

Engelder

**Stress Regimes in
the Lithosphere**

Princeton

Stress Regimes in the Lithosphere

Terry Engelder

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Designed by Laury A. Egan

To

All my friends who have shared in the excitement of working
at the Lamont-Doherty Geological Observatory
of Columbia University, one of the world's finest
earth science institutes

Preface

The debate over the magnitude and orientation of stress in the lithosphere has continued for several decades. As a student I was impressed by the seemingly irreconcilable differences between the high differential stresses required by models for lithospheric flexure and the low differential stresses found during hot creep experiments on rock. More recently, in situ stress data from California suggest that the frictional strength of the San Andreas fault is lower than expected by predictions using the general rock friction law to model shear stress along fault zones. Other contentious issues concerned the choice of a reference state of stress, the extrapolation of near-surface measurements to depth, the origin of platewide stress fields, and the significance of residual stress. The purpose of this monograph is to acquaint the geoscientist with these and many other issues associated with the debate over stress in the lithosphere.

My goal is to provide a broad understanding of stress in the lithosphere while touching some of the specific details involved in the interpretation of stress data generated by the most commonly used measurement techniques. Although the discussion of stress measurement techniques lacks a cookbook form, I illustrate some of the subtle aspects of the measurements, often drawing upon my own experience in making stress measurements. Given the breadth of the subject and a limit on space, I had to strike a balance on several counts. I wrote for the senior undergraduate or first-year graduate student who has a modest background in either structural geology or mechanics including an introduction to stress as a tensor (e.g., Means, 1976; Suppe, 1985). Space limitations meant that an encyclopedic referencing of literature on the subject was impractical. Yet, the student will find enough material to easily continue a search for additional references. I attempted to balance the most recent references with some time-honored literature. The need for conciseness led to the introduction of many theories with the assumption that the student would return to the original reference for a complete development of the concepts. Nevertheless, some theories are treated in detail according to the following outline.

An understanding of stress in the lithosphere starts with an introduction to nomenclature based on three reference states of stress (chap. 1). Chapters 2 through 4 cover the role of rock strength as a governor for stress magnitude. Stress regimes in the lithosphere are identified according to the particular failure mechanism (crack propagation, shear rupture, ductile flow, or frictional slip) which controls the magnitude of stress at a particular time and place in the lithosphere. After introducing the various stress regimes, their extent in the

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upper crust is demarcated by direct measurements of four types: hydraulic-fracture; borehole-logging; strain-relaxation; and rigid-inclusion measurements (chaps. 5 through 8). The relationship between lithospheric stress and the properties of rocks is then presented in terms of microcrack-related phenomena (chap. 9) and residual stress (chap. 10). Chapter 11 deals with lithospheric stress as inferred from the analysis of earthquakes. Finally, lithospheric stress is placed in the context of large-scale stress fields and plate tectonics (chaps. 12 and 13).

Terry Engelder
Boalsburg, PA

Acknowledgments

This monograph is, in part, an account of my experience while attempting to measure lithospheric stress during my twelve years as a research scientist at the Lamont-Doherty Geological Observatory of Columbia University. My experience started during the summer of 1973 when Marc Sbar and I had a conversation which ended with a simple understanding that we were going to make a concerted effort to “measure” stress. Early encouragement from Chris Scholz and Lynn Sykes was important. Dick Plumb participated in several of the field experiments discussed within this monograph. One of Dick’s important contributions was the development of the Lamont strain cell, a “doorstopper”-like module used in more than two hundred near-surface stress measurements. When funds were finally in hand for hydraulic fracture stress measurements, Keith Evans took the lead in the meticulous organization of several field experiments. Practical advice in many technical aspects of the field experiments was provided by Ted Koczyński. Others who helped with field work include Pat Barns (Lamont); John Barry (Penn State); Olaf Befeld (Bochem); Eric Bergman (Arizona); Steve Brown (Lamont); Tom Chen (Lamont); Peter Dahlgren (Arizona); Tom Engelder (Penn State); Zoe Engelder (Boalsburg); Chris Flaccus (Arizona); Peter Geiser (Connecticut); Larry Hooper (SUNY); Robert Kranz (Lamont); Tony Lomando (CUNY); Steve Marshak (Lamont); Irene Meglis (Penn State); Jim Mori (Lamont); Craig Nickolson (Lamont); Stu Nishenko (Lamont); Kevin Powell (Columbia); Randy Richardson (Arizona); Chuck Rine (Lamont); Fritz Rummel (Bochem); and Dave Stocker (Penn State).

