

LECTURE NOTES
IN PHYSICS

Kang-Sin Choi
Jihn E. Kim

Quarks and Leptons From Orbifolded Superstring



Springer

Kang-Sin Choi Jihn E. Kim

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Preface

Using the successful standard model of particle physics but without clear guidance beyond it, it is a difficult task to write a physics book beyond the standard model from a phenomenological point of view. At present, there is no major convincing inner space related experimental evidence against the standard model. The neutrino oscillation phenomena can be considered part of it by including a singlet field in the spectrum. Only the outer space observations on matter asymmetry, dark matter, and dark energy hint at the phenomenological need for an extension; however, there has been theoretical need for almost three decades, chiefly because of the gauge hierarchy problem in the standard model.

Thus, it seems that going beyond the standard model hinges on the desirability of resolving the hierarchy problem. At the field theory level, it is fair to say that the hierarchy problem is not as desperate as the nonrenormalizability problem present in the old $V-A$ theory of weak interactions on the road to the standard model. An extension beyond the standard model can easily be ruled out as witnessed in the case of technicolor. However, a consistent framework with supersymmetry for a resolution of the hierarchy problem has been around for a long time. Even its culprit “superstring” has been around for twenty years, and the most remarkable thing about this supersymmetric extension is that it is still alive. So the time is ripe for phenomenologists to become acquainted with superstring and its contribution toward the minimal supersymmetric standard model in four space-time dimensions.

This book is a journey toward the minimal supersymmetric standard model (MSSM) down the orbifold road. After some field theoretic orbifold attempts in recent years, there has been renewed interest in the physics of string orbifolds and it is time to revisit them. In this book, we take the viewpoint that the chirality of matter fermions is essential toward revealing the secrets of Nature. Certainly, orbifolds are an easy way to get the chirality from higher dimensions.

Strings and their orbifold compactification are presented for the interests of phenomenologists, sacrificing mathematical rigor. They are presented in

such a way that an orbifold model can be constructed by applying the rules included here. At the end of Chap. 10, we construct a \mathbf{Z}_{12} orbifold which contains all imaginable complications. Also, we attempt to correct any incompleteness in the rules presented before in the existing literature. In the final chapter we tabulate the simplest and most widely used orbifold \mathbf{Z}_3 with $\mathcal{N}=1$ supersymmetry, completely in the phenomenological sense of obtaining three families. These tables encompass all noteworthy models available with two Wilson lines. Since three Wilson line \mathbf{Z}_3 orbifolds do not automatically give three families, in a practical manner these tables close a chapter on \mathbf{Z}_3 orbifolds.

This book is not as introductory as a textbook, nor is it as special as a review article on a superstring topic. Instead, we aimed at an interim region so that a phenomenologist can read and directly commence building an orbifold model.

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Seoul
November, 2005

Kang-Sin Choi
Jihn E. Kim

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Introduction and Summary

During and since the second half of the twentieth century, enormous progress has been made in understanding our universe in terms of fundamental particles and their interactions, namely in the language of quantum field theory. The advent of the standard model (SM) of particle physics has been the culmination of quantum field theory in all its full glory. The dawn of this successful particle physics era was opened with the unexpected discovery of *parity violation* in weak interaction phenomena [1]. It had long been known that weak interactions change the electromagnetic charge, i.e. electron (e) to electron type neutrino (ν_e), neutron (n) to proton (p). But, until the mid-1950s it had never occurred to the leading minds [2] that “parity might be violated”, chiefly because the atomic and nuclear transitions did not reveal any such possibility before that time. For nuclear transitions, both weak and electromagnetic phenomena contribute but at that time there were not sufficient data to fully conclude on the nature of parity operation in weak interactions [1]. For atomic transitions, the fundamental interaction is of electromagnetic origin and the experimental confirmation of parity conservation in atomic phenomena convinced most physicists that parity is conserved in the universe. In hindsight, parity conservation should have been imposed only on electromagnetic interactions, as the discovery of parity violation in weak interactions started a new era for weak interactions. There is still no experimental evidence that strong and electromagnetic interactions violate parity. Therefore, we know that parity violation in weak interactions is at the heart of making our universe as it is now, because the SM assumes from the outset the existence of massless chiral fields.¹

