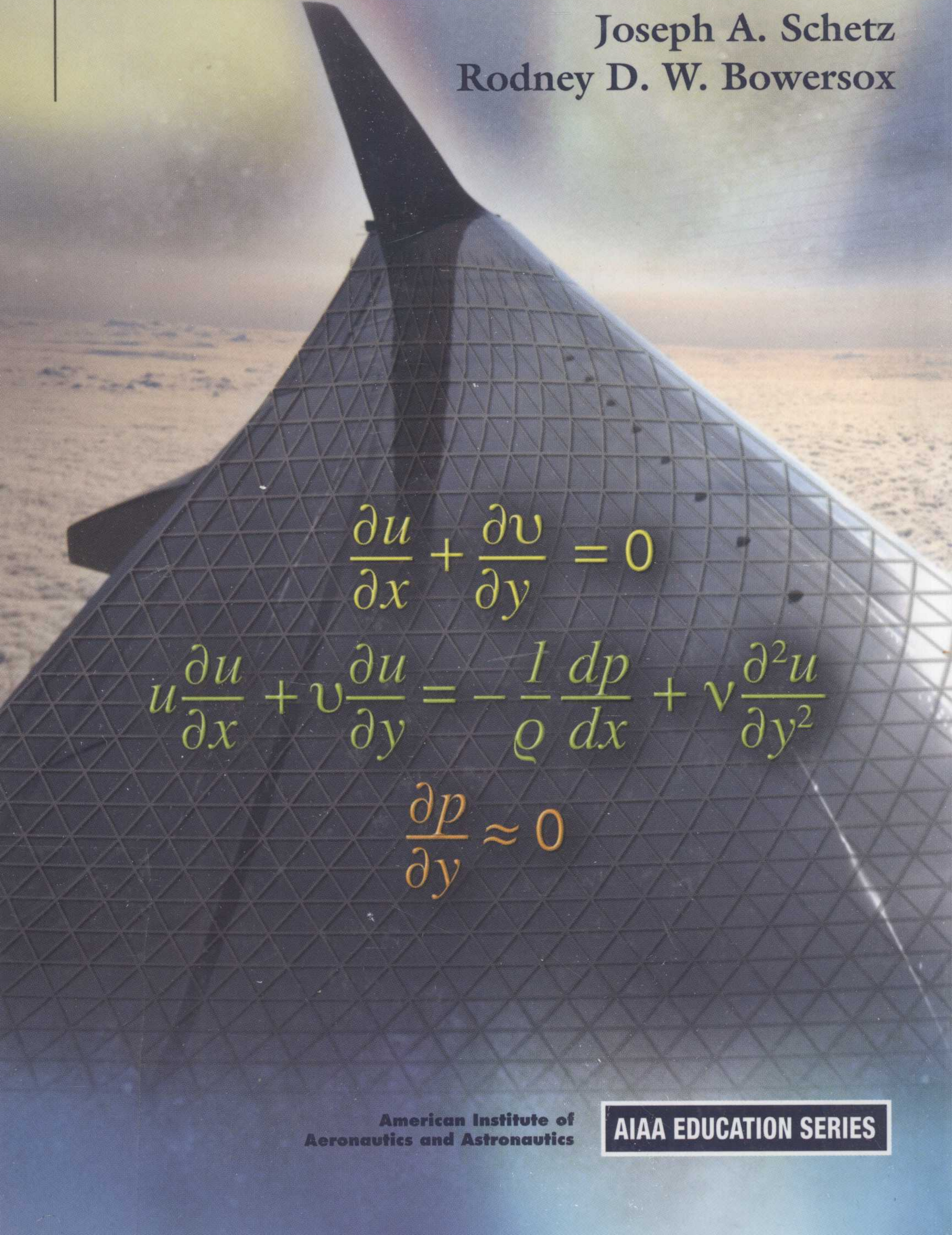
Boundary Layer Analysis

Second Edition

Joseph A. Schetz
Rodney D. W. Bowersox



$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\varrho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} \approx 0$$

American Institute of
Aeronautics and Astronautics

AIAA EDUCATION SERIES

Boundary Layer Analysis

Second Edition

Joseph A. Schetz
Virginia Polytechnic Institute
and State University
Blacksburg, Virginia

Rodney D. W. Bowersox
Texas A&M University
College Station, Texas

AIAA
EDUCATION SERIES

Michael R. Mendenhall
Guest Series Editor
Nielsen Engineering and Research
Santa Clara, California

Published by the
AMERICAN INSTITUTE OF AERONAUTICS AND ASTRONAUTICS, INC.
1801 ALEXANDER BELL DRIVE, RESTON, VIRGINIA 20191-4344

American Institute of Aeronautics and Astronautics, Inc., Reston, Virginia

1 2 3 4 5

Library of Congress Cataloging-in-Publication Data

Schetz, Joseph A.

