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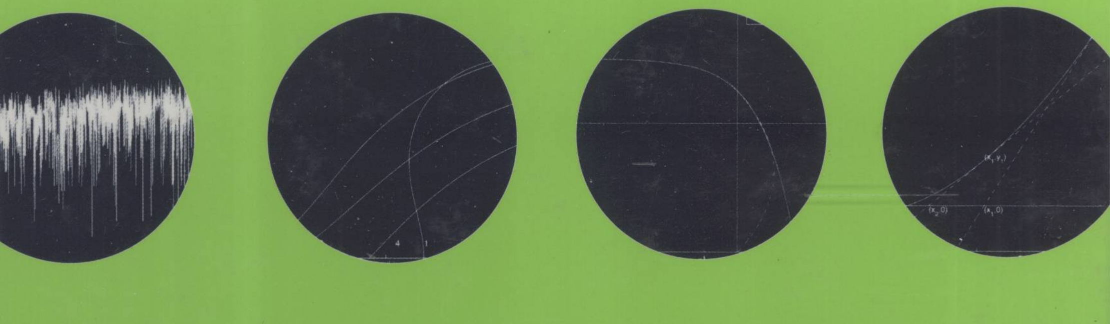
Revised Edition

AN INTRODUCTION TO NUMERICAL METHODS AND ANALYSIS



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JAMES F. EPPERSON

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Revised Edition

James F. Epperson Mathematical Reviews

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AN INTRODUCTION TO NUMERICAL METHODS AND ANALYSIS

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To Frank Crosby and Ed Croteau, Peter Duren and Fred Gehring, Richard MacCamy and (most especially) George J. Fix; teachers all, mathematicians all.

PREFACE

Preface to the Revised Edition

First, I would like to thank John Wiley for letting me do a Revised Edition of *An Introduction to Numerical Methods and Analysis*, and in particular I would like to thank Susanne Steitz and Laurie Rosatone for making it all possible.

So, what's new about this edition? A number of things. For various reasons, a large number of typographical and similar errors managed to creep into the original edition. These have been aggressively weeded out and fixed in this version. I'd like to thank everyone who emailed me with news of this or that error. In particular, I'd like to acknowledge Marzia Rivi, who translated the first edition into Italian and who emailed me with many typos, Prof. Nicholas Higham of Manchester University, Great Britain, and Mark Mills of Central College in Pella, Iowa. I'm sure there's a place or two where I did something silly like reversing the order of subtraction. If anyone finds any error of any sort, please email me at `jfe@ams.org`.

I considered adding sections on a couple of new topics, but in the end decided to leave the bulk of the text alone. I spent some time improving the exposition and presentation, but most of the text is the same as the first edition, except for fixing the typos.

I would be remiss if I did not acknowledge the support of my employer, the American Mathematical Society, who granted me a study leave so I could finish this project. Executive Director John Ewing and the Executive Editor of Mathematical Reviews, Kevin Clancey, deserve special mention in this regard. Amy Hendrikson of TeXnology helped with some LaTeX issues, as did my colleague at Mathematical Reviews, Patrick Ion. Another colleague, Maryse Brouwers, an extraordinary grammarian, helped greatly with the final copyediting process.

The original preface has the URL for the text website wrong; just go to `www.wiley.com` and use their links to find the book. The original preface also has my old professional email. The updated email is `jfe@ams.org`; anyone with comments on the text is welcome to contact me.

But, as is always the case, it is the author's immediate family who deserve the most credit for support during the writing of a book. So, here goes a big thank you to my wife, Georgia, and my children, Elinor and Jay. Look at it this way, kids: The end result will pay for a few birthdays.

Preface (To the First Edition)

This book is intended for introductory and advanced courses in numerical methods and numerical analysis, for students majoring in mathematics, sciences, and engineering. The book is appropriate for both single-term survey courses or year-long sequences, where students have a basic understanding of at least single-variable calculus and a programming language. (The usual first courses in linear algebra and differential equations are required for the last four chapters.)

To provide maximum teaching flexibility, each chapter and each section begins with the basic, elementary material and gradually builds up to the more advanced material. This same approach is followed with the underlying theory of the methods. Accordingly, one can use the text for a "methods" course that eschews mathematical analysis, simply by not covering the sections that focus on the theoretical material. Or, one can use the text for a survey course by only covering the basic sections, or the extra topics can be covered if you have the luxury of a full year course.

The objective of the text is for students to learn where approximation methods come from, why they work, why they sometimes don't work, and when to use which of many techniques that are available, and to do all this in a style that emphasizes readability and usefulness to the beginning student. While these goals are shared by other texts, it is the development and delivery of the ideas in this text that I think makes it different.

