

Quantitative Techniques for Managerial Decision Making

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T.R.Vagg**

Forecasting and Corporate Modelling • Linear Programming • The Transportation Method
al Programming • Network Analysis • Matrix Algebra and Markov Analysis • Queuing Theory
Simulation • Inventory Control • Decision Analysis and the Evaluation of Information
Integer Programming and Dynamic Programming • Cost Estimation and Prediction

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PREFACE

While the first edition of this book, titled *Operations Research in Business: An Introduction* (1974), was primarily designed to provide material for an introductory undergraduate course in Operations Research, the second edition, under the new title, is intended to serve two purposes:

1. To continue to meet the original objective of providing a concise text for an introductory course in Operations Research.
2. To be used in conjunction with the standard management accounting texts to supplement the coverage of quantitative techniques for students of management accounting.

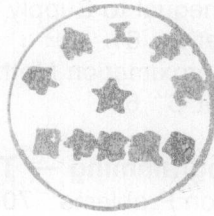
The first purpose is met by presenting a number of topics in a reasonably simple manner so that the principles and methods can be easily grasped. Any rigorous mathematics or mathematical notation has been avoided as far as possible, so that a reader without a particularly strong mathematical background can come to grips with the subject. Readers wishing to pursue particular topics in greater depth are advised to consult some of the more detailed texts on Operations Research.

For management accounting students the book should provide a valuable addition to the material presented in accounting texts. We believe that many of the problems, some of which are sufficiently challenging to sustain interest, will provide valuable insight into the use of quantitative techniques for management decision making. We hope that students will become sufficiently familiar with the techniques covered to be able to participate actively in designing accounting information systems capable of producing input information for use in a variety of decision models. The text should also

provide a basis for understanding some of the more advanced articles in the current accounting literature.

To a large extent the sequence of chapters is arbitrary, except that Chapters 4 and 5 assume a knowledge of the material in Chapter 2. Each chapter contains some problems, the key answers to which appear in the appendix.

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INTRODUCTION

ORIGINS OF OPERATIONS RESEARCH

The term *operations research* was coined during World War II. Simply defined, operations research means 'scientific approach to decisions'. Obviously, then, operations research was practised long before 1939, but it was for military purposes that teams of scientists were brought together and became known as operations research groups. Examples of the use of operations research during the war were:

1. How to most effectively use radar to locate enemy aircraft.
2. Programming bomber flights to ensure maximum destruction.
3. Calculation of the optimum size for shipping convoys so as to minimize the risk of torpedo damage.
4. Programming searches for pilots ditched in the sea.

After World War II there was a gradual movement of operations research workers into industry where management problems were tackled. The advent of electronic computers in the 1950s meant that new problems could be solved and provided the means for employing new techniques. Operations research in business may be defined as the application of mathematical models and statistical techniques to business decisions; or the application of the scientific method to the study of the operations of organizations.

ROLE OF MANAGEMENT AND THE O.R. TEAM IN DECISION MAKING

When confronted with a problem a business manager has to make a decision to follow some course of action which is consistent with the attainment of

organizational objectives. The decision making process consists of the following steps:

1. Define the problem.
2. Discover alternative strategies which may be adopted, and the different states of nature which may be expected.
3. Select the 'best' strategy or solution.
4. Implement that solution.

Operations research is concerned with steps (2) and (3). From the operations research point of view, the approach to the problem is:

- (a) Formulate the problem by determining the factors and relationships that exist, and by finding a measure of effectiveness such as profit maximization or cost minimization.
- (b) Construct a mathematical model.
- (c) Derive an optimum solution.
- (d) Test the model and the solution, perhaps with a pilot run.
- (e) Establish controls within which variable values and relationships may vary without rendering the solution or model invalid.

TYPES OF PROBLEMS THAT OPERATIONS RESEARCH TACKLES

Allocation of Resources

When there are limited resources available for carrying out a number of activities, often by alternative means, there is a problem in allocating the scarce resources most effectively. The allocation problem is to maximize the effective use of resources to achieve some objective such as profit maximization or cost minimization. Mathematical programming techniques are used to solve these problems, the most common technique being linear programming.

One of the simplest types of allocation models is the assignment problem; for example, assigning men to different jobs or assigning jobs to be run on machines with different efficiencies. The transportation method is a special case of the allocation problem and is useful in more complex assignment problems where jobs require the use of more than one resource or resources can be used for more than one job.

Queuing

Queuing theory is concerned with queues lined up for service. There is randomness in joining the queue and the problem is to determine the

optimum number of service facilities or personnel required by matching service costs with the costs of time lost by having employees waiting or with the cost of customers who leave rather than wait for service.

Sequencing

This is similar to queuing but the order of servicing is an important consideration. Network analysis fits into this category of problems.

Routing

Routing problems are dynamic versions of allocation. The classic example is the problem of the travelling salesman selecting a route such that he does not finish far from his base but minimizes the distance travelled.

Inventory

Inventory models answer the questions (i) how much to order, and (ii) when to order. If too much inventory is held carrying costs are high, but if too little is ordered there is a greater risk of stockouts. The timing of orders depends upon the lead time required to receive delivery and the anticipated demand during the lead time.