Boundary layer analysis / Joseph A. Schetz, Rodney D. W. Bowersox.—2nd ed.

p. cm.

Includes bibliographical references and index.

ISBN 978-1-60086-823-8

1. Turbulent boundary layer. 2. Boundary element methods. I. Bowersox, Rodney D. W. II. Title.

TA357.5.T87S34 2012

620.1'064—dc23

2011021473

ISBN 978-1-60086-823-8

This book is dedicated to our very gracious, patient, and supportive wives, Katherine Schetz and Selina Bowersox.

PREFACE

This book is intended as a text for courses in viscous fluid flow at high Reynolds numbers for advanced undergraduate and beginning graduate engineering students. Numerous homework problems ranging from the simple, but instructive, to the challenging are included. The emphasis is on understanding and analyzing flows of engineering interest. Thus, turbulent flows receive primary coverage, and a modern understanding of the physics of turbulent shear flows and turbulence models is discussed in detail. Further, the accurate analysis of practical flow problems, especially those involving turbulence, requires the use of computerized methods, so numerical methods are treated in depth. Coverage of the older analytical methods is also included, to aid in developing understanding and because such methods are still widely used in preliminary design, especially for design optimization studies.

Several computer codes for both modern numerical and older analytical methods in the form of applets written in the JAVA language have been developed and made available on the Internet (see the Supporting Materials page) to accompany this text. Numerous worked examples are provided to illustrate the use and capability of these applets. The applets were designed primarily for solving homework problems, but working professionals have also found them useful.

A second major theme of this volume is a concurrent treatment of the transfer of momentum, heat, and mass. This is primarily a book on viscous fluid dynamics, but the processes of convective heat and mass transfer are so closely connected to momentum transfer that a unified presentation was deemed valuable. Such a presentation allows the scope of the book to span the entire range from low-speed to hypersonic flows. In addition, including mass transfer permits a discussion of viscous flows with chemical reactions.

An introductory coverage of simple non-Newtonian fluids of the *power law* and *Bingham plastic* types is also provided to orient the student to some of the important differences found with such fluids, compared to the more usual Newtonian fluids.

Chapter 1 is devoted to an introduction to the subject of viscous flow and why it is important to the engineer. The relevant physical phenomena and properties and dimensionless numbers are discussed. Then, the exact equations of motion for a constant-density fluid and the boundary layer assumptions are derived. The phenomena of flow separation and the *Kutta condition* are described next. The last section introduces the basic ideas associated with turbulent flows.

The subject matter of Chapter 2 is the approximate solution to laminar boundary layer problems, including heat and mass transfer, based on the integral forms of the equations. Worked examples and JAVA applets for the Thwaites-Waltz method and a corresponding integral method for heat transfer are presented.

Chapter 3 is concerned with the derivation of the boundary layer equations expressing conservation of total mass, momentum, energy, and mass of species, including variable density and properties.

The coverage in Chapter 4 begins with a few of the exact solutions for special cases of laminar flow. Then, *similar solutions* are discussed. The remainder of the chapter deals with numerical solutions of boundary layer problems using the finite difference method and the finite element method. Worked examples and JAVA applets are included.

Chapter 5 discusses high-speed laminar flows. Viscous heating, compressible transformations, numerical solutions, and viscous-inviscid interactions are covered. Again, worked examples and a JAVA applet are given.

Transition to turbulent flow is treated in Chapter 6, which has been expanded from the previous edition. Hydrodynamic stability and the e^N method are described. Finally, selected empirical information is presented to illustrate the influence of roughness, pressure gradients, injection or suction, supersonic flow, and the like.

Updated from the previous edition, a detailed discussion of the nature of the turbulent, wall-bounded flows and modern turbulence models is the focus of Chapter 7. The discussion is limited to constant-density flows for clarity. The problems of analyzing turbulent flow problems are emphasized. Integral and numerical methods are discussed, and extensive comparisons of predictions with experiment are provided. Worked examples and JAVA applets are presented.

The important subject of laminar and turbulent internal flows has been highlighted by devoting a separate chapter, Chapter 8, to that topic alone.

Chapter 9 covers free shear flows, such as wakes, jets, and shear layers. Both laminar and turbulent cases are considered over the range from low to high speeds. Worked examples as well as a JAVA applets are provided.

Wall-bounded turbulent flows of variable density and properties are discussed in Chapter 10. Both low- and high-speed cases are included. Worked examples and JAVA applets are presented.

