a good companion to a credit risk course, at both introductory or advanced levels.

We are very grateful to Roger Bowden, Michael Kunisch and Alina Maurer for their comments on new parts of the book. One of us (Peter) benefited from discussions with a lot of people in the credit market, among them Nick Atkinson, David Kupfer and Marion Schlicker. Georg Haas taught him everything a trader needs to know, and Josef Gruber provided him with valuable insights to the practice of risk management. Several readers of the first edition pointed out errors or potential for improvement. We would like to use this opportunity to

thank them again and to encourage readers of the second edition to send us their comments (email: comment@loeffler-posch.com). Finally, special thanks to our team at Wiley: Andrew Finch, Brian Burge and our editors Aimee Dibbens and Karen Weller.

At the time of writing it is June. The weather is fine. We are looking forward to devoting more time to our families again.

Preface to the 1st Edition

This book is an introduction to modern credit risk methodology as well as a cookbook for putting credit risk models to work. We hope that the two purposes go together well. From our own experience, analytical methods are best understood by implementing them.

Credit risk literature broadly falls into two separate camps: risk measurement and pricing. We belong to the risk measurement camp. Chapters on default probability estimation and credit portfolio risk dominate chapters on pricing and credit derivatives. Our coverage of risk measurement issues is also somewhat selective. We thought it better to be selective than to include more topics with less detail, hoping that the presented material serves as a good preparation for tackling other problems not covered in the book.

We have chosen Excel as our primary tool because it is a universal and very flexible tool that offers elegant solutions to many problems. Even Excel freaks may admit that it is not their first choice for some problems. But even then, it is nonetheless great for demonstrating how to put models to work, given that implementation strategies are mostly transferable to other programming environments. While we tried to provide efficient and general solutions, this was not our single overriding goal. With the dual purpose of our book in mind, we sometimes favored a solution that appeared more simple to grasp.

Readers surely benefit from some prior Excel literacy, e.g., knowing how to use a simple function such as AVERAGE(), being aware of the difference between SUM(A1:A10) SUM($A1:$A10) and so forth. For less experienced readers, there is an *Excel for beginners* video on the DVD, and an introduction to VBA in the Appendix; the other videos supplied on the DVD should also be very useful as they provide a step-by-step guide more detailed than the explanations in the main text.

We also assume that the reader is somehow familiar with concepts from elementary statistics (e.g., probability distributions) and financial economics (e.g., discounting, options). Nevertheless, we explain basic concepts when we think that at least some readers might benefit from it. For example, we include appendices on maximum likelihood estimation or regressions.

We are very grateful to colleagues, friends and students who gave feedback on the manuscript: Oliver Blümke, Jürgen Bohrmann, André Güttler, Florian Kramer, Michael Kunisch, Clemens Prestele, Peter Raupach, Daniel Smith (who also did the narration of the videos with great dedication) and Thomas Verchow. An anonymous reviewer also provided a lot of helpful comments. We thank Eva Nacca for formatting work and typing video text. Finally, we thank our editors Caitlin Cornish, Emily Pears and Vivienne Wickham.

Any errors and unintentional deviations from best practice remain our own responsibility. We welcome your comments and suggestions: just send an email to `comment@loeffler-posch.com` or visit our homepage at `www.loeffler-posch.com`.

We owe a lot to our families. Before struggling to find the right words to express our gratitude we rather stop and give our families what they missed most, our time.

Some Hints for Troubleshooting

We hope that you do not encounter problems when working with the spreadsheets, macros and functions developed in this book. If you do, you may want to consider the following possible reasons for trouble:

- We repeatedly use the Excel Solver. This may cause problems if the Solver Add-in is not activated in Excel and VBA. How this can be done is described in Appendix A2. Apparently, differences in Excel versions can also lead to situations in which a macro calling the Solver does not run even though the reference to the Solver is set.
- In Chapters 10 and 11, we use functions from the *AnalysisToolpak* Add-in. Again, this has to be activated. See Chapter 10 for details.
- Some Excel 2003 functions (e.g., BINOMDIST or CRITBINOM) have been changed relative to earlier Excel versions. We've tested our programs on Excel 2003 and Excel 2010. If you're using an older Excel version, these functions might return error values in some cases.
- All functions have been tested for the demonstrated purpose only. We have not strived to make them so general that they work for most purposes one can think of. For example:

  - some functions assume that the data is sorted in some way, or arranged in columns rather than in rows;
  - some functions assume that the argument is a range, not an array. See Appendix A1 for detailed instructions on troubleshooting this issue.

A comprehensive list of all functions (Excel's and user-defined) together with full syntax and a short description can be found in Appendix A5.