Soon afterward, the parity-violating weak interactions were neatly summarized as a four fermion (charged current) \times (charged current) weak interaction where the charged current J_μ^{CC} is of the “V–A” type [3].

$$\mathcal{H}_{\text{weak}} = \frac{G_F}{\sqrt{2}} J_\mu^{CC \dagger}(x) J_\mu^{CC}(x) , \quad (1.1)$$

¹ Massless compared to the Planck mass M_P .

The “V–A” charged current of weak interactions indicates two important things: (1) only the left-handed fermions participate in the charge changing weak interactions, and (2) being current, the fundamental interaction at a deeper level may need a vector boson. Here, we note that the chiral nature of weak interactions is still the mystery among all mysteries of particle physics in the search for a fundamental theory at a very high energy scale using the low energy SM. Several years after this effective low energy (current) \times (current) four-fermion interaction was proposed, a modest attempt via a more fundamental interaction through a heavy spin-1 charged intermediate vector boson (IVB) W_μ^\pm was put forth [4]. Its coupling to the charged current was given by

$$\frac{g}{2\sqrt{2}} J_\mu^{CC} W^\mu + \text{h.c.} \quad (1.2)$$

The mass of W_μ was supposed to be heavy so that the four-fermion interaction mediated by the IVB is weak compared to the strong interactions. However, this IVB idea had several problems which have since been resolved by the standard model of particle physics.

A spin-1 field coupling to the fermion current had already been known in electromagnetic interactions, i.e. the photon A_μ coupling to the electromagnetic current through $e\bar{\psi}\gamma^\mu\psi A_\mu$. This electromagnetic interaction can be formulated in terms of U(1) gauge theory [5], where one uses the covariant derivative \mathcal{D}_μ instead of the ordinary partial derivative ∂_μ ,

$$\partial_\mu \longrightarrow \mathcal{D}_\mu \equiv \partial_\mu - ieA_\mu$$

which introduces the minimal gauge coupling of A_μ to charged fields. In quantum mechanics, the additive conservation of electromagnetic charge implies a global U(1) symmetry and generalizing it to a local U(1) leads to the above covariant derivative. This is our first example of how a bigger symmetry might be discovered from a representation of matter, i.e. starting from the electron in the above example of quantum electrodynamics.

Consider the generalization of this gauge principle to the IVB. Since the IVB changes the electromagnetic charge, we must start from a defining state in Hilbert space which contains at least two components differing by one-unit of the electromagnetic charge. This doublet is a kind of matter which, in the doublet representation, necessarily introduces a nonabelian gauge group. This is our second example in which matter can indicate a bigger symmetry. In general, one can introduce the covariant derivative using the nonabelian gauge fields A_μ^i ($i = 1, 2, \dots, N_A$), with the size N_A (e.g. 3 for SU(2)) dependent upon the matter representation. Yang and Mills were the first to show that a consistent construction along this line needs nonlinear couplings between gauge fields.

In the late 1960s the standard model of particle physics was constructed, employing the nonabelian gauge group. The group structure is SU(2) \times U(1) [6], and the covariant derivative is

$$\mathcal{D}_\mu = \partial_\mu - igT^i A_\mu^i - ig'YB_\mu \quad (1.3)$$

where T^i ($i = 1, 2, 3$) are the $SU(2)$ generators and Y is the electroweak hypercharge generator. The Gell-Mann-Nishijima type definition of the electromagnetic charge is $Q_{\text{em}} = T_3 + Y$. All leptons and quarks are put into left-handed doublets and right-handed singlets, and the charged current IVB mediation violates parity symmetry by construction from the outset. For example, the left-handed electron and its neutrino are put into a doublet $l_L = (\nu_e, e)_L^T$, where $L(R)$ represents the left(right)-handed projection $L = \frac{1+\gamma_5}{2}$, or $\psi_L = \frac{1+\gamma_5}{2}\psi$. Since the quarks carry the additional degree called *color* coming in three varieties, the first family $(l_L, e_R, q_L, u_R, d_R)$ contains 15 two-component chiral fields. In addition, these fifteen fields repeat three times, making a total of 45 chiral fields, all of which have been observed in high energy accelerators.