Chapter 11 contains a detailed presentation of current knowledge for truly three-dimensional boundary layer flows, including three-dimensional jets.

A key addition to the second edition is a completely new chapter, Chapter 12, that provides an introduction to the numerical solution of high-Reynolds-number viscous flow problems using the full Navier-Stokes equations rather than invoking the boundary layer approximation. The primary focus is on the finite volume methods. The important subject of grid generation is discussed. A convenient JAVA applet has been developed and is presented, and this permits the student to make computations using both boundary layer and Navier-Stokes equations formulations and to compare the results, limitations, and computational burden. The use of the Navier-Stokes applet is illustrated with a worked example. Finally, this applet allows the student to perform simulations that involve separated flows.

Throughout the first eleven chapters of the book, separating, but not separated, flows are discussed because the analysis of the latter cannot be handled with the usual boundary layer equations. Separated flows can be handled only with the methods presented in Chapter 12. Last, unsteady flows are not covered, and that is justified by the observation that those phenomena are usually omitted in courses at the level intended here.

The goal of this effort was to write a book appropriate for mechanical, aerospace, civil, ocean, and chemical engineering students. The treatment assumes that the student has taken at least one general undergraduate course in fluid mechanics and one in mathematics with partial differential equations. It also assumes that the student is computer literate. The computer codes accompanying this book are written in JAVA and made available on the Internet (see the Supporting Materials page). A course that would encompass the bulk of the material in this volume would likely be a two-semester graduate course. A good one-semester graduate course at the undergraduate or graduate level can be formed with a judicious selection of all the material supplied. The selection of topics to be covered in a given course will clearly be strongly influenced by the subject area of the students. It is suggested, however, that all students receive some coverage of numerical solution techniques and turbulence models.

Joseph A. Schetz
Blacksburg, Virginia

Rodney D. W. Bowersox
College Station, Texas
August 2011

NOMENCLATURE

a	= speed of sound and amplification factor
A	= area or constant
A, B	= inviscid flux Jacobians ($\partial f/\partial Q$, $\partial g/\partial Q$)
A_ν, B_ν	= viscous flux Jacobians ($\partial f_\nu/\partial Q$, $\partial g_\nu/\partial Q$)
$b_{1/2}$	= half-width
B_1, B'_2, B'_1, B'_2	= constants
$B_{f,h}$	= injection/suction parameters
c	= average speed of molecules
c'_i	= fluctuating value of species concentration
c_i	= species concentration
C_i	= mean value of species concentration
C_1, C_2, C_μ, etc.	= constants
c_{b1}, c_{b2}, c_{w1}, etc.	= constants in Spalart-Allmaras turbulence model
c_p	= specific heat at constant pressure
c_v	= specific heat at constant volume
c_r, c_i	= real and imaginary parts of the phase velocity
C_f	= skin friction coefficient
$\overline{C}_f$	= average skin friction coefficient
C_p	= pressure coefficient
C_D	= drag coefficient
D, d	= diameter and minor axis in three-dimensional jets
D	= dissipation term in the finite volume flux evaluation
$\mathcal{D}$	= drag
D_h	= hydraulic diameter
D_{ij}	= binary diffusion coefficient
D_T	= turbulent diffusion coefficient
e	= internal energy
$E_1(k_1)$	= kinetic energy of axial fluctuations at wave number k_1
f	= friction factor
$f(\cdot)$	= function of (·)
f_i	= body force vector
f_1, f_2, f_μ	= factors in $K\varepsilon$ model
$f_{\nu 1}, f_{\nu 2}, f_{t2}, f_w$	= quantities in Spalart-Allmaras turbulence model
f, g, h	= inviscid flux vectors in the x-, y-, and z-directions, respectively

