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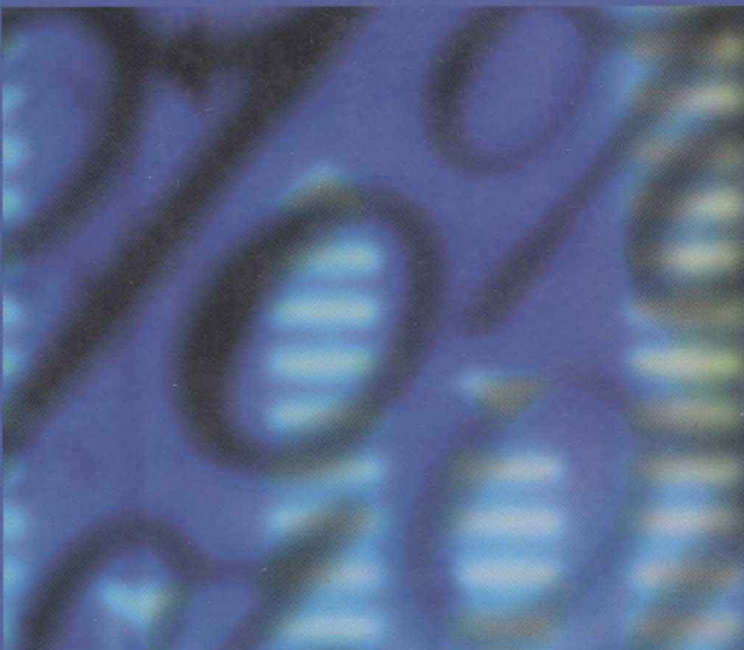
interest rate, term structure, and valuation modeling



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Interest Rate, Term Structure, and valuation modeling

FRANK J. FABOZZI

EDITOR

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preface

The valuation of fixed-income securities and interest rate derivatives, from the most simple structures to the complex structures found in the structured finance and interest rate derivatives markets, depends on the interest rate model and term structure model used by the investor. *Interest Rate, Term Structure, and Valuation Modeling* provides a comprehensive practitioner-oriented treatment of the various interest rate models, term structure models, and valuation models.

The book is divided into three sections. Section One covers interest rate and term structure modeling. In Chapter 1, Oren Cheyette provides an overview of the principles of valuation algorithms and the characteristics that distinguish the various interest rate models. He then describes the empirical evidence on interest rate dynamics, comparing a family of interest rate models that closely match those in common use. The coverage emphasizes those issues that are of principal interest to practitioners in applying interest rate models. As Cheyette states: "There is little point in having the theoretically ideal model if it can't actually be implemented as part of a valuation algorithm."

In Chapter 2, Peter Fitton and James McNatt clarify some of the commonly misunderstood issues associated with interest rate models. Specifically, they focus on (1) the choice between an arbitrage-free and an equilibrium model and (2) the choice between risk neutral and realistic parameterizations of a model. Based on these choices, they classify interest rate models into four categories and then explain the proper use of each category of interest rate model.

Stochastic differential equations (SDE) are typically used to model interest rates. In a one-factor model, an SDE is used to represent the short rate; in two-factor models an SDE is used for both the short rate and the long rate. In Chapter 3 Gerald Buetow, James Sochacki, and I review no-arbitrage interest rate models highlighting some significant differences across models. The most significant differences are those due to the underlying distribution and, as we stress in the chapter, indicates the need to calibrate models to the market prior to their use. The models covered are the Ho-Lee model, the Hull-White model, the Kalotay-

Williams-Fabozzi model, and the Black-Derman-Toy model. The binomial and trinomial formulations of these models are presented.

Moorad Choudhry presents in Chapter 4 an accessible account of the various term structure theories that have been advanced to explain the shape of the yield curve at any time. While no one theory explains the term structure at all times, a combination of two of these serve to explain the yield curve for most applications.

