FOUNDATIONS OF APPLIED ELECTRODYNAMICS

$$\nabla \times \mathbf{E}$$

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

$$\phi_{\mathbf{E}} = -\int_{S} \mathbf{E} \cdot d\mathbf{s}$$

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FOUNDATIONS OF APPLIED ELECTRODYNAMICS

To my parents

To Jun and Lan

Preface

Electrodynamics is an important course in both physics and electrical engineering curricula. The graduate students majoring in applied electromagnetics are often confronted with a large number of new concepts and mathematical techniques found in a number of courses, such as *Advanced Electromagnetic Theory*, *Field Theory of Guided Waves*, *Advanced Antenna Theory*, *Electromagnetic Wave Propagation*, *Network Theory and Microwave Circuits*, *Computational Electromagnetics*, *Relativistic Electronics*, and *Quantum Electrodynamics*. Frequently, students have to consult a large variety of books and journals in order to understand and digest the materials in these courses, and this turns out to be a time-consuming process. For this reason, it would be helpful for the students to have a book that gathers the essential parts of these courses together and treats them according to the similarity of mathematical techniques.

Engineers, applied mathematicians and physicists who have been doing research for many years often find it necessary to renew their knowledge and want a book that contains the fundamental results of these courses with a fresh and advanced approach. With this goal in mind, inevitably this is beyond the conventional treatment in these courses. For example, the completeness of eigenfunctions is a key result in mathematical physics but is often mentioned without rigorous proof in most books due to the involvement of generalized function theory. As a result, many engineers lack confidence in applying the theory of eigenfunction expansions to solve practical problems. In order to fully understand the theory of eigenfunction expansions, it is imperative to go beyond the classical solutions of partial differential equations and introduce the concept of generalized solutions.

The contents of this book have been selected according to the above considerations, and many topics are approached in contemporary ways. The book intends to provide a whole picture of the fundamental theory of electrodynamics in most active areas of engineering applications. It is self-contained and is adapted to the needs of graduate students, engineers, applied physicists and mathematicians, and is aimed at those readers who wish to acquire more advanced analytical techniques in studying applied electrodynamics. It is hoped that the book will be a useful tool for readers saving them time and effort consulting a wide range of books and technical journals. After reading this book, the readers should be able to pursue further studies in applied electrodynamics without too much difficulty.

The book consists of ten chapters and four appendices. Chapter 1 begins with experimental laws and reviews Maxwell equations, constitutive relations, as well as the important properties derived from them. In addition, the basic electromagnetic theorems are summarized. Since most practical electromagnetic signals can be approximated by a temporal or a spatial wavepacket, the theory of wavepackets and various propagation velocities of wavepackets are also examined.

In applications, the solution of a partial differential equation is usually understood to be a classical solution that satisfies the smooth condition required by the highest derivative in the equation. This requirement may be too stringent in some situations. A rectangular pulse is not smooth in the classical sense yet it is widely used in digital communication systems. The first derivative of the Green's function of a wave equation is not continuous, but is broadly accepted by physicists and engineers. Chapter 2 studies the solutions of Maxwell equations. Three main analytical methods for solving partial differential equations are discussed: (1) the separation of variables; (2) the Green's function; and (3) the variational method. In order to be free of the constraint of classical solutions, the theory of generalized solutions of differential equation is introduced. The Lagrangian and Hamiltonian formulations of Maxwell equations are the foundations of quantization of electromagnetic fields, and they are studied through the use of the generalized calculus of variations. The integral representations of the solutions of Maxwell equations and potential theory are also included.

Eigenvalue problems frequently appear in physics, and have their roots in the method of separation of variables. An eigenmode of a system is a possible state when the system is free of excitation, and the corresponding eigenvalue often represents an important quantity of the system, such as the total energy and the natural oscillation frequency. The theory of eigenvalue problems is of fundamental importance in physics. One of the important tasks in studying the eigenvalue problems is to prove the completeness of the eigenmodes, in terms of which an arbitrary state of the system can be expressed as a linear combination of the eigenmodes. To rigorously investigate the completeness of the eigenmodes, one has to use the concept of generalized solutions of partial differential equations. Chapter 3 discusses the eigenvalue problems from a unified perspective. The theory of symmetric operators is introduced and is then used to study the interior eigenvalue problems in electromagnetic theory, which involves metal waveguides and cavity resonators. This chapter also treats the mode theory of spherical waveguides and the method of singular function expansion for scattering problems, which are useful in solving exterior boundary value problems.