Ideas rarely have a single source in the literature; usually they evolve through discussion among colleagues at national meetings and conferences. Ideas arising from such fora are often the most difficult to acknowledge or credit; my memory of the details of specific discussions are now fuzzy at best. Nevertheless, some of the richest discussions happened at the following meetings: the 1971 Penrose Conference on “Earthquake Source Mechanisms and Fracture Mechanics”; the 1974 Penrose Conference on “Earthquake Source Mechanisms and Fracture Mechanics”; the 1976 Chapman Conference on “Stress in the Lithosphere”; the 1979 U.S. Geological Survey Workshop on “Stress and Strain Measurements Related to Earthquake Prediction”; the 1979 U.S. Geological Survey Workshop on “Analysis of Actual Fault Zones in Bedrock”; the 1980 U.S. Geological Survey Workshop on “Magnitude of Deviatoric Stresses in the Earth’s Crust and Upper Mantle”; the 1981 U.S. Geological Survey Workshop on “Hydraulic Fracture Stress Measurements”; the 1984

EPRI Workshops on “Tectonic Processes of Intraplate Stress Generation and Concentration”; and the 1988 NSF/GRI Workshop on “Hydraulic Fracturing Stress Measurements.”

Although it is customary to acknowledge agencies supporting work on a particular project, the effort of individual contract monitors is often overlooked. Silent partners in my earth stress field experiments include: Jack Evernden for the U.S. Geological Survey; Jerry Harbor for the U.S. Nuclear Regulatory Commission; George Kolstad for the U.S. Department of Energy-Washington; Chuck Komar for the U.S. Department of Energy-Morgantown; Louis Lacy for EXXON Production Research; Dick Plumb for Schlumberger-Doll Research; Carl Stepp for the Electric Power Research Institute; Paul Westcott for the Gas Research Institute; and Leonard Johnson, Mike Mayhew, Dan Weill, and Thomas Wright for the National Science Foundation. Logistical support during the writing of this book came from EPRI contract R.P. 2556-24 and GRI contract 5088-260-1746.

Initial drafts of this book were written as a syllabus for a graduate-level course at The Penn State. One class exercise was the chapter-by-chapter review of a draft of this monograph by R. Cornelius, V. Lee, I. Meglis, P. Scott, and D. Srivastava. I am grateful to several colleagues for chapter reviews including Z. T. Bieniawski; D. Dunn; K. Evans; M. Friedman; N. Gay; R. Goodman; M. Gross; B. Haimson; R. Hatcher; S. Hickman; J. Logan; S. Mackwell; S. Marshak; R. A. Plumb; D. Pollard; R. Richardson; M. Sbar; C. H. Scholz; H. Swolfs; C. Thornton; M. D. Zoback; and M. L. Zoback.

In addition to many colleagues who have allowed me to modify or reprint their figures herein, I would like to thank the following copyright holders for graciously permitting me to modify or reprint material in this book.

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List of Symbols

The following lists the symbols used in this book. Their order is alphabetical using the Latin and then Greek alphabets. The equation in which the symbol first appears or is first defined is given in brackets unless otherwise indicated.