In Chapter 5, David Audley, Richard Chin, and Shrikant Ramamurthy review the approaches to term structure modeling and then present an eclectic mixture of ideas for term structure modeling. After describing some fundamental concepts of the term structure of interest rates and developing a useful set of static term structure models, they describe the approaches to extending these into dynamic models. They begin with the discrete-time modeling approach and then build on the discussion by introducing the continuous-time analogies to the concepts developed for discrete-time modeling. Finally, Audley, Chin, and Ramamurthy describe the dynamic term structure model.

The swap term structure is a key benchmark for pricing and hedging purposes. In Chapter 6, Uri Ron details all the issues associated with the swap term structure derivation procedure. The approach presented by Ron leaves the user with enough flexibility to adjust the constructed term structure to the specific micro requirements and constraints of each primary swap market.

There have been several techniques proposed for fitting the term structure with the technique selected being determined by the requirements specified by the user. In general, curve fitting techniques can be classified into two types. The first type models the yield curve using a parametric function and is therefore referred to as a parametric technique. The second type uses a spline technique, a technique for approximating the market discount function. In Chapter 7, Rod Pienaar and Moorad Choudhry discuss the spline technique, focussing on cubic splines and how to implement the technique in practice.

Critical to an interest rate model is the assumed yield volatility or term structure of yield volatility. Volatility is measured in terms of the standard deviation or variance. In Chapter 8, Wai Lee and I look at how to measure and forecast yield volatility and the implementation issues related to estimating yield volatility using observed daily percentage changes in yield. We then turn to models for forecasting volatility, reviewing the latest statistical techniques that can be employed.

The three chapters in Section Two explain how to quantify fixed-income risk. Factor models are used for this purpose. Empirical evidence indicates that the change in the level and shape of the yield curve are the major source of risk for a fixed-income portfolio. The risk associated with changes in the level and shape of the yield curve are referred to as term structure risk. In Chapter 9, Robert Kuberek reviews some of the leading approaches to term structure factor modeling (arbitrage models, principal component models, and spot rate and functional models), provides the examples of each type of term structure factor model, and explains the advantages and disadvantages of each.

While the major source of risk for a fixed-income portfolio is term structure risk, there are other sources of risk that must be accounted for in order to assess a portfolio's risk profile relative to a benchmark index. These non-term structure risks include sector risk, optionality risk, prepayment risk, quality risk, and volatility risk. Moreover, the risk of a portfolio relative to a benchmark index is measured in terms of tracking risk. In Chapter 10, Lev Dynkin and Jay Hyman present a multi-factor risk model that includes all of these risks and demonstrates how the model can be used to construct a portfolio, rebalance a portfolio, and control a portfolio's risk profile relative to a benchmark.

A common procedure used by portfolio and risk managers to assess the risk of a portfolio is to shift or "shock" the yield curve. The outcome of this analysis is an assessment of a portfolio's exposure to term structure risk. However, there is a wide range of potential yield curve shocks that a manager can analyze. In Chapter 11, Bennet Golub and Leo Tilman provide a framework for defining and measuring the historical plausibility of a given yield curve shock.

Section Three covers the approaches to valuation and the measurement of option-adjusted spread (OAS). Valuation models are often referred to as OAS models. In the first chapter of Section III, Chapter 12, Philip Obazee explains the basic building blocks for a valuation model.

In Chapter 13, Andrew Kalotay, Michael Dorigan, and I demonstrate how an arbitrage-free interest rate lattice is constructed and how the lattice can be used to value an option-free bond. In Chapter 14, we apply the lattice-based valuation approach to the valuation of bonds with embedded options (callable bonds and putable bonds), floaters, options, and caps/floors. In Chapter 15, Gerald Buetow and I apply the lattice-based valuation approach to value forward start swaps and swaptions. A methodology for applying the lattice-based valuation approach to value path-dependent securities is provided by Douglas Howard in Chapter 16.

