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BENFORD'S LAW

Applications for Forensic Accounting, Auditing, and Fraud Detection

MARK J. NIGRINI, PhD

FOREWORD BY JOSEPH T. WELLS

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Benford's Law

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To the Hutchinson Family:
Martha, Paul, Kevin, Jim, Tom IV, Peter, and Martha-Ann.
Thanks for believing in me and my Benford's Law work all those years ago
in Cincinnati, Ohio, and for all your continued support and encouragement until now.

Foreword

NUMBERS. DO THEY TELL the truth? Back in my own Dark Ages, I was struggling through calculus in college, simply trying to grasp it. (That didn't happen.) My professor was an Asian fellow with a very thick accent. He reminded our class frequently that "mathematics is . . . leality." The professor had as much trouble pronouncing his *r*'s as I had in understanding abstract math.

But I was struck with the fact that calculus could be principally credited to one man, Sir Isaac Newton (1642–1727), to assist in addressing the issue of planetary motion. In short, when the mathematics of the era did not solve the problem, Newton helped invent one that did. Although some of his early work has been superseded (most notably by Einstein and his theory of general relativity), the majority of it has stood the test of time for five centuries. The reason is that numbers don't lie; Newton's didn't, and neither do Benford's.

However, fraudsters falsely use numbers to give credibility to their multitudinous scams. And Frank Benford (1883–1948) inadvertently gave us a way to catch many of these crooks. As you will read in the following pages, Benford was a brilliant if somewhat eccentric engineer whose hobby from an early age was mathematics. (That alone should prove he was a bit odd; as a child, my hobby was collecting baseball cards.)

Benford's Law essentially states that the sequence of multiple numbers from real-life sources is likely to be distributed in a specific, nonuniform way. This counterintuitive result can apply to disparate lists such as stock prices, death rates, population numbers, and stock prices, to name just a few.

Why Frank Benford had the interest to pursue this topic is anyone's guess and perhaps is lost to history. There is no question his hobby was time consuming. In an era before computers or even calculators, he spent untold hours doing more than 20,000 calculations entirely by hand. One thing is for sure: Benford's fascination with mathematics had little to do with developing a method to detect fraud. However, it works. By the same token, inventing Teflon was not related to its application for nonstick cookware. But it works too.

When it comes to fraud, I know more than a little bit; detecting and preventing it has been my life's work. After two years of toiling in the ledgers of an auditing firm to fulfill my requirements to be a CPA, I was appointed a Special Agent of the U.S. Federal Bureau of Investigation. Over the next decade, I investigated a plethora of fraud schemes, ranging from nickel-and-dime con artists to Watergate.

These investigations were done like Frank Benford's early calculations—the hard way. In my case, it was with much shoe leather and lots of pencils and paper. I knew

from long experience that fraudsters often make up numbers out of thin air. After investigating fraud for a living, I became chairman of the Association of Certified Fraud Examiners (ACFE), the world's largest antifraud organization with more than 60,000 members in 140 nations.

Enter Mark Nigrini, ACFE member. He took Benford's original data and developed it into solid analytical methods to detect a variety of white-collar misdeeds: vendor, payroll, and sales tax frauds; fraudulent medical and insurance claims; check fraud and tax evasion, among others. In other words, Dr. Nigrini has put Benford's Law on the map. With the passage of time, he and others will find additional useful applications for this powerful tool.

As you read the following pages, do not be daunted if you aren't a mathematician in the vein of Benford or Nigrini; you can still tell time without knowing how to build a watch. The important thing is to understand enough to apply these techniques to detect and deter fraud. And by doing so, you are helping make the world a better place.

