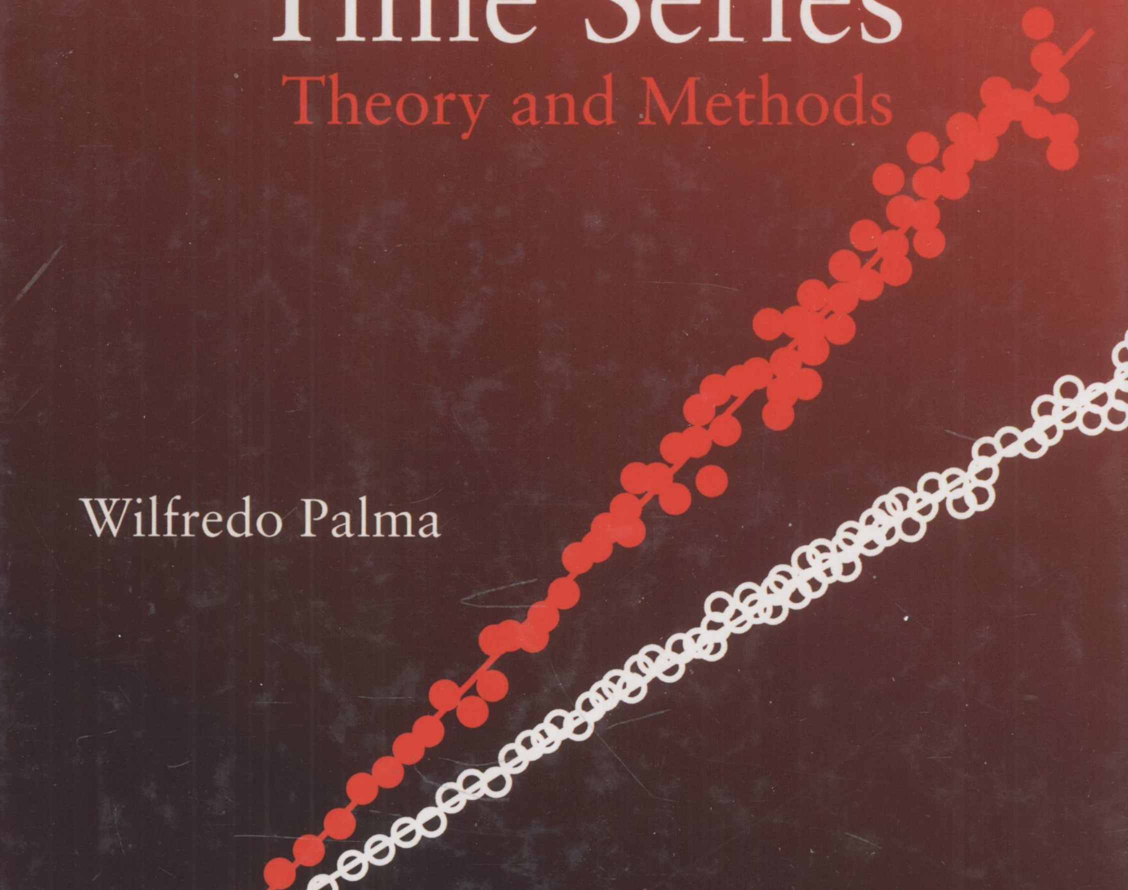
Long-Memory Time Series

Theory and Methods

Wilfredo Palma



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Wilfredo Palma

Pontificia Universidad Católica de Chile

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LONG-MEMORY TIME SERIES

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PREFACE

During the last decades long-memory processes have evolved into a vital and important part of the time series analysis. Long-range-dependent processes are characterized by slowly decaying autocorrelations or by a spectral density exhibiting a pole at the origin. These features change dramatically the statistical behavior of estimates and predictions. As a consequence, many of the theoretical results and methodologies used for analyzing short-memory time series, for instance, ARMA processes, are no longer appropriate for long-memory models.

This book aims to provide an overview of the theory and methods developed to deal with long-range-dependent data as well as describe some applications of these methodologies to real-life time series. It is intended to be a text for a graduate course and to be helpful to researchers and practitioners. However, it does not attempt to cover all of the relevant topics in this field.

Some basic knowledge of calculus and linear algebra including derivatives, integrals and matrices is required for understanding most results in this book. Apart from this, the text intends to be self-contained in terms of other more advanced concepts. In fact, Chapter 1 of this book offers a brief discussion of fundamental mathematical and probabilistic concepts such as Hilbert spaces, orthogonal projections, stationarity, and ergodicity, among others. Definitions and basic properties are presented and further readings are suggested in a bibliographic notes section. This chapter ends with a number of proposed exercises. Furthermore, Chapter 2 describes some fundamental concepts on state space systems and Kalman filter equations. As discussed in this chapter, state space systems offer an alternative representation of time series models which may be very useful for calculating estimates and predictors, especially in the presence of data gaps. In particular, we discuss applications of state space techniques to parameter estimation in Chapter 4, to missing values in Chapter 11, and to seasonal models in Chapter 12.