The Monte Carlo simulation approach to valuing residential mortgage-backed securities—agency products (passthroughs, collateralized mortgage obligations, and mortgage strips), nonagency products, and real-estate backed asset-backed securities (home equity loan and manufactured housing loan-backed deals) is demonstrated by Scott Richard, David Horowitz, and me in Chapter 17. An alternative to the Monte Carlo simulation approach for valuing mortgage products is presented in Chapter 18 by Alexander Levin. The approach he suggests uses low-dimensional grids.

In the last chapter, Chapter 19, the effect of mean reversion on the value of a security and the option-adjusted spread is discussed by David Audley and Richard Chin.

I believe this book will be a valuable reference source for practitioners who need to understand the critical elements in the valuation of fixed-income securities and interest rate derivatives and the measurement of interest rate risk.

I wish to thank the authors of the chapters for their contributions. A book of this type by its very nature requires the input of specialists in a wide range of technical topics and I believe that I have assembled some of the finest in the industry.

Frank J. Fabozzi

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contents

SECTION
one

Interest Rate and Term Structure Modeling

CHAPTER 1

Interest Rate Models

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An interest rate model is a probabilistic description of the future evolution of interest rates. Based on today's information, future interest rates are uncertain: An interest rate model is a characterization of that uncertainty. Quantitative analysis of securities with rate dependent cash flows requires application of such a model in order to find the present value of the uncertainty. Since virtually all financial instruments other than default- and option-free bonds have interest rate sensitive cash flows, this matters to most fixed-income portfolio managers and actuaries, as well as to traders and users of interest rate derivatives.

For financial instrument valuation and risk estimation one wants to use only models that are arbitrage free and matched to the currently observed term structure of interest rates. "Arbitrage free" means just that if one values the same cash flows in two different ways, one should get the same result. For example, a 10-year bond putable at par by the holder in 5 years can also be viewed as a 5-year bond with an option of the holder to extend the maturity for another 5 years. An arbitrage-free model will produce the same value for the structure viewed either way. This is also known as the *law of one price.* The term structure matching condition means that when a default-free straight bond is valued according to the model, the result should be the same as if the bond's cash flows are simply discounted according to the current default-free term structure. A model that fails to satisfy either of these conditions cannot be trusted for general problems, though it may be usable in some limited context.

For equity derivatives, lognormality of prices (leading to the Black-Scholes formula for calls and puts) is the standard starting point for option calculations. In the fixed-income market, unfortunately, there is no equally natural and simple assumption. Wall Street dealers routinely use a multiplicity of models based on widely varying assumptions in different markets. For example, an options desk most likely uses a version of the Black formula to value interest rate caps and floors, implying an approximately lognormal distribution of interest rates. A few feet away, the mortgage desk may use a normal interest rate model to evaluate their passthrough and CMO durations. And on the next floor, actuaries may use variants of both types of models to analyze their annuities and insurance policies.

It may seem that one's major concern in choosing an interest rate model should be the accuracy with which it represents the empirical volatility of the term structure of rates, and its ability to fit market prices of vanilla derivatives such as at-the-money caps and swaptions. These are clearly important criteria, but they are not decisive. The first criterion is hard to pin down, depending strongly on what historical period one chooses to examine. The second criterion is easy to satisfy for most commonly used models, by the simple (though unappealing) expedient of permitting predicted future volatility to be time dependent. So, while important, this concern doesn't really do much to narrow the choices.

A critical issue in selecting an interest rate model is, instead, ease of application. For some models it is difficult or impossible to provide efficient valuation algorithms for all financial instruments of interest to a typical investor. Given that one would like to analyze all financial instruments using the same underlying assumptions, this is a significant problem. At the same time, one would prefer not to stray too far from economic reasonableness—such as by using the Black-Scholes formula to value callable bonds. These considerations lead to a fairly narrow menu of choices among the known interest rate models.

The organization of this chapter is as follows. In the next section I provide a (brief) discussion of the principles of valuation algorithms. This will give a context for many of the points made in the third section, which provides an overview of the various characteristics that differentiate interest rate models. Finally, in the fourth section I describe the empirical evidence on interest rate dynamics and provide a quantitative comparison of a family of models that closely match those in common use. I have tried to emphasize those issues that are primarily of interest for application of the models in practical settings. There is little point in having the theoretically ideal model if it can't actually be implemented as part of a valuation algorithm.

