Three-Dimensional Analysis of Crack Growth

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Preface

The objective of this book is to formulate, implement and apply the dual boundary element method to three-dimensional crack problems in linear elastic fracture mechanics. The dual boundary element method has been developed to overcome the ill–conditioning problems associated with the conventional boundary element method (BEM). By applying the displacement boundary integral equation on one of the crack surfaces and the traction boundary integral equation on the other, the general mixed mode crack problems can be solved in a single region. The focus of the numerical implementation is on the efficient evaluation of singular and near-singular integrals associated with the formulation. The engineering applications presented in this book concentrate on the simulation and analysis of general mixed mode crack growth in three dimensions.

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Contents

Chapter 1

Introduction

1.1 General

All structures contain flaws to some extent, mainly as a result of the manufacturing processes, fabrication or due to localized in–service damage. These flaws act as sites of stress elevation; subsequent growth due to operating loads could lead to degradation in the structural strength with the possibility of catastrophic consequences (i.e. fracture). The costs of fracture can be significantly large. One example is the collapse of the Norwegian semi–submersible oil rig, Alexander L. Kieland [8, 56], where 123 out of 212 men on board lost their lives. The failure was ultimately due to fatigue cracking of a non–redundant brace. Also, accidents caused by the fracture of structures in the aircraft industry are, in general, catastrophic. Structural failure of the pressure cabin through metal fatigue cracking claimed the total loss of two of the newly developed British Comet jet airliners in 1954 [23, 154]. Another example occurred on 28 April 1988. The roof of the forward cabin of an Aloha Airlines Boeing 737 was torn off at 24 000 feet [119]. The Boeing 737 landed safely with the loss of one life and several passengers injured. The cause of the accident was the growth of fatigue cracks from rivet fastener holes under the repeated stress of thousands of take-offs and landings. In the United States, the costs of fracture were estimated in a report [38, 135] to be $119 billion per year, which was about 4 per cent of the gross national product in 1982.

Fracture mechanics is essentially the study of cracked bodies under load. It tries to answer four main questions:

- Will the crack grow?
- Will it grow in a fast unstable, or slow stable manner?
- If it will grow stably, at what rate will it grow?
- Before becoming unstable, to what size can the crack grow, and for what period of time?

Since the pioneering work of Griffith [51] and through its rapid development in the 1960s and 1970s, fracture mechanics technology has become the primary approach for analysing and preventing failures in structures. It was stated in a report [38, 135] that 'An estimated $35 billion (out of the total cost of $119 billion by fracture per year in the United States) per year could be saved through the use of currently available technology. Costs could be further reduced by much as $28 billion per year through fracture related research'. The significance of fracture mechanics research is apparent. A similar report [41] by the Commission of European Countries estimated 80 billion ECU per year could be saved through the use of fracture mechanics technology.

Linear elastic fracture mechanics (LEFM) is a part of the discipline of fracture mechanics that can be used for the analysis of cracked linear elastic bodies or those with limited plastic deformation. One of the most important parameters in linear elastic fracture mechanics is the stress intensity factor which is a measure of the strength of the singularity of the stresses near the crack front (tip). It is a function of the applied load, the geometry of the cracked structure and the position of the point concerned along the crack front. The fundamental postulate of linear elastic fracture mechanics is that the crack behaviour is determined solely by the values of the stress intensity factors. A knowledge of this parameter allows one to determine the crack growth and the life expectancy of engineering structures through the fatigue criteria. Therefore, the determination of stress intensity factors is a fundamental task for the application of the linear elastic fracture mechanics to practical engineering problems.

Over the years, a great amount of work has been done to evaluate stress intensity factors for different loads and geometries with a variety of both computational and experimental methods proposed to determine them. Extensive collections of stress intensity factors have been published in handbook form by Tada, Paris & Irwin [166], Rooke & Cartwright [140] and Murakami *et al.* [116]. Recently, a database of stress intensity factors in the form of a flexible software package has been developed by Aliabadi [5]. Most of these solutions are for two-dimensional problems, and also problems with infinite domains.