Dr. Joseph T. Wells, CFE, CPA
Austin, Texas
February 2012

Preface

IF WE TOOK THE payment amounts for a refinery, would you guess that the first digit proportions were absolutely random from month to month? So, in one month we could have 20 percent of all numbers starting with a *7* and in the next month 5 percent of all numbers starting with a *7*. Or would you guess that the digit patterns are stable with the proportions being specific to *that* refinery? The last option is that the digit patterns are stable at (say) 5.8 percent first digit *7*'s per month, and this proportion is the *same* for *all* refineries. The last possibility means that the proportions are not only the same for all refineries, irrespective of the oil company, but the same for all refineries, for all oil companies, and for all countries. The last scenario is the true scenario in that the digit proportions are generally the same across all refineries and across all companies. The surprising fact is that the digit *1* is expected to occur a little more than six times as often as *9* as a first digit.

My excursion into the world of digit frequencies began after reading about Benford's Law in a statistical decision theory course in 1989 during my PhD coursework at the University of Cincinnati. On page 86 the author noted, almost in passing, that if you believed that the digits were equally likely, this would be a noninformative prior and also an improper prior because of what he called the "table entry" problem. James Berger, the author of the textbook, went on to show that the correct expectation of the digits was Benford's Law, and he called this fact "intriguing."

The digit probabilities fascinated me, and I rushed to the library and photocopied the 1938 paper written by Frank Benford. It was April 1989, and I read the article from start to finish over and over again in a mild disbelief. Benford set out in precise terms the expected single digit and digit combination frequencies for lists of data. These expected frequencies are now known as Benford's Law. It occurred to me that auditors might be able to use the formulas while auditing client data. PhD students were often exposed to papers that found a technique in the literature and suggested using the technique in an auditing context. The author always talked about the paper being an exploratory study and the data analyzed, or subjects used, or the settings were less complex than anything in reality. But I saw that these papers got published.

I had an idea, but my 286 computer could analyze only 2,000 numbers at a time, and then very slowly. With such a "large" data set, I would have to continually change formulas to values to keep the spreadsheet size within the limits of my 2 megabytes of random access memory. Mainframes were very difficult to use, and the possibility of loading corporate data onto the mainframe and analyzing it was impossible for two reasons. No Cincinnati company would part with its data for an exploratory study by an

unknown student. Even with the data, the university computer people would not go to any trouble for a *student.* I wasn't even an untenured assistant professor.

So I had to at least temporarily forget about anything practical. I focused on satisfying myself that if the digits actually had stable patterns, these patterns would be the patterns of Benford's Law. Since I could not prove Benford's Law, I took the line that I should find out what others thought. I went to the library and worked to find *every* published article on Benford's Law. Fortune was on my side. There were several papers by mathematicians and statisticians. I contacted as many people as I could who had written a paper on the subject. Ralph A. Raimi was then the most cited author in the field. His 1969 *Scientific American* paper listed his affiliation as being the University of Rochester. It was now October 1989. Would he still be there? I called and the secretary told me that Professor Raimi was still there. I left a message. Me, an insignificant *student*, left a message for *the* Professor Raimi. In the hierarchy of academia, students rank below the bottom of the totem pole. Professors who have *published* in anything with a cover on it avoid all forms of direct contact unless you hold some promise of writing a paper with (actually "for") them.

Professor Raimi soon returned the call. I was so nervous my voice was breaking. I explained that I was looking at Benford's Law and thought that auditors could use it. I asked him whether Benford's Law was true or not. Professor Raimi told me that "it's not a mirage." That was all I needed to jump for joy that the past six months were not wasted. Over the course of the next few years, Professor Raimi was a pillar of strength. He reviewed my papers. They often came back in the mail with more red ink than typed words. I worked longer and harder each time to reduce the Raimi red ink. Later on there were often some unmarked pages.