Contents

1

Estimating Credit Scores with Logit

Typically, several factors can affect a borrower's default probability. In the retail segment, one would consider salary, occupation and other characteristics of the loan applicant; when dealing with corporate clients, one would examine the firm's leverage, profitability or cash flows, to name but a few items. A scoring model specifies how to combine the different pieces of information in order to get an accurate assessment of default probability, thus serving to automate and standardize the evaluation of default risk within a financial institution.

In this chapter, we show how to specify a scoring model using a statistical technique called *logistic regression* or simply *logit*. Essentially, this amounts to coding information into a specific value (e.g., measuring leverage as debt/assets) and then finding the combination of factors that does the best job in explaining historical default behavior.

After clarifying the link between scores and default probability, we show how to estimate and interpret a logit model. We then discuss important issues that arise in practical applications, namely the treatment of outliers and the choice of functional relationship between variables and default.

An important step in building and running a successful scoring model is its validation. Since validation techniques are applied not just to scoring models but also to agency ratings and other measures of default risk, they are described separately in Chapter 8.

LINKING SCORES, DEFAULT PROBABILITIES AND OBSERVED DEFAULT BEHAVIOR

A score summarizes the information contained in factors that affect default probability. Standard scoring models take the most straightforward approach by linearly combining those factors. Let x denote the factors (their number is K) and b the weights (or coefficients) attached to them; we can represent the score that we get in scoring instance i as

$$\text{Score}_i = b_1 x_{i1} + b_2 x_{i2} + \ldots + b_K x_{iK} \tag{1.1}$$

It is convenient to have a shortcut for this expression. Collecting the bs and the xs in column vectors $\mathbf{b}$ and $\mathbf{x}$ we can rewrite (1.1) to

$$\text{Score}_i = b_1 x_{i1} + b_2 x_{i2} + \ldots + b_K x_{iK} = \mathbf{b}'\mathbf{x}_i, \quad \mathbf{x}_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \cdots \\ x_{iK} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_K \end{bmatrix} \tag{1.2}$$

If the model is to include a constant b_1, we set $x_{i1} = 1$ for each i.

Assume for simplicity that we have already agreed on the choice of the factors $\mathbf{x}$ – what is then left to determine is the weight vector $\mathbf{b}$. Usually, it is estimated based on observed default

Table 1.1 Factor values and default behavior

			Default indicator for year +1	Factor values from the end of year			
Scoring instance i	Firm	Year	y_i	x_{i1}	x_{i2}		x_{iK}
1	XAX	2001	0	0.12	0.35	...	0.14
2	YOX	2001	0	0.15	0.51	...	0.04
3	TUR	2001	0	−0.10	0.63	...	0.06
4	BOK	2001	1	0.16	0.21	...	0.12
...	...	...	...	...	...	...	...
912	XAX	2002	0	−0.01	0.02	...	0.09
913	YOX	2002	0	0.15	0.54	...	0.08
914	TUR	2002	1	0.08	0.64	...	0.04
...	...	...	...	...	...	...	...
N	VRA	2005	0	0.04	0.76	...	0.03

behavior.[1] Imagine that we have collected annual data on firms with factor values and default behavior. We show such a data set in Table 1.1.[2]

Note that the same firm can show up more than once if there is information on this firm for several years. Upon defaulting, firms often stay in default for several years; in such cases, we would not use the observations following the year in which default occurred. If a firm moves out of default, we would again include it in the data set.

The default information is stored in the variable y_i. It takes the value 1 if the firm defaulted in the year following the one for which we have collected the factor values, and zero otherwise. N denotes the overall number of observations.

The scoring model should predict a high default probability for those observations that defaulted and a low default probability for those that did not. In order to choose the appropriate weights **b**, we first need to link scores to default probabilities. This can be done by representing default probabilities as a function F of scores:

$$\text{Prob}(\text{Default}_i) = \text{Prob}(y_i = 1) = F(\text{Score}_i) \tag{1.3}$$

Like default probabilities, the function F should be constrained to the interval from zero to one; it should also yield a default probability for each possible score. The requirements can be fulfilled by a cumulative probability distribution function, and a distribution often considered for this purpose is the logistic distribution. The logistic distribution function $\Lambda(z)$ is defined as $\Lambda(z) = \exp(z)/(1 + \exp(z))$. Applied to (1.3) we get

$$\text{Prob}(\text{Default}_i) = \Lambda(\text{Score}_i) = \frac{\exp(\mathbf{b}'\mathbf{x}_i)}{1 + \exp(\mathbf{b}'\mathbf{x}_i)} = \frac{1}{1 + \exp(-\mathbf{b}'\mathbf{x}_i)} \tag{1.4}$$