Even though it seems to be a general agreement that in order to have long memory a time series must exhibit slowly decaying autocorrelations, the formal definition of a long-range-dependent process is not necessarily unique. This issue is discussed in Chapter 3 where several mathematical definitions of long memory are reviewed. Chapter 4 is devoted to the analysis of a number of widely used parameter estimation methods for strongly-dependent time series models. The methodologies are succinctly presented and some asymptotic results are discussed. Since a critical problem with many of these estimation methods is their computational implementation, some specific aspects such as algorithm efficiency and numerical complexity are also analyzed. A number of simulations and practical applications complete this chapter. The statistical analysis of the large-sample properties of the parameter estimates of long-memory time series models described in Chapter 4 is different and usually more complex than for short-memory processes. To illustrate this difference, Chapter 5 addresses some of the technical aspects of the proof of the consistency, central limit theorem, and efficiency of the maximum-likelihood estimators in the context of long-range-dependent processes.

Chapter 6 and Chapter 7 deal with heteroskedastic time series. These processes, frequently employed to model economic and financial data, assume that the conditional variance of an observation given its past may vary with time. While Chapter 6 describes several widely used heteroskesdatic models, Chapter 7 characterizes these processes in terms of their memory and the memory of some of their nonlinear transformations. On the other hand, Chapter 8 discusses Bayesian methods for dealing with strongly dependent data. Special attention is dedicated to iterative procedures such the Metropolis-Hastings algorithm and the Gibbs sampler. Prediction of long-memory time series models is reviewed in Chapter 9. This chapter summarizes several results on the prediction of stationary linear processes and discusses some specific methods for heteroskedastic time series. Linear regression models with strongly dependent disturbances are addressed in Chapter 10. In particular, some large sample statistical properties of least squares and best linear unbiased estimators are analyzed, including consistency, asymptotic normality, and efficiency. Furthermore, these results are applied to the estimation of polynomial and harmonic regressions.

Most of the methods reviewed up to Chapter 10 are only applicable to complete time series. However, in many practical applications there are missing observations. This problem is analyzed in Chapter 11 which describes some state space techniques for dealing with data gaps. Finally, Chapter 12 examines some methodologies for the treatment of long-memory processes which display, in addition, cyclical or seasonal behavior. Apart from discussing some theoretical and methodological aspects of the maximum-likelihood and quasi-maximum-likelihood estimation, this chapter illustrates the finite sample performance of these techniques by Monte Carlo simulations and a real-life data application. It is worth noting that similarly to Chapter 1, every chapter of this book ends with a bibliographic notes section and a list of proposed problems.

I wish to express my deep gratitude to Jay Kadane for many insightful discussions on time series statistical modeling, for encouraging me to write this book, and for valuable comments on a previous version of the manuscript. I am also indebted to many coauthors and colleagues, some of the results described in this text reflect part of that fruitful collaboration. I would like to thank Steve Quigley, Jacqueline Palmieri, Christine Punzo, and the editorial staff at John Wiley & Sons for their continuous support and for making the publication of this book possible. Special thanks to Anthony Brockwell for several constructive suggestions on a preliminary version of this work. I am also grateful of the support from the Department of Statistics and the Faculty of Mathematics at the Pontificia Universidad Católica de Chile. Several chapters of this book evolved from lecture notes for graduate courses on time series analysis. I would like to thank many students for useful remarks on the text and for trying out the proposed exercises. Financial support from Fondecyt Grant 1040934 is gratefully acknowledged.

W. PALMA

Santiago, Chile
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ACRONYMS

ACF	autocorrelation function
AIC	Akaike's information criterion
ANOVA	analysis of variance
AR	autoregressive
ARCH	autoregressive conditionally heteroskedastic
ARFIMA	autoregressive fractionally integrated moving-average
ARMA	autoregressive moving-average
BLUE	best linear unbiased estimator
DFA	detrended fluctuation analysis
DWT	discrete wavelet transform
EGARCH	exponential generalized autoregressive conditionally heteroskedastic
fBm	fractional Brownian motion
FFT	fast Fourier transform
fGn	fractional Gaussian noise

FI	fractionally integrated
FIGARCH	fractionally integrated generalized autoregressive conditionally heteroskedastic
FIEGARCH	fractionally integrated exponential generalized autoregressive conditionally heteroskedastic
FN	fractional noise
GARCH	generalized autoregressive conditionally heteroskedastic
GARMA	Gegenbauer autoregressive moving-average
HTTP	hyper text transfer protocol
IG	inverse gamma distribution
IM	intermediate memory
LM	long memory
LMGARCH	long-memory generalized autoregressive conditionally heteroskedastic
LMSV	long-memory stochastic volatility
LSE	least squares estimator
MA	moving average
MCMC	Markov chain Monte Carlo algorithm
ML	maximum likelihood
MLE	maximum-likelihood estimator
MSE	mean-squared error
MSPE	mean-squared prediction error
PACF	partial autocorrelation function
QMLE	quasi-maximum-likelihood estimator
R/S	rescaled range statistics
SARFIMA	seasonal autoregressive fractionally integrated moving-average
SD	standard deviation
SM	short memory
SV	stochastic volatility

CONTENTS