One morning in 1990 I spoke to a tax professor, Jeff Patterson, who told me that the accounting department owned a database of 100,000 tax returns. A breakthrough! We had numbers to analyze. By the end of 1990, I had planned a dissertation titled "A Statistical Approach to the Detection of Income Tax Evasion." There was more good news in that the Internal Revenue Service research department in Washington, DC, would allow me to present my research in its offices. I made the presentation in December 1990. I showed lots of graphs of the first and second digits of taxpayer numbers for various tax fields (e.g., AGI and balance due). The University of Cincinnati's College of Business paid for my Greyhound bus trip and motel. I forgot my neat shoes at home and only realized it that morning. In downtown Washington, DC, there are no stores. I gave the presentation in a suit and sneakers. I hope no one noticed. I thought that if David Letterman could wear a suit and sneakers, so too could I. The IRS showed me the same enthusiasm that anyone else would to a brand-new idea based on a law that I probably didn't fully understand at that time. Nothing further ever came of that meeting.

In early 1991 I had quite a few graphs of taxpayer data. I knew that page after page of first and second digit graphs would not cut it as a PhD dissertation. John Bryant of the Quantitative Analysis department told me that I needed a mathematical model of how people cheat and how this would be detected by Benford's Law. He wrote his notes on a napkin during a Friday evening social get-together for PhD students at a bar on Calhoun Street. I needed to show how exactly people cheat and how exactly Benford's

Law would pick this up. Then I would have a dissertation. For months I wrestled with this problem and for months I saw a Benford's Law dissertation slipping away. I had a contingency plan. I would do a dissertation on how to measure complexity. What makes something complex? Was it time taken, errors made, and mental fatigue? The excitement of unraveling this for auditors put me to sleep. In fact, I usually yawned when I told people about this idea.

One day while walking in the parking garage from the library to Lindner Hall, I asked myself how the sums of numbers with similar first digits would compare. In a data set that followed Benford's Law, was the sum of all the numbers with a first digit *1* smaller or larger than the sum of all numbers with a first digit *2*? Was the first digit *2* sum bigger or smaller than the first digit *3* sum? That evening I walked home wondering about this relationship. I fired up my computer, made a set of 1,000 numbers that followed Benford's Law, and set about getting the machine to calculate the sums of the numbers with similar first digits. The sums turned out to be equal! My mind raced. If people make numbers bigger, a first digit *3* might become a first digit *9*. The sum of the 9s (a high digit) would be more than the sum of the *3*s (a low digit). If they reduced a number from (say) 829 to 229, the sums of the *8*s (a high digit) would be less than the sum of the *2*s. Within weeks I developed something that I called *bias*. Amit Raturi told me in a seminar that the term *bias* has a special meaning to statisticians and I need to change the name. So, the only real criticism of a statistics professor was the name that I had given to the formula. The distortion factor model was then born. This was the theoretical advance that would allow me to do a Benford's Law dissertation.

I will always owe Marty Levy a huge debt of gratitude for all those Friday afternoons that we spent at Denny's going over the results and the models. Not only did he give up his time, but he paid for his own coffee. Denny's charged about a dollar for a cup and, with refills, we probably drank a pot each. The models were so complex that my advisor, Wallace Wood, insisted that a statistics professor formally vet each and every statistical move in the analysis.

I submitted my proposal in May 1992. In August 1992 (three months later), I defended my dissertation, titled "The Detection of Income Tax Evasion through an Analysis of Digital Distributions." That summer I worked through the nights to 7:00 A.M. each morning watching CBS News rerunning the same stories through the night. At 7:00 I would leave to go to the department to laser print my work of that night. At about 9:00 I would go to sleep until about 3:00 P.M., when it was back to the digits. They knew me by name in the mainframe control center and in the printout room. Most of the floor space of my house was covered with oversize green-lined printouts. The final product on the digits used by taxpayers was 309 pages long.

Then came the drought of 1993. I couldn't disseminate the technology. It didn't even have a name. At Wayne State University, my friend and colleague, Tom Buttross, would smile every time that I told him how much time I was spending spreading the digital word. I summarized my overly complex dissertation to a 50-page paper that was supposed to impress readers. Perhaps they stopped reading after page 1. With hindsight, I shouldn't be surprised. I had no software and no experience to speak of. Everything was absolutely new.