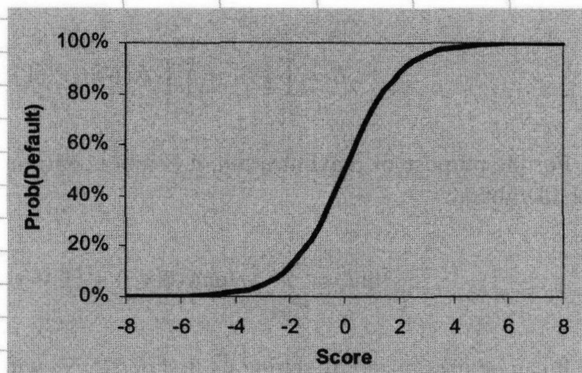
Models that link information to probabilities using the logistic distribution function are called *logit* models.

[1] In qualitative scoring models, however, experts determine the weights.

[2] Data used for scoring are usually on an annual basis, but one can also choose other frequencies for data collection as well as other horizons for the default horizon.

Table 1.2 Scores and default probabilities in the logit model

	A	B	C	D	E	F	G	H
1	**Score**	**Prob(Default)**						
2	-8	0.03%	=1/(1+EXP(-A2))					
3	-7	0.09%	*(can be copied into B3:B18)*					
4	-6	0.25%						
5	-5	0.67%						
6	-4	1.80%						
7	-3	4.74%						
8	-2	11.92%						
9	-1	26.89%						
10	0	50.00%						
11	1	73.11%						
12	2	88.08%						
13	3	95.26%						
14	4	98.20%						
15	5	99.33%						
16	6	99.75%						
17	7	99.91%						
18	8	99.97%						



In Table 1.2, we list the default probabilities associated with some score values and illustrate the relationship with a graph. As can be seen, higher scores correspond to a higher default probability. In many financial institutions, credit scores have the opposite property: they are higher for borrowers with a lower credit risk. In addition, they are often constrained to some set interval, e.g., zero to 100. Preferences for such characteristics can easily be met. If we use (1.4) to define a scoring system with scores from -9 to 1, but want to work with scores from 0 to 100 instead (100 being the best), we could transform the original score to $myscore = -10 \times score + 10$.

Having collected the factors $\mathbf{x}$ and chosen the distribution function F, a natural way of estimating the weights $\mathbf{b}$ is the maximum likelihood (ML) method. According to the ML principle, the weights are chosen such that the probability (=likelihood) of observing the given default behavior is maximized (see Appendix A3 for further details on ML estimation).

The first step in maximum likelihood estimation is to set up the likelihood function. For a borrower that defaulted, the likelihood of observing this is

$$\text{Prob}(\text{Default}_i) = \text{Prob}(y_i = 1) = \Lambda(\mathbf{b}'\mathbf{x}_i) \tag{1.5}$$

For a borrower that did not default, we get the likelihood

$$\text{Prob}(\text{No default}_i) = \text{Prob}(y_i = 0) = 1 - \Lambda(\mathbf{b}'\mathbf{x}_i) \tag{1.6}$$

Using a little trick, we can combine the two formulae into one that automatically gives the correct likelihood, be it a defaulter or not. Since any number raised to the power of zero evaluates to one, the likelihood for observation i can be written as

$$L_i = (\Lambda(\mathbf{b}'\mathbf{x}_i))^{y_i}(1 - \Lambda(\mathbf{b}'\mathbf{x}_i))^{1-y_i} \tag{1.7}$$

Assuming that defaults are independent, the likelihood of a set of observations is just the product of the individual likelihoods:[3]

$$L = \prod_{i=1}^{N} L_i = \prod_{i=1}^{N} (\Lambda(\mathbf{b}'\mathbf{x}_i))^{y_i}(1 - \Lambda(\mathbf{b}'\mathbf{x}_i))^{1-y_i} \tag{1.8}$$

For the purpose of maximization, it is more convenient to examine ln L, the logarithm of the likelihood:

$$\ln L = \sum_{i=1}^{N} y_i \ln(\Lambda(\mathbf{b}'\mathbf{x}_i)) + (1 - y_i)\ln(1 - \Lambda(\mathbf{b}'\mathbf{x}_i)) \tag{1.9}$$

It can be maximized by setting its first derivative with respect to **b** to zero. This derivative (like **b**, it is a vector) is given by

$$\frac{\partial \ln L}{\partial \mathbf{b}} = \sum_{i=1}^{N} (y_i - \Lambda(\mathbf{b}'\mathbf{x}_i))\mathbf{x}_i \tag{1.10}$$