A	area [2–26]; $\alpha_b \frac{1-2\nu}{1-\nu}$ [5–20]; real area of contact [11–3]
a	variable [6–10]; constant [11–3]
b	half length of the short axis of a crack [2–1]; variable [6–11]
b	the Burgers vector [4–10]
C_{ijkl}	stiffness tensor [#] [1–11]
C₀	uniaxial compressive strength [3–12]
c	half length of the long axis of a crack [2–1]; variable [6–12]; half length of the slot [8–2]
c₀	half length of a flatjack [8–5]
D	average displacement along the faulted area [11–7]; flexural rigidity [13–27]
D₀	normalized deviatoric tensor [3–35]
d	grain diameter [4–15]; variable [6–13]; lateral distance in lithosphere [13–1]
E	Young's modulus [1–10]; total work (energy) during an earthquake rupture [11–10]
E	Young's modulus of a rock containing joints [2–31]
E_g	Young's modulus of the borehole inclusion device [8–1]
E_h	Young's modulus of the host rock [8–1]
E_s	energy dissipated as seismic waves [11–11]
F	shear force [11–4]
F_i	components of force on lithospheric plates [13–9]
f	normalized stress intensity function [5–12]
G	energy release rate per unit length of crack tip [2–20]
G_c	critical energy release rate per unit length of crack tip [2–24]
g	gravitational acceleration [1–17]; normalized stress intensity function [5–12]
H*	activation enthalpy [4–2]
h	topographic relief [13–19]
h_i	material constants [3–16]
h_o, h_a	normalized stress intensity functions [5–12]

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I	unit tensor [3–35]
i, j, k, l	subscripts [#] [1–5]
i_h	angle of emergence [11–1]
K_I	stress intensity factor [2–16]
K_{Ic}	fracture toughness [2–18]
k	diffusivity constant (cm ² /sec) [7–22]
k_o	ratio of S _h to S _v [1–15]
M	mean of a data set of orientation data [12–1]; mass per unit area of a column of oceanic lithosphere [13–2]
M_b	bending moment [13–24]
M_o	seismic moment [11–8]
N	normal force [11–3]
n	amount of a substance [2–5]
n	unit vector normal to the fault plane [3–41]
P_b	breakdown pressure [5–9]
P_c	confining pressure [1–13b]; radial pressure [7–19]
P_c^e	crack closure pressure [9–3]
P_f	fluid pressure inside a borehole [5–6]
P_f^H	wellbore pressures at which horizontal fracture initiation is predicted [5–23]
P_f^V	wellbore pressures at which vertical fracture initiation is predicted [5–24]
P_i	crack driving pressure [2–15]
P_j	flatjack pressure [8–7]
P_l	pressure in the lithosphere [13–12]
P_m	magma pressure [1–13a]
P_p	pore pressure [1–43]
P_{ro}	fracture reopening pressure [5–13]
P_t	fluid inclusion trapping pressure [2–27]
p_i	penetration hardness measured at unit time [11–3]
Q	rock stress parallel to the flatjack [8–4]
R	gas constant [2–5]; the stress ratio [3–36]; radius of a borehole [5–1]; radius of curvature [13–21]
R^ˆ	the stress ratio [3–50]
R_e	radius of the earth [13–17]
R_h	least stress ratio, S _h /S _v [chap. 5]
r	distance from the crack tip [2–16]; distance between dislocations [4–10]; distance from the center of a borehole [5–1]; characteristic length associated with the narrowest dimension of the fault [11–7]
r_c	radius of curvature [2–1]
S	stress tensor [3–32]
S_c	actual stress under grain contacts with a crack or container wall under dry conditions [1–45]