f^+, g^+, h^+ = positive eigenvalue inviscid flux vectors
f^-, g^-, h^- = positive eigenvalue inviscid flux vectors
f_v, g_v, h_v = viscous flux vectors in the x-, y-, and z-directions, respectively
F_i = element force vector
g = acceleration of gravity, Clauser similarity variable, Johnson-King variable, Spalart-Allmaras quantity, and $G/(1-G_w)$
G = stagnation enthalpy ratio
h = enthalpy
$\hbar$ = film coefficient
$\hbar_D$ = film coefficient for diffusion
h_1, h_3, h_c = metric coefficients
H = stagnation enthalpy
$H(\Lambda)$ = shape factor
i $\equiv \sqrt{-1}$
j = index
J = integrated momentum flux
k = thermal conductivity and average roughness size
k_m = Crocco-Lees mixing constant
k_T = turbulent thermal conductivity
k_1 = wave number of fluctuations
K = turbulent kinetic energy
$K(T)$ = equilibrium constant
K_1, K_2, etc. = constants and geodesic curvatures
K_{ij} = element stiffness matrix
ℓ = turbulent length scale and arc length
ℓ_m = mixing length
ℓ_1 = major axis in three-dimensional jets
L_m = length scale in Johnson-King model
Le = Lewis number
Le_T = turbulent Lewis number
m = index along surface
$\dot{m}_i$ = diffusive mass flux of species i
M = maximum value of index m and Mach number
M_{c1} = convective Mach number
M_τ = Mach number in compressibility correction
n = index across layer, frequency, and transverse streamline coordinate
n_y = y-component of normal vector to surface
N = maximum value of index n
Nu = Nusselt number
Nu_{Diff} = Nusselt number for diffusion

N_{He} = Hedstrom number
N_{Pl} = plasticity number
p = pressure and exponent for power law fluids
p_i = partial pressure of species i
p' = fluctuating pressure
P = mean pressure, scaled pressure, and perimeter
P_t = total pressure
P_c, P_T, P_V = power law decay exponents
Pr = Prandtl number
Pr_T = turbulent prandtl number
$\mathcal{P}$ = production of turbulent kinetic energy
q_i = heat flux vector
q_T = turbulent heat flux
q_w = wall heat transfer rate
q^* = friction velocity in three-dimensional flows
Q = total velocity in three-dimensional flows and state vector for the conservation laws
$Q_{n,m}$ = state vector at the finite volume cell n, m center
r = radial coordinate, recovery factor, element coordinate, and quantity defined in Spalart-Allmaras turbulence model
$r_0(x)$ = body radius
$r_{1/2}$ = half-radius
R = pipe radius, gas constant, and radius of curvature
$R_{n,m}$ = residual for the finite volume discretization for cell n, m
Ri = Richardson number
Re = Reynolds number
R_0 = body nose radius of curvature
R_t = turbulent Reynolds number
$s, \bar{s}$ = transformed streamwise and streamline coordinate and element coordinate
Δs = face length of finite volume cell
S = eigenvector (columns) matrix
S^+ = positive eigenvector matrix
S^- = negative eigenvector matrix
$\tilde{S}$ = quantity defined in Spalart-Allmaras turbulence model
Sc = Schmidt number
Sc_T = turbulent Schmidt number
St = Stanton number
St_{Diff} = Stanton number for diffusion
$S(\Lambda)$ = Shear parameter
t = time
Δt_{CFL} = inviscid Courant, Friedrichs and Lewy condition

T	= static temperature
$\mathbf{T}_{x, y, z}$	= surface force vector
T_b	= bulk temperature
T^*	= reference temperature
T_t	= total (stagnation) temperature
T_0	= time period
$\overline{T}$	= mean temperature
T'	= fluctuating temperature
T_*	= heat transfer temperaure
T^+	$\equiv (T_w - \overline{T})T_*$
u	= streamwise velocity
u_{ave}	= average velocity
u_i	= nodal velocity in FEM
$\overline{u}$	= transformed velocity and velocity from previous iteration in FEM
$\overline{\overline{u}}$	= mass-weighted mean streamwise velocity
u'	= fluctuating velocity
u_*	= friction velocity
u^+	$\equiv U/u_*$
u_c	= convective velocity
U	= mean streamwise and scaled velocity
U_p	= primary direction velocity in a channel
U_w	= velocity of moving wall
v	= transverse or radial velocity
v_w	= transverse velocity at the wall
v_i	= nodal velocity in FEM
$\overline{v}$	= transformed velocity and velocity from previous iteration in FEM
$\overline{\overline{v}}$	= mass-weighted mean transverse velocity
v_0^+	= dimensionless transverse velocity at the wall
v'	= fluctuating transverse velocity
V	= mean transverse, scaled and general velocity
V_0	= entrainment velocity
$V_{n,m}$	= volume of finite volume cell n, m
$V_{C,N}$, $V_{C,M}$	= contravariant velocities
V_ϕ	= velocity from scalar potential in a channel
V_ψ	= velocity from vector potential in a channel
w	= spanwise velocity in three-dimensional flows
w'	= fluctuating spanwise velocity
W	= mean spanwise velocity in three-dimensional turbulent flows and channel half-height
W_i	= molecular weight
$W(y/\delta)$	= wake function

w_A = reaction source term
x = streamwise coordinate
$\overline{x}$ = transformed streamwise coordinate
x_1, x_3 = coordinates in three-dimensional flows
x_{PC} = length of potential core
X_i = mole fraction of species i
X = scaled streamwise coordinate
y = transverse coordinate
$\overline{y}$ = transformed transverse coordinate
y^+ = transverse coordinate for the law of the wall, $\equiv yu_*/\nu$
y_{max} = length scale in Baldwin-Lomax model
Y = transformed and scaled transverse coordinate
$Y_{1/2}, Z_{1/2}$ = half-widths in three-dimensional jets
z = Spanwise coordinate in three-dimensional flows
z_A = dimensionless mass fraction
Z $\equiv k^m \ell^n$
$Z(p, T)$ = compressibility factor