An antenna is a device for radiating or receiving radio waves. It is an overpass connecting a feeding line in a wireless system to free space. The antenna is characterized by a number of parameters such as gain, bandwidth, and radiation pattern. The free space may be viewed as a spherical waveguide, and the spherical wave modes excited by the antenna depend on the antenna size. The bigger the antenna size, the more the propagating modes are excited. For a small antenna, most spherical modes turn out to be evanescent, making the stored energy around the antenna very large and the gain of the antenna very low. For this reason, most of the antenna parameters are subject to certain limitations. From time to time, there arises a question of how to achieve better antenna performance than previously obtained. Chapter 4 attempts to answer this question and deals with the fundamentals of radiation theory. The most important antenna parameters are reviewed and summarized. A complete theory of spherical vector wave functions is introduced, and is then used to study the upper bounds of the product of gain and bandwidth for an arbitrary antenna. In this chapter, the Foster reactance theorem for an ideal antenna without Ohmic loss, and the relationship between antenna bandwidth and antenna quality factor are investigated. In addition, the methods for evaluating antenna quality factor are also developed.

Electromagnetic boundary value problems can be characterized either by a differential equation or an integral equation. The integral equation is most appropriate for radiation and scattering problems, where the radiation condition at infinity is automatically incorporated in the formulation. The integral equation formulation has certain unique features that a differential equation formulation does not have. For example, the smooth requirement for the solution of integral equation is weaker than the corresponding differential equation. Another feature is that the discretization error of the integral equation is limited on the boundary of the solution region, which leads to more accurate numerical results. Chapter 5 summarizes integral equations for various electromagnetic field problems encountered in microwave and antenna engineering, including waveguides, metal cavities, radiation, and scattering problems by conducting and dielectric objects. The spurious solutions of integral equations are examined. Numerical methods generally applicable to both differential equations and integral equations are introduced.

Field theory and circuit theory are complementary to each other in electromagnetic engineering, and the former is the theoretical foundation of the latter while the latter is much easier to master. The circuit formulation has removed unnecessary details in the field problem and has preserved most useful overall information, such as the terminal voltages and currents. Chapter 6 studies the network representation of electromagnetic field systems and shows how the network parameters of multi-port microwave systems can be calculated by the field theory through the use of reciprocity theorem, which provides a deterministic approach to wireless channel modeling. Also discussed in this chapter is the optimization of power transfer between antennas, a foundation for wireless power transfer.

The wave propagation in an inhomogeneous medium is a very complicated process, and it is characterized by a partial differential equation with variable coefficients. The inhomogeneous waveguides are widely used in microwave engineering. If the waveguides are bounded by a perfect conductor, only a number of discrete modes called guided modes can exist in the waveguides. If the waveguides are open, an additional continuum of radiating modes will appear. In order to obtain a complete picture of the modes in the inhomogeneous waveguides, one has to master a sophisticated tool called spectral analysis in operator theory. Chapter 7 investigates the wave propagation problems in inhomogeneous media and contains an introduction to spectral analysis. It covers the propagation of plane waves in inhomogeneous media, inhomogeneous metal waveguides, optical fibers and inhomogeneous metal cavity resonators.

Time-domain analysis has become a vital research area in recent years due to the rapid progress made in ultra-wideband technology. The traditional time-harmonic field theory is based on an assumption that a monotonic electromagnetic source turns on at $t = -\infty$ so that the initial conditions or causality are ignored. This assumption does not cause any problems if the system has dissipation or radiation loss. When the system is lossless, the assumption may lead to physically unacceptable solutions. In this case, one must resort to time-domain analysis. Chapter 8 discusses the time-domain theory of electromagnetic fields, including the transient fields in waveguides and cavity resonators, spherical wave expansion in time domain, and time-domain theory for radiation and scattering.