LIST OF SYMBOLS **xxi**

S_c^a	average rock stress normal to a crack or container wall under dry conditions [1–45]
S_H	maximum horizontal stress [1–13]
\bar{S}_H	effective maximum horizontal stress [6–16]
S_h	minimum horizontal stress [1–13]
S_h^T	horizontally induced thermal stress [1–22]
S_i^*	stress concentration at the end of a borehole [7–17]
S_N	radial profile of normal stress across the plane of the fracture [5–18]
S_n	rock stress normal to the flatjack [8–2]
S_R	reduced stress tensor [3–44]
S_t	circumferential stress tangent to the edge of an elliptical crack [2–1]
S_v	vertical stress [1–23]
S_{vw}	vertical stress at the borehole wall [5–23]
s	slip lineation vector [3–45]
s_o	standard deviation of a data set [12–3]
T	temperature [1–22]; the travel time of the P-wave [11–1]
T_a	amplitude of surface temperature variation [7–24]
T_e	elastic thickness of the lithosphere [13–27]
T_m	temperature of the mantle [13–6]
T_0	initial temperature [1–22]; uniaxial tensile strength [2–11] mean annual surface temperature [7–24]
T_s	surface temperature [1–22]
T_z	temperature variation at depth [7–22]
T	torque generated by a lithosphere plate [13–8]
t	period (seconds) [7–22]; time of maximum annual surface temperature [7–24]; time of contact [11–3]
\mathbf{t}	unit vector associated with τ [3–43]
U_E	strain energy [2–6]
U_E^{el}	strain energy due to an external load [2–12]
U_E^c	strain energy due to crack wall displacement [2–13]
U_s	surface energy [2–6]
U_T	total energy [2–5]
u	displacement [7–10]; velocity of spreading from midocean ridge [13–6]
u_y	crack wall displacement [2–28]
V, V_i	volume [2–5]
V_{out}	volume of flowback {fig. 5–5}
W_c	work is done by the fluid in the crack to move the crack wall [2–13]
W_j	half displacement caused by raising jack pressure [8–5]
W_R	work on rock surrounding the rock-crack system [2–6]
W_s	work on expansion of a gas [2–5]
W_y	half displacement across open flatjack slot [8–6]
W_0	half displacement during flatjack slot cutting [8–2]

W_1	half displacement due to finite slot width [8–3]
W_2	half displacement due to biaxial stress around a flatjack slot [8–4]
w	change in depth of isotatic compensation due to erosion [1–18]; the dislocation strain energy per unit length in the grain volume [4–18]
Y	crack modification factor [2–17]
x, y, z	subscripts representing orthogonal coordinates with z vertical for faults [fig. 1–3] or for vertical joints [fig. 2–2]
y	distance of measuring pins from the major axis of a flatjack slot [8–2]
y_0	half width of the slot [8–3]
z_1	thickness of the lithosphere [1–19]
z	depth within the earth [1–17]; thickness of eroded lithosphere [1–18]
α_b	Biot's poroelastic parameter [1–49]
α_m	volumetric thermal expansion coefficient of the mantle [13–5]
α_T	linear thermal expansion coefficient [$\mu\epsilon/^\circ\text{C}$] [1–22]
β	compressibility [1–14]; pore pressure coefficient [5–22]
β_b	the bulk compressibility of the solid with cracks and pores [1–49]
β_i	intrinsic compressibility [1–49]
β_{ij}	cosine between fault and stress tensor coordinate systems [3–37]
γ	free surface energy per unit surface area [2–10]; latitude relative to pole of rotation [13–7]
γ_{dis}	the dislocation strain energy per unit area in the grain boundary [4–8]
γ_τ	engineering shear strain [4–14]
Δ	symbol for differentiation [1–14]; the distance in degrees from the recording station to the epicenter [11–1]
ΔP_f	absolute magnitude of the difference between the formation pore pressure, P_p , and the borehole fluid pressure [6–3]
$\Delta\epsilon$	volumetric strain [1–4]
$\Delta\sigma$	stress change in the host rock [8–1]
$\Delta\sigma_g$	stress change in the rigid-inclusion gauge [8–1]; stress change in a grain [10–6]
$\Delta\hat{\sigma}_h^*$	change in horizontal stress as a consequence of lithospheric thinning [1–21]
$\Delta\tau$	shear stress drop during stick slip [11–2]
δ	dip of a fault plane
δ_{ij}	Kronecker delta [#] [1–43]