Other

α = wave number, amplification factor, atom mass fraction, constant in Wilcox $K\omega$ turbulence model, and dissipation model coefficient
α_T $\equiv k_T/\rho c_p$
$\alpha(p)$ = friction law coefficient for power law fluids
β = pressure gradient parameter, wave number, quantity in Wilcox $K\omega$ turbulence model, and dissipation model coefficient
β^* = constant in Wilcox $K\omega$ turbulence model
β_w = wall streamline angle
β_ξ, β_ω = pressure gradient parameters in three dimensions
$\beta(p)$ = friction law function for power law fluids
χ = quantity defined in Spalart-Allmaras turbulence model
χ_ω = vortex stretching parameter in Wilcox $K\omega$ turbulence model
$\overline{\chi}$ = hypersonic interaction parameter
ψ = planar stream function
Ψ = axisymmetric stream function
$\hat{\psi}$ = disturbance stream function
ε = dissipation of turbulent energy
$\varepsilon_{n,m}$ = truncation error
ε_{xy} = strain
ρ = density
λ = Pohlhausen pressure gradient parameter, pipe resistance coefficient, and second viscosity coefficient

λ^* = mean free path between molecules
Λ = Thwaites-Walz pressure gradient parameter, convective spectral radius, and diagonal eigenvalue matrix
Λ^+ = positive diagonal eigenvalue matrix
Λ^- = negative diagonal eigenvalue matrix
τ, τ_{xy} = shear
τ_T = turbulent shear
τ_0 = yield stress for plastic fluid
Ω = intermittency and vorticity parameter
Ω_e = element area
Γ_e = element perimeter
μ = laminar viscosity
μ_a = apparent viscosity for non-Newtonian fluid
μ_{BP} = viscosity factor for Bingham plastic fluid
μ_{PL} = viscosity factor for power law fluid
μ_T = turbulent viscosity
μ_{Tx} = turbulent viscosity in the streamwise direction
μ_{Tz} = turbulent viscosity in the crossflow direction
κ = constant in the law of the wall and inverse of channel radius of curvature and Q vector interpolation function parameter
κ_T = constant in the temperature law of the wall
ν = laminar kinematic viscosity
ν_T = turbulent kinematic viscosity
$\tilde{\nu}$ = variable in Spalart-Allmaras turbulence model
ϕ_i = element interpolation function
ϕ = amplitude function and dummy variable and Q vector interpolation function parameter
$\phi_{1,2}$ = deformation angle
δ = boundary layer thickness
δ_T = thermal boundary layer thickness
δ_c = concentration boundary layer thickness
δ^*, Δ^* = displacement thickness
δ_k^* = kinematic displacement thickness
δ_1^*, δ_2^* = displacement thicknesses in three-dimensional flows
Δ_{PS} = Perry-Schofield length scale
Δ = Clauser integral boundary layer thickness
θ = momentum thickness
$\theta_{11}, \theta_{12}, \theta_{21}, \theta_{22}$ = momentum thicknesses in three-dimensional flows
Θ, Θ_r = excess temperature
ζ $\equiv \delta_T/\delta$
Π = wake parameter
ξ = dummy variable and stretched time

$\sigma_d, \sigma_K, \sigma_\omega$	= constants in Wilcox $K\omega$ turbulence model
η	= similarity variable
$\bar{\eta}$	= transformed transverse coordinate
ω	= dimensionless frequency, variable in $K\omega$ turbulence model, and transformed lateral coordinate
$\omega_x, \omega_y, \omega_z$	= components of vorticity
$\sigma_{K,\varepsilon,\tau}$	= Prandtl numbers for K, ε, τ
Γ	= see Eq. (10.8)
γ	= ratio of specific heats and stability parameter in three-dimensional flows
ξ	= transformed streamwise coordinate
ш	= exponent in laminar viscosity relation

Subscripts

c	= values on the centerline
e	= values at the edge of the boundary layer
j	= initial values in a jet
t	= stagnation values
w	= wall values
∞	= conditions in the approach flow

CONTENTS