Newton's method (see Appendix A3) does a very good job in solving equation (1.10) with respect to **b**. To apply this method, we also need the second derivative, which we obtain as

$$\frac{\partial^2 \ln L}{\partial \mathbf{b} \partial \mathbf{b}'} = -\sum_{i=1}^{N} \Lambda(\mathbf{b}'\mathbf{x}_i)(1 - \Lambda(\mathbf{b}'\mathbf{x}_i))\mathbf{x}_i\mathbf{x}_i' \tag{1.11}$$

ESTIMATING LOGIT COEFFICIENTS IN EXCEL

Excel does not contain a function for estimating logit models, and so we sketch how to construct a user-defined function that performs the task. Our complete function is called LOGIT. The syntax of the LOGIT command is equivalent to the LINEST command: LOGIT(*y*,*x*,[*const*],[*statistics*]), where [] denotes an optional argument.

The first argument specifies the range of the dependent variable, which in our case is the default indicator y; the second parameter specifies the range of the explanatory variable(s). The third and fourth parameters are logical values for the inclusion of a constant (1 or omitted if a constant is included, 0 otherwise) and the calculation of regression statistics (1 if statistics are to be computed, 0 or omitted otherwise). The function returns an array, therefore, it has to be executed on a range of cells and entered by [ctrl]+[shift]+[enter].

[3] Given that there are years in which default rates are high, and others in which they are low, one may wonder whether the independence assumption is appropriate. It will be if the factors that we input into the score capture fluctuations in average default risk. In many applications, this is a reasonable assumption.

Table 1.3 Application of the LOGIT command to a data set with information on defaults and five financial ratios

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Firm ID	Year	Default	WC/ TA	RE/ TA	EBIT/ TA	ME/ TL	S/ TA		CONST	WC/ TA	RE/ TA	EBIT/ TA	ME/ TL	S/ TA
2	1	1999	0	0.50	0.31	0.04	0.96	0.33	b	-2.543	0.414	-1.454	-7.999	-1.594	0.620
3	1	2000	0	0.55	0.32	0.05	1.06	0.33		{=LOGIT(C2:C4001,D2:H4001,1,0)}					
4	1	2001	0	0.45	0.23	0.03	0.80	0.25		*(applies to J2:O2)*					
5	1	2002	0	0.31	0.19	0.03	0.39	0.25							
6	1	2003	0	0.45	0.22	0.03	0.79	0.28							
7	1	2004	0	0.46	0.22	0.03	1.29	0.32							
8	2	1999	0	0.01	-0.03	0.01	0.11	0.25							
9	2	2000	0	-0.11	-0.12	0.03	0.15	0.32							
...															
108	21	1996	1	0.36	0.06	0.03	3.20	0.28							
...															
4001	830	2002	1	0.07	-0.11	0.04	0.04	0.12							

Before delving into the code, let us look at how the function works on an example data set.[4] We have collected default information and five variables for default prediction; Working Capital (WC), Retained Earnings (RE), Earnings Before Interest and Taxes (EBIT) and Sales (S), each divided by Total Assets (TA); and Market Value of Equity (ME) divided by Total Liabilities (TL). Except for the market value, all these items are found in the balance sheet and income statement of the company. The market value is given by the number of shares outstanding multiplied by the stock price. The five ratios are the ones from the widely known *Z*-score developed by Altman (1968). WC/TA captures the short-term liquidity of a firm, RE/TA and EBIT/TA measure historic and current profitability, respectively. S/TA further proxies for the competitive situation of the company and ME/TL is a market-based measure of leverage.

Of course, one could consider other variables as well; to mention only a few, these could be: cash flows over debt service, sales or total assets (as a proxy for size), earnings volatility, stock price volatility. In addition, there are often several ways of capturing one underlying factor. Current profits, for instance, can be measured using EBIT, EBITDA (=EBIT plus depreciation and amortization) or net income.

In Table 1.3, the data is assembled in columns A to H. Firm ID and year are not required for estimation. The LOGIT function is applied to range J2:O2. The default variable that the LOGIT function uses is in the range C2:C4001, while the factors x are in the range D2:H4001. Note that (unlike in Excel's LINEST function) coefficients are returned in the same order as the variables are entered; the constant (if included) appears as the leftmost variable. To interpret the sign of the coefficient b, recall that a higher score corresponds to a higher default probability. The negative sign of the coefficient for EBIT/TA, for example, means that default probability goes down as profitability increases.

Now let us have a close look at important parts of the LOGIT code. In the first lines of the function, we analyze the input data to define the data dimensions: the total number of observations `N` and the number of explanatory variables (including the constant) `K`. If a

[4] The data is hypothetical, but mirrors the structure of data for listed US corporates.