$\delta\sigma_i$	deviatoric stress [#] [1–41]
$\varepsilon, \varepsilon_i, \varepsilon_{ij}$	strain [#] [1–1]
ε_{ij}^i	strain in the intrinsic rock (i.e., the uncracked solid) [9–4]
$\dot{\varepsilon}$	strain rate [4–2]
$\dot{\varepsilon}_s$	a steady-state creep rate [4–2]
ζ	Lame's constant which is also called the modulus of rigidity [1–1] and the shear modulus [4–10]
ζ_{ij}	cumulative crack strain [9–14]
ζ_v	total crack porosity [9–17]
η	seismic efficiency [11–11]
η_{ij}	strain due to the presence of cracks [9–4]
Θ	angular distance in polar coordinates [2–16]
θ	angle from plane of crack [2–39]; angle between σ_1 and normal to a plane [3–4]; angle measured clockwise from S_H at a borehole [5–1]
κ	bulk modulus [1–14]; the angles between σ_2 and the normal to the e -twin plane [4–6]; thermal diffusivity of the lithosphere [13–6]
λ	Lame's constant [1–1]; rake of the slip lineation on a fault; angle between σ_1 and the normal to the slip plane [4–1]
Λ	critical slip distance [11–6]
μ_o	initial coefficient of sliding friction [11–6]
μ_d	coefficient of dynamic friction [11–2]
μ_s	coefficient of static friction [3–24]
$\bar{\mu}_s$	static friction coefficient at unit time of contact [11–5]
μ^*	coefficient of internal friction [3–2]
$\mu\varepsilon$	microstrain [7–1]
ν	Poisson's ratio [1–12]
$\xi_{ij}(P_c)$	total crack spectra [9–8]
ρ	density of the overburden [1–17]
ρ_c	spatial density of cracks [2–35]
ρ_{dis}	the steady state dislocation density in the grain volume [4–18]
ρ_l	density of lithosphere [1–18]
ρ_m	density of mantle [1–18]
ρ_w	density of water [13–2]
Σ	Surface traction [3–41]
$ \bar{\sigma}_3^c $	critical effective stress for crack propagation [2–11]
σ_i, σ_{ij}	applied stress [#] [1–1]; total stress [#] {1–44}
$\bar{\sigma}, \bar{\sigma}_{ij}$	effective stress [#] [1–43]
σ_d	differential stress [1–40]
σ_d^H	differential stress in the horizontal plane [5–27]
σ_d^s	steady-state flow stress [4–2]
σ_e	excess stress [1–32]
σ_c^i	cement stress just after cementation [10–5]
σ_g^i	intragranular stress after cementation [10–5]

$\bar{\sigma}_h$	average horizontal stress in the lithosphere [1–19]
$\bar{\sigma}_h^*$	average horizontal stress in a thinned lithosphere [1–20]
σ_{ij}	stress in the vicinity of a crack tip [2–16]
σ_{int}	internal stress within the host grain [4–9]
σ_m	lithostatic stress in the mantle [1–19]; mean stress [1–42]
σ_n	normal stress [3–2]
σ_r	radial stress near a borehole [5–1]
σ_θ	circumferential stress tangent to a borehole [5–1]
σ_t	tectonic stress added to the uniaxial-strain reference state [1–30]
σ_t^*	tectonic stress added to the lithostatic reference state [1–29]
τ	shear stress [3–2]; shear stress from viscous drag [13–1]
τ	shear traction on a fault plane [3–42]
τ_a	average stress along the fault [11–10]
τ_c	critical resolved shear stress [4–1]
τ_f	shear stress on a fault plane after an earthquake rupture [11–10]
τ_i	shear stress on the fault plane before an earthquake [11–10]
τ_m	maximum shear stress [3–13]
τ_o	cohesion of the material [3–2]; cohesive strength of a fault zone [3–25]
τ_{oct}	octahedral shear stress [6–1]
τ_r	resolved shear stress [4–1]
τ_θ	shear stress near a borehole [5–3]
ϕ	rock porosity [1–46]; angle of internal friction [3–15]; angle between σ_1 and the slip direction [4–1]
ϕ_p	the angle measured in the counterclockwise direction from the $\theta = 0^\circ$ axis of the strain rosette [7–6]
φ	relative magnitude of stress [3–42]; crack aspect ratio [9–3]
χ	the angle between σ_2 and the e -twin glide line [4–6]; material property [10–13]
Ω_o	$\cos\lambda\cos\phi$ [4–1]

#—Subscripts have ranges from one to three: $i, j, k, l = 1, 2, 3$.

Kronecker delta δ_{ij} is defined as
$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}.$$

Einstein summation convention is followed. Repeated indices in any single term mean that the term is to be summed over the full range of the term. For example, $\sigma_{ij} = \sigma_{i1}x_1 + \sigma_{i2}x_2 + \sigma_{i3}x_3$.