Modern physics has its origins deeply rooted in electrodynamics. A cornerstone of modern physics is relativity, which is composed of both special relativity and general relativity. The special theory of relativity studies the physical phenomena perceived by different observers traveling at a constant speed relative to each other, and it is a theory about the structure of space–time. The general theory studies the phenomena perceived by different observers traveling at an arbitrary relative speed and is a theory of gravitation. The relativity, especially the special relativity, is usually considered as an integral part of electrodynamics. Relativity has many practical applications. For example, in the design of the global positioning system

(GPS), the relativistic effects predicted by the special and general theories of relativity must be taken into account to enhance the positioning precision. Chapter 9 deals with both special relativity and general relativity. The tensor algebra and tensor analysis on manifolds are used throughout the chapter.

Another cornerstone of modern physics is quantum mechanics. Quantum electrodynamics is a quantum field theory of electromagnetics, which describes the interaction between light and matter or between two charged particles through the exchange of photons. It is remarkable for its extremely accurate predictions of some physical quantities. Quantum electrodynamics is especially needed in today's research and education activities in order to understand the interactions of new electromagnetic materials with the fields. Chapter 10 provides a short introduction to quantum electrodynamics and a review of the fundamental concepts of quantum mechanics. The interactions of fields with charged particles are investigated by use of the perturbation method, in terms of which the dielectric constant for atom media is derived. Furthermore, the Klein–Gordon equation and the Dirac equation in relativistic mechanics are briefly discussed.

The book features a wide coverage of the fundamental topics in applied electrodynamics, including microwave theory, antenna theory, wave propagation, relativistic and quantum electrodynamics, as well as the advanced mathematical techniques that often appear in the study of theoretical electrodynamics. For the convenience of readers, four appendices are also included to present the fundamentals of set theory, vector analysis, special functions, and the SI unit system. The prerequisite for reading the book is advanced calculus. The SI units are used throughout the book. A $e^{j\omega t}$ time variation is assumed for time-harmonic fields. A special symbol □ is used to indicate the end of an example or a remark.

During the writing and preparation of this book, the author had the pleasure of discussing the book with many colleagues and cannot list them all here. In particular, the author would like to thank Prof. Robert E. Collin of Case Western Reserve University for his comments and input on many topics discussed in the book, and Prof. Thomas T. Y. Wong of Illinois Institute of Technology for his useful suggestions on the selection of the contents of the book.

Finally, the author is grateful to his family. Without their constant support, the author could not have made this book a reality.

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Maxwell Equations

> Ten thousand years from now, there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics.
>
> —Richard Feynman (American physicist, 1918–1988)

To master the theory of electromagnetics, we must first understand its history, and find out how the notions of electric charge and field arose and how electromagnetics is related to other branches of physical science. Electricity and magnetism were considered to be two separate branches in the physical sciences until Oersted, Ampère and Faraday established a connection between the two subjects. In 1820, Hans Christian Oersted (1777–1851), a Danish professor of physics at the University of Copenhagen, found that a wire carrying an electric current would change the direction of a nearby compass needle and thus disclosed that electricity can generate a magnetic field. Later the French physicist André Marie Ampère (1775–1836) extended Oersted's work to two parallel current-carrying wires and found that the interaction between the two wires obeys an inverse square law. These experimental results were then formulated by Ampère into a mathematical expression, which is now called Ampère's law. In 1831, the English scientist Michael Faraday (1791–1867) began a series of experiments and discovered that magnetism can also produce electricity, that is, electromagnetic induction. He developed the concept of a magnetic field and was the first to use lines of force to represent a magnetic field. Faraday's experimental results were then extended and reformulated by James Clerk Maxwell (1831–1879), a Scottish mathematician and physicist. Between 1856 and 1873, Maxwell published a series of important papers, such as 'On Faraday's line of force' (1856), 'On physical lines of force' (1861), and 'On a dynamical theory of the electromagnetic field' (1865). In 1873, Maxwell published 'A Treatise on Electricity and Magnetism' on a unified theory of electricity and magnetism and a new formulation of electromagnetic equations since known as Maxwell equations. This is one of the great achievements of nineteenth-century physics. Maxwell predicted the existence of electromagnetic waves traveling at the speed of light and he also proposed that light is an electromagnetic phenomenon. In 1888, the German physicist Heinrich Rudolph Hertz (1857–1894) proved that an electric signal can travel through the air and confirmed the existence of electromagnetic waves, as Maxwell had predicted.

Foundations of Applied Electrodynamics Geyi Wen