In August 1993 I moved to Halifax to take up a position at Saint Mary's University. In March 1994 I published a short piece in *The White Paper*. In the summer of 1994 the Dutch Ministry of Finance agreed to a seminar on Benford's Law at its offices. I was elated. At their offices in The Hague we used some SAS programs, and found that some of the Dutch tax data followed Benford's Law closely. Also that summer I worked with Linda Mittermaier to analyze the tax returns of President Clinton. His returns had been the subject of much publicity, and I wondered whether his numbers followed Benford's Law. For two years those Clinton graphs were the only Benford results in my presentations. In December 1994 I visited a company called Trelleborg AB in Sweden. I paid for my own airfare. It was a start. I gave a presentation to Arthur Andersen in Malmo and also to KPMG in Copenhagen. Both were interested, but I had no software and no documentation and no experience.

A Halifax journalist wrote a story on my work called "To Catch a Cheat" and also had a story published in the May 1995 issue of *CA Magazine*. The articles brought in a few information requests from the mathematically curious. It was, however, enough to pique the interest of a reporter at the *Wall Street Journal* (*WSJ*).

Since 1993 I had regularly written to writers at the *WSJ*. Every time I saw an article that had the word *accounting* or *audit* in it, I would call or write to the journalist. I had no luck for a long time. Then in April 1995 Lee Berton returned my call. In May he called me again, and for two hours I went through the toughest grilling of my life. He questioned me from left to right and upside down and right side up. After the call was over I sat on my sofa and asked myself who I thought I was that I would be important to the finest newspaper in the United States. I now know that our phone conversation would not have lasted for two hours if my story was weak. Within two weeks I traveled to New York City, and Lee Berton and I had coffee together. He told me that my digital work was quite interesting, but he could not guarantee a story. By then I'd done some work with Bob Burton of the Brooklyn District Attorney's Office. Two companies listed on the New York Stock Exchange, AIG and ARMCO, had also shown an interest in my work. Things were moving very slowly. I was 37 years old. At this rate I could see that I might be in my 50s, long-faced and disillusioned, before anything digital happened in professional circles.

In July 1995, while I was doing the taxpayer analysis work in The Hague, our department secretary, Cathy Golden, sent me a note that the *WSJ* had published an article about my work and calls were coming in rapidly. The number of calls that came in that first morning (about 20) was more than I had received about Benford's Law in my life. I cut short my European trip and went back to Halifax. Calls were coming in faster than I could answer them. I couldn't even listen to my messages without the system telling me that another call had been added to my mailbox.

The article led to more interviews and these to yet more interviews in the press, radio, and TV. Several people wanted to make some money by marketing my product. The most important call came one month later from Ernst & Young in Cleveland, Ohio. In 1997 I left academia for a stint with Ernst & Young working on including Benford's Law as a part of the firm's new audit methodology.

The previous four pages are the (slightly edited) preface of my 1997 self-published book on Benford's Law. In 2000 a slightly edited version of that book was published by

Global Audit Publications. A few years later the recession of the early 2000s caused the company to its doors.

It is now October 2011, and Benford's Law has moved ahead in leaps and bounds from the 1990s. A Google search for *Benford's Law* shows about 40,000 hits. I remember when my searches in the mid-1990s using search engines such as Yahoo and AltaVista returned 300 hits of which only 100 were actual Benford's Law hits. Benford's Law has been included as audit routines in IDEA and ACL, two audit software platforms used by internal and external auditors throughout the world. Benford's Law is regularly used by internal auditors at large reputable public companies and by large and small external auditing firms. Benford's Law is also used by forensic accountants and fraud examiners. Personal computers have improved to the stage where we can now analyze 20 million records with relative ease, and Microsoft Excel has also helped out because we can now use its 1,048,576 rows to analyze numeric data.