The SM representation is written in such a way that the intermediate vector boson W^+ transforms the lower elements of l and q to their upper elements. For example e_L to ν_{eL} and d_L to u_L , and hence there exists the coupling

$$\frac{g}{\sqrt{2}} \bar{\nu}_e \gamma^\mu \frac{1+\gamma_5}{2} e W_\mu^+.$$

For each representation, we can assign the Y quantum number to match the electromagnetic charges of the fields in the representation through the following formula,²

$$Q_{\text{em}} = T_3 + Y. \quad (1.4)$$

Thus, the standard model is certainly left(L)–right(R) asymmetric in that the interchange $L \leftrightarrow R$ does not give the original representation. This is called a chiral theory.³ In a chiral theory, one cannot write down a mass term for the fermions. Under the SM group $SU(2) \times U(1)$, for example, one cannot write down a gauge invariant mass term for e ($l = \mathbf{2}_{-\frac{1}{2}}, e_R = \mathbf{1}_{-1}$ where the weak hypercharges are written as subscripts in the usual way). The SM is designed such that chiral fermions can obtain mass after the gauge group $SU(2) \times U(1)$ is spontaneously broken down to $U(1)_{\text{em}}$, and then one has to consider only the gauge invariance of the unbroken gauge group $U(1)_{\text{em}}$. This makes it possible to write

$$-m_e \bar{e} e = -m_e (\bar{e}_R e_L + \bar{e}_L e_R).$$

This way of rendering mass to SM chiral fields is assumed throughout this book, and the fundamental question is *how such chiral fields arise in the beginning*. For spontaneous symmetry breaking leading to $G \rightarrow H$,⁴ one needs a singlet member under the Lorentz group and a singlet under the unbroken

² l_L has $Y = -\frac{1}{2}$, e_R has $Y = -1$, q_L has $Y = \frac{1}{6}$, u_R has $Y = \frac{2}{3}$, and d_R has $Y = -\frac{1}{3}$.

³ The converse is not necessarily true: the $SU(2)_L \times SU(2)_R \times U(1)$ model is $L - R$ symmetric but chiral [7].

⁴ In the above example, $G = SU(2) \times U(1)_Y$ and $H = U(1)_{\text{em}}$.

gauge group H , but there should also be a non-singlet under G . In the Hilbert space, such a member as a fundamental field is a neutral scalar transforming nontrivially under both $SU(2)$ and $U(1)$. The simplest such representation is a spin-0 Higgs doublet with $Y = \frac{1}{2}$ [8],

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (1.5)$$

A more complicated mechanism for spontaneous symmetry breaking is the use of a composite field which is a neutral scalar transforming nontrivially under both $SU(2)$ and $U(1)$. The simplest such composite field is one that assumes a new confining force, the so-called *techni-color* confining around the TeV scale, and composites of techni-quarks realize this idea [9]. This neutral scalar component can develop a vacuum expectation value (VEV) which certainly breaks G but leaves H invariant. Breaking the gauge symmetry through the VEV of scalar fields is the *Higgs mechanism* [10]. The SM is a chiral theory based on $SU(2) \times U(1)$ with the above Higgs mechanism employed.

Below the spontaneous symmetry breaking scale, the unbroken gauge symmetry is H , and the gauge bosons of G corresponding to G/H obtain mass of order (gauge coupling) \times (VEV). This process applied to the SM renders three IVB (W^\pm, Z) masses at the electroweak scale: $M_W \simeq 80$ GeV, $M_Z \simeq 91$ GeV. Of course, the photon A_μ remains massless. The origin of the fermion masses in the SM is not from the $SU(2) \times U(1)$ invariant mass term, which cannot be written down anyway, but originates from the gauge invariant Yukawa couplings of the fermions with the spin-0 Higgs doublet. Then, fermion masses are given by (Yukawa couplings) \times (VEV). A variety of fermion masses is attributed to the variety of the Yukawa couplings.