However, cracks often occur in structures where the geometry is very complicated. Typical examples of cracks in such structural components include surface, interior and through cracks. In each of these cases the near front stress fields are generally three-dimensional in character. In general, for three-dimensional crack problems, the geometry and loading encountered is too complex for the stress intensity factor to be solved analytically. The problem is further complicated since this parameter is known to be a function of the position on the crack front and not a single constant as in the case of two-dimensional problems. None of the handbooks can meet the requirements for such varied configurations of problems. To this end, numerical methods, in conjunction with powerful computers, have been developed as possible solutions. The other area in fracture mechanics where numerical methods have to be used is in the determination of the rate of crack growth. Apart from unstable crack growth, which will lead to instant component failure, stable and subcritical crack growth may happen and result in fracture failure. When crack growth processes are simulated with an incremental crack extension analysis, stress intensity factors are required for each increment. Generally, numerical methods are employed to meet this requirement.

The finite element method (FEM) first appeared in structural mechanics and has since been the most widely used numerical technique for the solution of general mechanics problems, see Zienkiewicz [188]. Based on an energy interpretation, it can be used to approximate many physical phenomena including crack problems. However, the generality is achieved at the expense of significant approximation requirements. Among the many drawbacks of the finite element method, there are two that are most significant for its application to fracture mechanics. The first is that the problems with steep solution gradients in elastic bodies cannot be accurately approximated by the method, unless extremely refined meshes are used. This is the case with crack problems since stresses tend to infinity at the crack tip. The second is that the continuous re–meshing requirement for crack growth simulation makes the finite element method, a type of domain discretization method, less competitive, in particular for three-dimensional problems.

The boundary element method, one of the powerful general tools to solve the boundary value problems arising from various disciplines [17], can be applied to fracture mechanics problems without the limitations associated with the finite element method, see Aliabadi & Rooke [4]. Problems with solution gradients varying rapidly in elastic bodies can be modelled accurately with reasonably fine meshes. One reason for this is that, in the boundary element method, an analytical approach toward the real solutions has been made prior to the discretization. Moreover, by virtue that only boundary discretization is required for the application of the boundary element method, re–meshing work for the simulation of the crack growth process is reduced dramatically compared with the finite element method. This advantage of the boundary element method becomes more apparent when three-dimensional crack growth is modelled. However, there is a fundamental difficulty in the direct application of the boundary element method to fracture mechanics. This difficulty is due to the modelling of two coplanar crack surfaces. Collocating the boundary integral equation on the crack surfaces gives a system of non–definite linear equations, that is, a system with fewer independent linear equations than unknowns. Special techniques were proposed to overcome this difficulty. Among them, the dual boundary element method is one of the most important general approaches. The efficient systematic implementation of the dual boundary element method in three dimensions for general crack problems has been reported by Mi & Aliabadi [108]. The application of the method to the simulation of three-dimensional crack growth problems was presented by the same authors, see Ref. [110].

1.2 The application of the boundary element method to fracture mechanics

Development of the boundary element method in elasticity is briefly reviewed in this section. Also presented is a brief historical perspective of the development, formulation and application of the boundary element method in linear elastic fracture mechanics.

1.2.1 The boundary element method in elasticity

The boundary element method is based on the boundary integral equation representation of physical phenomena. The boundary integral equation formulation for the elasticity problem begins with a restatement of the equilibrium equations in integral form, using the reciprocal work theorem presented in the pioneering research of Betti [12]. Basing his work upon the fundamental solutions of the linear homogeneous governing equations of the elasticity problem, Somigliana [160] is credited with the first use of the Betti's reciprocal work theorem to derive the integral solution identities. With the contribution of the mathematicians, Muskhelishvili [118], and Kupradze [83], both the theory and the application of the mathematics of integral formulations were derived for elasticity problems. However, only a few attempts were made to find numerical solutions of the integral boundary equation before the appearance and the extended use of computers. Early examples of numerical solutions include the work of Friedman & Shaw [45], Jaswon [73], Symm [165], and Massonet [104] mainly in the sixties. The formulation used by these authors is the so-called 'indirect formulation'. The direct formulation in elasticity was reported by Rizzo [139] and Cruse & Rizzo [35]. It was certainly one of most critical steps in the development of numerical solutions for the singular elasticity formulation. Brebbia & Dominguez [15] introduced the weighted residual theorem into the derivation of the boundary element formulation. Parametric representations for both the geometry and the field variables were introduced by Lachat [85], and Lachat & Watson [86]. The numerical solution of integral equation formulations has significantly expanded and extended into almost every engineering discipline, see Brebbia, Telles & Wrobel [17].