The most profound developments have been in the academic publications arena. The online database www.benfordonline.net lists about 750 published papers on Benford's Law. I estimate that we'll have 1,500 papers by the end of 2015. In 1975 there were only 50 published papers on Benford's Law and by 2000 there were about 150 papers. Academic researchers and finance professionals can now write articles about Benford's Law without wondering whether it really is true. By 1975 only one article had been published in a publication with a wide circulation (*Scientific American*). By now we've had several articles published in publications with large circulations, such as *USA Today*, the *WSJ*, and the *Journal of Accountancy*. There have been solid developments on the mathematical frontier, and in 2011 Berger and Hill published their 126-page "Basic Theory of Benford's Law." (Note that the number of pages has a first digit *1*.) This theory paper has a one-and-a-half-page summary of the symbols, some 60 references, and 25 figures. Imagine how long it would be if it wasn't a *basic* theory of Benford's Law. The first Benford's Law conference was held in December 2007 in Santa Fe, New Mexico. These developments have all exceeded my most optimistic expectations.

I have enjoyed writing this book aimed at professionals and academics who plan to use Benford's Law as a test of data integrity and authenticity. Readers do not need a knowledge of advanced statistics. This book has some refresher paragraphs and references where needed. Ideally, though, most users will have some experience in obtaining transactional data and with the basic concepts of data analysis, such as working with tables, combining (appending) or selecting (extracting subsets) data, and performing calculations across rows or down columns. Users should understand the basics of either Excel or Access. Chapters 1 to 3 of the book set out the mathematical foundations of Benford's Law. Chapters 4 to 7 describe the primary, advanced, and associated Benford's Law tests. These chapters include demonstrations of how to run the tests in Excel and Access. Chapter 8 describes a potpourri of interesting fraud findings. Chapters 9 to 12 describe applications of Benford's Law, including the detection of fraudulent financial reporting and the detection of Ponzi schemes. Chapter 13, the final chapter, describes where we are now with Benford's Law and sets an agenda for future research that focuses on the My Law concept, how to identify invented numbers, and makes some suggestions for future Benford's Law research. This chapter, like the others,

includes some interesting data including the charitable gift donations made by Lehman Brothers just prior to bankruptcy.

I look forward to having even more progress to report for the next edition of this book, given the ease with which data can now be accessed, downloaded, and processed. I hope that the main thrust going forward in the literature is data-driven papers. These papers would show either conformity or nonconformity for various types of financial, scientific, and simulated data (e.g., data from a log-normal distribution). The true contributions will come from papers that have detected some form of irregularity by using Benford's Law.

The companion site for the book is www.nigrini.com/benfordslaw.htm. The Web site includes many of the data sets used in the book. Users can run the tests on the same data and check their results against the results shown in the book. The Web site also includes Excel templates that will make your results exactly match the results in the book. The templates were prepared in Excel 2007. Over time, more sections will be added to the Web site. These might include links to useful resources and updates to sections in the book.

In a letter to me dated August 19, 1993, Harry Benford (a son of Frank Benford) wrote:

> I don't recall much about my father's paper on anomalous numbers except that he was led to his investigation after noting that his log tables were dirtier in way of the low digits. I also recall that he enjoyed telling friends about his findings.

Thank you Frank Benford, for your wonderful discovery ☺. It has been a special part of my life now for many years. Thanks too to the Benford family for the Frank A. Benford memorabilia and a special thanks to Jim Benford and Frank Benford (a grandson of FAB), whom I write to from time to time. Like Frank Benford, I still enjoy telling friends, students, colleagues, and seminar attendees about my findings, and I've enjoyed writing this book for you. It's been a 110,000-word trip down the digital highway (note the first digit *1*). Without Benford's Law I'd still be busy trying to figure out what makes something complex.