VALUATION

Valuation algorithms for rate dependent contingent claims are usually based on a risk neutral formula, which states that the present value of an uncertain cash flow at time T is given by the average over all interest rate scenarios of the scenario cash flow divided by the scenario value at time T of a money market investment of \$1 today.[1] More formally, the value of a security is given by the expectation (average) over interest rate scenarios

$$P = E\left[\sum_i \frac{C_i}{M_i}\right] \tag{1}$$

where C_i is the security's cash flows and M_i is the money market account value at time t_i in each scenario, calculated by assuming continual reinvestment at the prevailing short rate.

The probability weights used in the average are chosen so that the expected rate of return on any security over the next instant is the same, namely the short rate. These are the so-called "risk neutral" probability weights: They would be the true weights if investors were indifferent to bearing interest rate risk. In that case, investors would demand no excess return relative to a (riskless) money market account in order to hold risky positions—hence equation (1).

It is important to emphasize that the valuation formula is not dependent on any *assumption* of risk neutrality. Financial instruments are valued by equation (1) *as if* the market were indifferent to interest rate risk *and* the correct discount factor for a future cash flow were the inverse of the money market return. Both statements are false for the real world, but the errors are offsetting: A valuation formula based on probabilities implying a nonzero market price of interest rate risk and the corresponding scenario discount factors would give the same value.

There are two approaches to computing the average in equation (1): by direct brute force evaluation, or indirectly by solving a related differential equation. The brute force method is usually called the Monte Carlo method. It consists of generating a large number of possible interest rate scenarios based on the interest rate model, computing the cash flows and money market values in each one, and averaging. Properly speaking, only path generation based on random numbers is a Monte Carlo method. There are other scenario methods—e.g., complete sampling of a tree—that do not depend on the use of random numbers.

[1] The money market account is the *numeraire*.

Given sufficient computer resources, the scenario method can tackle essentially any type of financial instrument.[2]

A variety of schemes are known for choosing scenario sample paths efficiently, but none of them are even remotely as fast and accurate as the second technique. In certain cases (discussed in more detail in the next section) the average in equation (1) obeys a partial differential equation—like the one derived by Black and Scholes for equity options—for which there exist fast and accurate numerical solution methods, or in special cases even analytical solutions. This happens only for interest rate models of a particular type, and then only for certain security types, such as caps, floors, swaptions, and options on bonds. For securities such as mortgage passthroughs, CMOs, index amortizing swaps, and for some insurance policies and annuities, simulation methods are the only alternative.

MODEL TAXONOMY

The last two decades have seen the development of a tremendous profusion of models for valuation of interest rate sensitive financial instruments. In order to better understand these models, it is helpful to recognize a number of features that characterize and distinguish them. These are features of particular relevance to practitioners wishing to implement valuation algorithms, as they render some models completely unsuitable for certain types of financial instruments.[3] The following subsections enumerate some of the major dimensions of variation among the different models.

One- versus Multi-Factor

In many cases, the value of an interest rate contingent claim depends, effectively, on the prices of many underlying assets. For example, while the payoff of a caplet depends only on the reset date value of a zero coupon bond maturing at the payment date (valued based on, say, 3-month LIBOR), the payoff to an option on a coupon bond depends on the exercise date values of all of the bond's remaining interest and principal payments. Valuation of such an option is in principle an inherently multidimensional problem.

Fortunately, in practice these values are highly correlated. The degree of correlation can be quantified by examining the covariance matrix of

[2] This is true even for American options. For a review see P. Boyle, M. Broadie, and P. Glasserman, "Monte Carlo Methods for Security Pricing," *Journal of Economic Dynamics and Control* (1997), pp. 1267–1322.

[3] There is, unfortunately, a version of Murphy's law applicable to interest rate models, which states that the computational tractability of a model is inversely proportional to its economic realism.