A course in numerical computation—whether it emphasizes the theory or the methods—requires that students think quite differently than in other mathematics courses, yet students are often not experienced in the kind of problem-solving skills and mathematical judgment that a numerical course requires. Many students react to mathematics problems by pigeon-holing them by category, with little thought given to the meaning of the answer. Numerical mathematics demands much more judgment and evaluation in light of the underlying theory, and in the first several weeks of the course it is crucial for students to adapt their way of thinking about and working with these ideas, in order to succeed in the course.

To enable students to attain the appropriate level of mathematical sophistication, this text begins with a review of the important calculus results, and why and where these ideas play an important role in this course. Some of the concepts required for the study of computational mathematics are introduced, and simple approximations using Taylor's theorem are treated in some depth, in order to acquaint students with one of the most common and basic tools in the science of approximation. Computer arithmetic is treated in perhaps more detail than some might think necessary, but it is instructive for many students to see the actual basis for rounding error demonstrated in detail, at least once.

One important element of this text that I have not seen in other texts is the emphasis that is placed on "cause and effect" in numerical mathematics. For example, if we apply the trapezoid rule to (approximately) integrate a function, then the error should go down by a factor of 4 as the mesh decreases by a factor of 2; if this is not what happens, then almost surely there is either an error in the code or the integrand is not sufficiently smooth. While this is obvious to experienced practitioners in the field, it is not obvious to beginning students who are not confident of their mathematical abilities. Many of the exercises and examples are designed to explore this kind of issue.

Two common starting points to the course are root-finding or linear systems, but diving in to the treatment of these ideas often leaves the students confused and wondering what the point of the course is. Instead, this text provides a second chapter designed as a "toolbox" of elementary ideas from across several problem areas; it is one of the important innovations of the text. The goal of the toolbox is to acclimate the students to the culture of numerical methods and analysis, and to show them a variety of simple ideas before proceeding to cover any single topic in depth. It develops some elementary approximations and methods that the students can easily appreciate and understand, and introduces the students, in the context of very simple methods and problems, to the essence of the analytical and coding issues that dominate the course. At the same time, the early development of these tools allows them to be used later in the text in order to derive and explain some algorithms in more detail than is usually the case.

The style of exposition is intended to be more lively and "student friendly" than the average mathematics text. This does not mean that there are no theorems stated and proved correctly, but it does mean that the text is not slavish about it. There is a reason for this: The book is meant to be read by the students. The instructor can render more formal anything in the text that he or she wishes, but if the students do not read the text because they are turned off by an overly dry regimen of definition, theorem, proof, corollary, then all of our effort is for naught. In places, the exposition may seem a bit wordier than necessary, and there is a significant amount of repetition. Both are deliberate. While brevity is indeed better mathematical style, it is not necessarily better pedagogy. Mathematical textbook exposition often suffers from an excess of brevity, with the result that the students cannot follow the arguments as presented in the text. Similarly, repetition aids learning, by reinforcement.

Nonetheless I have tried to make the text mathematically complete. Those who wish to teach a lower-level survey course can skip proofs of many of the more technical results in order to concentrate on the approximations themselves. An effort has been made—not always successfully—to avoid making basic material in one section depend on advanced material from an earlier section.

The topics selected for inclusion are fairly standard, but not encyclopedic. Emerging areas of numerical analysis, such as wavelets, are not (in the author's opinion) appropriate for a first course in the subject. The same reasoning dictated the exclusion of other, more mature areas, such as the finite element method, although that might change in future editions should there be sufficient demand for it. A more detailed treatment of approximation theory, one of the author's favorite topics, was also felt to be poorly suited to a beginning text. It was felt that a better text would be had by doing a good job covering some of the basic ideas, rather than trying to cover everything in the subject.

The text is not specific to any one computing language. Most illustrations of code are made in an informal pseudo-code, while more involved algorithms are shown in a "macro-outline" form, and programming hints and suggestions are scattered throughout the text. The exercises assume that the students have easy access to and working knowledge of software for producing basic Cartesian graphs.

A diskette of programs is *not* provided with the text, a practice that sets this book at odds with many others, but which reflects the author's opinion that students must learn how to write and debug programs that implement the algorithms in order to learn the underlying mathematics. However, since some faculty and some departments structure their courses differently, a collection of program segments in a variety of languages is available on the text web site so that instructors can easily download and then distribute the code to their students. Instructors and students should be aware that these are program *segments*; none of them are intended to be ready-to-run complete programs. Other features of the text web site are discussed below. (*Note:* This material may be removed from the Revised Edition website.)