1.2.2 The boundary element methods in fracture mechanics

As mentioned in the section above, it is not possible to solve fracture mechanics problems with the direct application of the boundary element method. This was first noted by Cruse [34] and some solutions were proposed by him in two subsequent papers [27, 30] where symmetric geometry and loading conditions were used to overcome this difficulty. Since then, more special techniques have been developed, the most important among them being the crack Green's function method proposed by Snyder & Cruse [159], the displacement discontinuity method used by Crouch [24], the subregion method by Blandford, Ingraffea & Liggett [14], and the dual boundary element method by Portela, Aliabadi & Rooke [131]. The crack Green's function method is limited to problems with a single, straight, traction-free crack, although the need for discretization of the crack is avoided. The displacement discontinuity method, where the unknown functions on the crack surfaces are the displacement differences between the surfaces, can be used for crack problems with traction-free or known surface tractions on the crack surfaces. However, it introduces extra unknowns into the formulation in addition to the boundary displacements and tractions. The subregion method introduces artificial boundaries into the body, along the crack surfaces, partitioning the body into subregions without cracks. Then the standard boundary element method is applied to each of the subregions and a linear system of equations is formed by applying the displacement compatibility and the traction equilibrium to the unknown field variables on these artificial boundaries. The main drawback of the method is that the introduction of the artificial boundaries is not unique, which causes difficulties for its application to crack growth simulation.

The dual boundary element method incorporates the displacement and the traction boundary integral equations to solve the above problem. By applying the displacement boundary integral equation on one of the crack surfaces and the traction boundary integral equation on the other, the general mixed mode crack problems can then be solved in a natural single region way. Besides the displacement boundary integral equation, an additional independent boundary integral equation is required to constitute the dual integral equation formulation for solving fracture mechanics problems. Watson [178] used the normal derivative of the displacement equation as the additional equation, together with the displacement equation, to present a dual equation formulation. Choosing the traction boundary integral equation as the additional integral equation, Hong & Chen [62] presented the derivation of the dual boundary integral equation formulation and the properties of the involved kernel functions, although the traction boundary integral equation had been previously derived by Cruse [29]. In elastostatics, a two-dimensional result was reported by Watson [179] for an embedded crack problem using the dual equation formulation together with Hermitian cubic interpolation. Three-dimensional results were presented by Gray, Martha & Ingraffea [50] and Martha, Gray & Ingraffea [101] for embedded flat cracks using the analytical evaluation of singular integrals associated with the traction boundary integral equation.

The effective numerical implementation of the dual boundary element method in two dimensions was reported by Portela, Aliabadi & Rooke in a series of publications [129, 131, 130, 132]. The use of discontinuous elements and the analytical treatment of the singular integrals arising in the dual equation formulation make the method an effective tool for the evaluation of stress intensity factors and the simulation of the crack growth problems in two dimensions. The effective implementation and the application of the dual boundary element method in three dimensions for general crack problems have been reported by Mi & Aliabadi in [6, 107, 108, 109, 110, 111, 112, 113]. An integration technique based on the singularity subtraction was implemented to evaluate the singular integrals on general quadratic elements so that there is virtually no restriction on the shape of the crack. Sets of discontinuous elements were derived to fulfill the smoothness requirements for using the traction equation, and to give the necessary smooth transition from continuous elements to discontinuous elements. Therefore, both embedded and edge crack problems can be dealt with efficiently. After this work, it has been shown that the dual boundary element method is also an effective tool for three-dimensional fracture mechanics problems.

1.3 Overview of the work

The objective of the work is to formulate, implement and apply the dual boundary element method to three-dimensional crack problems in linear elastic fracture mechanics. Upon the successful accomplishment of the aim, an efficient numerical tool will be available, which can be used to evaluate stress intensity factors in three-dimensions for general mixed mode crack problems. Moreover, three-dimensional simulation of the crack growth will become a much more efficient and practical procedure. The following text is a short description of the work and its organization which are described in further detail in subsequent chapters.

The work presented is divided into eight chapters, the first of which is the Introduction. Chapter 2 presents an overview of the basic elasticity and fracture mechanics to the extent necessary to allow the reader to follow the work in later chapters.