I am thankful to my PhD dissertation committee Marty Levy, Linda Mittermaier, Amit Raturi, Tim Sale, and Wally Wood for their guidance and supervision of my Benford-based dissertation that was then a move into uncharted waters. Special thanks are also due to my small circle of Benford-related colleagues who kept me going when there were only a few believers. These include Ralph A. Raimi and Ted Hill and more recently Steven Miller of Williams College. I'm also grateful to the first internal audit directors, Jim Adams, Bob Bagley, and Steve Proesel, who used my forensic analytic services in the mid-1990s. I needed their vote of confidence to keep going. Final thanks go to the Wiley professionals, Timothy Burgard, Stacey Rivera, and Stefan Skeen, who worked hard to turn my manuscript into a quality finished product.

Mark J. Nigrini, PhD
Pennington, New Jersey,
October, 2011

About the Author

MARK J. NIGRINI, PHD, IS an associate professor at The College of New Jersey in Ewing, New Jersey, where he teaches auditing and forensic accounting. He has also taught at other institutions, including Southern Methodist University in Dallas, Texas.

Mark is a Chartered Accountant and holds a B.Com. (Hons) from the University of Cape Town and an MBA from the University of Stellenbosch. His PhD in accounting is from the University of Cincinnati, where he discovered Benford's Law. His dissertation was titled "The Detection of Income Tax Evasion through an Analysis of Digital Distributions." His minor was in statistics, and some of the concepts studied in those statistics classes are used in this book.

Mark is the author of *Forensic Analytics*, published by Wiley in 2011. *Forensic Analytics* reviews and describes substantive and rigorous tests that are used to detect fraud, errors, estimates, or biases in corporate and government data. The tests include high-level data overviews together with highly focused tests that give small samples of highly suspicious transactions. *Forensic Analytics* also shows how Access, Excel, and PowerPoint can be used in a forensic setting.

After Mark finished his dissertation, it took a while for corporate America to notice his work. The breakthrough came in 1995 when his work was publicized by an article titled "He's Got Their Number: Scholar Uses Math to Foil Financial Fraud" in the *Wall Street Journal.* This was followed by other articles on his work in the national and international media, including the *Financial Times*, the *New York Times*, *Der Spiegel*, *Businessweek*, and *USA Today.* A recent article that discussed Mark's forensic work was published in Canada's *Globe and Mail*, and he was also recently cited as a Benford's Law "expert" in the *Frankfurter Allgemeine Zeitung.* Mark's radio interviews have included the BBC in London and NPR in the United States. His television interviews have included an appearance on NBC's *Extra.*

Mark has published papers on Benford's Law, auditing, and accounting in academic journals, such as the *Journal of the American Taxation Association*, *Auditing: A Journal of Practice and Theory*, the *Journal of Accounting Education*, the *Review of Accounting and Finance*, the *Journal of Forensic Accounting*, and the *Journal of Emerging Technologies in Accounting.* He has also published in scientific journals, such as *Mathematical Geology*, and pure mathematics journals, such as the *International Journal of Mathematics* and *Mathematical Sciences.* Mark has also published articles in practitioner journals, such as *Internal Auditor* and the *Journal of Accountancy.* Mark's current research deals with

advanced theoretical work on Benford's Law and the legal process surrounding fraud convictions.

Mark has presented many academic and professional seminars for accountants in the United States and Canada with the audiences primarily made up of internal auditors, external auditors, and forensic accountants in the public and private sectors. He has presented several conference plenary and keynote sessions with his talk titled "Benford's Law: The Facts, the Fun, and the Future." The release date of *Benford's Law* is planned to coincide with a keynote session to be delivered by Mark at the 2012 Annual Williamsburg Fraud Conference in April, 2012. Mark has also presented seminars overseas in the United Kingdom, the Netherlands, Germany, Luxembourg, Sweden, Thailand, Malaysia, Singapore, and New Zealand. A PWC presentation in Zurich, Switzerland, is scheduled for May, 2012. Mark is also scheduled to present Benford's Law sessions in June, 2012 at the 2012 Annual ACFE Fraud Conference, and the 2012 NACVA Annual Consultants' Conference. Mark is available for seminars and presentations, and he can be contacted at ForensicAnalytics@gmail.com. Other contact information is given on his Web site, www.nigrini.com.

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