Replacement

There are two types of replacement problems:

1. Those involving equipment, machines, etc. which deteriorate over time, where the problem is to determine the optimum time to replace them.
2. Those items which fail after a period of time, such as electric light bulbs. The problem is to determine whether replacement should be effected as individual items fail or whether when one fails all items should be replaced.

This is not an exhaustive list of problems to which operations research techniques are applied but examples of some of the more common ones.

TYPES OF MODELS

Any classification of models can be difficult because different classification systems may be employed according to the attributes selected as the basis for segregating models into classes. The most common basis of classification is

by degree of abstraction, the three main types being iconic, analogue and symbolic models.

Iconic Models

The adjective iconic is derived from the noun 'icon' meaning image, statue, painting. An iconic model, therefore, looks like what it represents. This type of model can be either two or three dimensional and may be life-sized or scaled. Common examples are a photograph, a map, a statue, and scale models of aeroplanes, cars, houses, mountain ranges and human organs.

Analogue Models

An analogue model may be one which provides some resemblance to the attributes and relationships of the phenomenon being observed, or one which has similar characteristics to that phenomenon. It may be used because the phenomenon itself cannot be modelled visually. For instance, the relationship between temperature and time of day cannot be directly depicted visually, but may be exhibited on graph paper by means of a two-dimensional chart. An analogue model may also be used when experimenting with an actual system which the model represents is undesirable, so that substitutes exhibiting similar characteristics are used. Hence, rats, guinea pigs and monkeys are analogue models which may be used for medical experiments or space-flight observations.

Symbolic Models

A symbolic model uses symbols, usually mathematical symbols to represent the concepts and relationships of a system. Direct labour costs per time period in a production department might be considered to depend simply on the number of men directly engaged in production in that department, assuming all are paid at the same rate. Thus the relationship between cost and labour could be shown in a symbolic model by the equation

$$C = rX,$$

where C is total direct labour cost for the department, r is the wage rate and X the number of men employed.

If we then add that the number of men employed depends on the level of production, and that the level of production depends on expected sales, required inventory levels, the number of machines in the department and their processing times and the availability of raw materials, several dimen-

sions have been added to the problem. Provided, however, that these relationships can be expressed quantitatively, a symbolic model can be constructed to depict the situation, and the behaviour of total direct labour cost can be observed for different values of the independent variables.

Symbolic models tend to be the most useful type for business analysis and will be the type with which we are mainly concerned.

Chapter 1

FORECASTING AND CORPORATE MODELLING

For a business organization to be successful in a competitive environment, it must have managers who collectively make a sufficient number of good decisions. Timely availability of relevant information alone will not ensure that correct decisions are made. Good quality decisions, however, are unlikely to emerge in the absence of such information. Of course a great deal depends on the way in which the information is used in the decision making process.

Whilst management accountants have a responsibility to supply much of the information needed for decision making, other specialists, for example mathematicians, statisticians and operations researchers have developed quantitative techniques in many areas, aimed at improving the decision making process itself. These techniques inevitably depend on reliable input information.

One of the purposes of this book is to assist students of management accounting to become aware of how the techniques work. Only in this way will they later be able to design accounting systems capable of generating the

various categories of information needed as inputs for the decisions required in a particular organization.

Quantitative techniques developed to assist decision makers are frequently referred to collectively as decision models. The name is perhaps not ideal, implying, as it does, that the models generate decisions. Decisions are the responsibility and prerogative of people. It is only in special circumstances that the output from a model may be treated as a decision and acted upon uncritically. In most instances, the model output is used by the human decision maker in conjunction with other qualitative information in arriving at the decision, by a process which cannot be fully described or programmed in the mathematical sense.

Before proceeding to an examination of the models that have gained a measure of acceptance, and have proved of value to managers, it is worthwhile to consider some of the aspects of business decisions that ensure that human decision makers occupy a vital and ongoing role in business organizations.

The difficulties that prevent development of models capable of generating actual decisions include:

1. A decision is essentially a choice in the light of some objective. The precise specification of the objective presents a problem. The decision maker may have in mind a number of objectives, each of which can be served to a greater or lesser extent depending upon the course of action chosen. The priority or importance attaching to each of the objectives may change over time and from time to time, as indeed may the objectives included in the set of objectives. Further, the decision maker may be unable or unwilling even to rank the objectives in order of importance at any particular time, much less attach a numerical weighting to each. This means that the ultimate decision may have to be made subjectively through an informal model in the mind of the decision maker.
2. Qualitative factors important in the situation may not be quantifiable and therefore cannot be included in a formal decision model. For example, the productivity of employees may be affected by their reaction to the decision, hence labour cost may be a complex function partly dependent upon the final decision taken. The reaction of other interested groups may need to be considered in an informal way, for example customers, competitors, suppliers, and various governmental bodies.
3. The problem itself may be so complex that construction of a sufficiently detailed model becomes impractical, in which case output from the formal model must be considered having regard to the impact of those aspects of the problem not included in the model itself.
4. The problem being dealt with may be only a part of a broader, more complex structure of business affairs requiring many decisions which, to