In Chapter 3 the dual boundary element formulation for three-dimensional linear elastic crack problems is presented. The Kelvin fundamental solutions of elasticity are given first, together with the description of the notations used in the conventional boundary element formulation. The boundary integral equation is derived from Betti's reciprocity theorem. The discretization of the boundary integral equation, which produces the boundary element method, is briefly discussed. The dual boundary element method for crack problems is presented next. Finally, the modelling strategy for general three-dimensional crack problems is presented, along with shape functions for different elements including continuous, discontinuous and edge discontinuous, as well as quadrilateral and triangular elements.

The treatment of the singular and the near-singular integrals is presented in Chapter 4. The definition, classification and computation of singular integrals arising in the dual boundary element formulation are reviewed. The explicit expressions for the free terms are derived from the adopted limiting process of the integrals in the traction boundary integral equation. A method based on the singularity subtraction technique is used to directly evaluate the Cauchy and Hadamard principal value integrals. An example is presented to demonstrate the efficiency of the method for the integration of a hypersingular integral. The remainder of the chapter is devoted to the treatment of near-singular integrals, in particular the near-hypersingular problem. A semi–analytical integration scheme based on the Taylor expansion of the field variables on the projection of the integration element is presented to deal with the near hypersingular integral. It is an extension of the work by Cruse & Aithal [32].

The application of the dual boundary element method to three- dimensional fracture mechanics problems is presented in Chapter 5. Accurate evaluation of the stress intensity factors requires special techniques. Quarter point elements and special elements are used in this chapter to model the near tip field and evaluate the stress intensity factors. The possibility, and also the restrictions of the use of quarter point elements for the discontinuous element, is investigated. The special discontinuous elements are derived in order to remove the restrictions on the use of quarter point elements. Different examples are chosen to test the proposed methods.

The numerical simulation of crack growth using the dual boundary element method is presented in Chapter 6. The analysis performs an incremental crack extension process. The determination of two key parameters for the incremental analysis, namely the direction and size of the increment, is discussed. A re–meshing strategy for modelling the edge crack growth is proposed. Four examples of crack growth analysis are presented in the remainder of the chapter covering the pure Mode I and mixed mode, for embedded and edge crack growth cases.

The application of the dual boundary element method to fatigue crack growth analysis of a bridge girder is presented in Chapter 7. The numerical results obtained for the crack growth rates and shape of the crack front are compared with experiment measurement.

Finally, conclusions and recommendations for further research and extensions of the work are presented in Chapter 8.

Chapter 2

Basic elasticity and fracture mechanics

Fracture mechanics, although a relatively new field, has advanced to a stage where its value to engineering design is generally no longer in question. The first stage of the development of fracture mechanics is marked by Griffith's classic paper [51] in 1920 for the predominant mechanical representation of brittle fracture. This theory was based, in part, on results by Inglis [66] which provided a mathematical analysis for the stresses in the vicinity of a two-dimensional elliptical opening of arbitrary eccentricity. The fracture criterion proposed by Griffith is limited to ideally brittle materials. A series of documents by Irwin, the first of which is entitled 'Fracture Dynamics'[70] published in 1948, establishes fracture mechanics as an important engineering discipline. To date, fracture mechanics technology has become the primary approach for analysing and preventing fracture failures in structures and components.

Most of the studies on fracture mechanics have dealt with situations and materials for which the use of linear elasticity concepts are valid or constitute a good approximation. It is referred to as linear elastic fracture mechanics. The concept of the stress intensity factor is used in linear elastic fracture mechanics to quantify the severity of the stress in a cracked structure. Whether a crack will grow or not, and how fast it grows is determined by the stress intensity factor under linear elastic assumptions. Therefore, a knowledge of stress intensity factors is essential within the framework of linear elastic fracture mechanics.

In this chapter, the fundamental principles and basic equations of elasticity, as a basis of fracture mechanics, is first reviewed. The sources of information are the classic works of Love [93], Timoshenko & Goodier [169] and Fung [46]. The introduction of fracture mechanics starts from three deformation modes of a cracked body. Then, the formulae which describe the asymptotic expansion expressions of the elastic fields in the vicinity of the crack front are presented. Following this is the definition of stress intensity factors. Finally, basic aspects of crack growth and fracture mechanics criteria for the analysis of crack growth are discussed. The works of Broek [19], Kanninen & Popelar [80], Aliabadi & Rooke [4] and Sih [149] constitute the main sources of reference.