One can glimpse that the essence of the above description of nature in terms of the SM is that *the theory is chiral* until the SM gauge group is spontaneously broken at the electroweak scale of $v \simeq 247$ GeV. Since the fundamental theory may be given at the Planck scale

$$M_P = \frac{1.22 \times 10^{19} \text{ GeV}}{\sqrt{8\pi}} = 2.44 \times 10^{18} \text{ GeV} \quad (1.6)$$

our chief aim in the construction of the SM is to obtain the correct chiral spectrum from a fundamental theory such as string theory given near the Planck scale. As we move toward a chiral theory, the parity violating weak interaction phenomena guide us to the SM.

In the search for a fundamental theory, two approaches can be taken. One can be the accumulation of low energy observed evidence and the building of a theoretically satisfactory gigantic model describing all these phenomena. This is a bottom-up approach in which the model cannot be excluded experimentally and is hence physically sound. The other approach is to find a theoretically satisfactory model given near the Planck scale and compare its low energy manifestation with experimental data. This is known as the top-down approach. Sometimes, the bottom-up approach is mingled together with

the top-down approach because a fundamental theory can never be achieved using the bottom-up approach alone. In any case, one needs guidance for such a theory. From the theoretical point of view, the best guidance is the *symmetry principle*. In recent years, the top-down approach has gained momentum.

Looking back at the construction of the SM, it started from matter representation $|\Psi\rangle$ in the Hilbert space where $|\Psi\rangle$ symbolically stands for the L-handed electron doublet l and the R-handed electron singlet e_R . If we include quarks also in the matter, $|\Psi\rangle$ will include them as well. In this Hilbert space, operations by the weak charge and the electromagnetic charge are treated in a similar fashion, thus the SM is dubbed with the phrase, “unified theory of weak and electromagnetic interactions”. The key point to observe here is the role of matter representation $|\Psi\rangle$. It is the representation on which symmetry charges act. In this book we will generalize this symmetry concept, and adopt the *unification theme: unify all the matter representations if it is possible*.

The first top-down approach toward a more fundamental theory beyond the SM was the grand unified theory (GUT). In one attempt, among the representations in $|\Psi\rangle$ the lepton doublet l and the charge conjugated field d_L^c of the R-handed d_R quark are unified into a single representation [11]. Other SM representations are grouped together. This attempt succeeded in unifying the SM group into a simple group $SU(5)$. Another early attempt was to combine the quark doublet q and the lepton doublet l together into a single representation [7]. Then, the remaining SM representations are matched together with the attempted extended gauge group. This attempt succeeded with a semi-simple group $SU(4) \times SU(2)_L \times SU(2)_R$. It follows that, in these GUTs the strong and electroweak couplings are necessarily the same when the unification is valid.

Apparently, the strong, weak and electromagnetic couplings observed at low energy are not the same at the electroweak scale, and at first glance this idea of unification with the identical gauge couplings seems to contradict the observed phenomena. However, the size of the coupling constant looks different at different energy scales of the probing particle. This is due to the fact that a renormalizable theory intrinsically introduces a mass scale μ , and the energy dependence of the coupling is described by the renormalization group equation. Therefore, one can construct a GUT such that the gauge couplings are unified at a scale, say at M_{GUT} , which is supposed to be super-heavy so that the electroweak coupling and the strong coupling constants are sufficiently separated at the energy scales (~ 100 GeV) probed by the current accelerators [12]. For a significant separation through logarithmic dependence, one needs an exponentially large M_{GUT} [12] which should be smaller than the Planck mass so that gravitational corrections might be insignificant. Here, we should not forget that the construction of the simplest $SU(5)$ was possible after realizing that one can collect all the pieces of the fifteen chiral fields with