Exercises run the gamut from simple hand computations that might be characterized as "starter exercises" to challenging derivations and minor proofs to programming exercises designed to test whether or not the students have assimilated the important ideas of each chapter and section. Some of the exercises are taken from application situations, some are more traditionally focused on the mathematical issues for their own sake. Each chapter concludes with a brief section discussing existing software and other references for the topic at hand, and a discussion of material not covered in this text.

Historical notes are scattered throughout the text, with most named mathematicians being accorded at least a paragraph or two of biography when they are first mentioned. This not only indulges my interest in the history of mathematics, but it also serves to engage the interest of the students.

The web site for the text (http://www.wiley.com/epperson) will contain, in addition to the set of code segments mentioned above, a collection of additional exercises for the text, some application modules demonstrating some more involved and more realistic applications of some of the material in the text, and, of course, information about any updates that are going to be made in future editions. Colleagues who wish to submit exercises or make comments about the text are invited to do so by contacting the author at `epperson@math.uah.edu`.

A Note to the Student (from the First Edition)

This book was written to be read. Now I am under no illusions that this text will compete with the latest popular novel for interest and thrilling narrative. But I have tried very hard to write a book on mathematics that could be read by the students. So do not simply buy the book, work the exercises, and sell the book back to the bookstore at the end of the term. Read the text, think about what you have read, and ask your instructor questions about the things that you do not understand.

Numerical methods and analysis is a very different area of mathematics, certainly different from what you have seen in your previous courses. It is not harder, but the differentness of the material makes it seem harder. We worry about different issues than in other mathematics classes. In a calculus course you are typically asked to compute the derivative or antiderivative of a given function, or to solve some equation for a particular unknown. The task is clearly defined, with a very concrete notion of "the right answer." Here, we are concerned with computing approximations, and this involves a slightly different kind of thinking. We have to understand what we are approximating well enough to construct a reasonable approximation, and we have to be able to think clearly and logically enough to analyze the accuracy and performance of that approximation. One former student has characterized this course material as "rigorously imprecise" or "approximately precise." Both are appropriate descriptions. Rote memorization of procedures is not of use here; it is vital in this course that the student learn the underlying concepts. Numerical mathematics is also *experimental* in nature. A lot can be learned by simply trying something out and seeing how the computation goes.

Notation

Most notation is defined as it appears in the text, but here we include some commonplace items.

$\mathbf{R}$ — The real number line; $\mathbf{R} = (-\infty, \infty)$.

$\mathbf{R}^n$ — The vector space of real vectors of n components.

$\mathbf{R}^{n\times n}$ — The vector space of real $n \times n$ matrices.

$C([a,b])$ — The set of functions f which are defined on the interval $[a,b]$, continuous on all of (a,b), and continuous from the interior of $[a,b]$ at the endpoints.

$C^k([a,b])$ — The set of functions f such that f and its first k derivatives are all in $C([a,b])$.

$C^{p,q}(Q)$ — The set of all functions u that are defined on the two-dimensional domain $Q = \{(x,t) \mid a \le x \le b, 0 < t \le T\}$, and that are p times continuously differentiable in x for all t, and q times continuously differentiable in t for all x.

$\approx$ — Approximately equal. When we say that $A \approx B$ we mean that A and B are approximations to each other. See §1.2.2.

$\equiv$ — Equivalent. When we say that $f(x) = g(x)$ we mean that the two functions agree at the single point x. When we say that $f(x) \equiv g(x)$, we mean that they agree at all points x. The same thing is said by using just the function names, i.e., $f = g$.

$\mathcal{O}$ — On the order of ("big O of"). We say that $A = B + \mathcal{O}(D(h))$ whenever $|A - B| \le CD(h)$ for some constant C and for all h sufficiently small. See §1.2.3.

$\mathbf{u}$ — Machine epsilon. The largest number such that, in computer arithmetic, $1 + \mathbf{u} = 1$. Architecture dependent, of course. See §1.3.

sgn — Sign function. The value of $\text{sgn}(x)$ is $1, -1$, or 0 depending on whether or not x is positive, negative, or zero, respectively.

CONTENTS

CHAPTER 1

INTRODUCTORY CONCEPTS AND CALCULUS REVIEW

It is best to start this book with a question: What do we mean by "Numerical Methods and Analysis"? What kind of mathematics is this book about?

Generally and broadly speaking, this book covers the mathematics and methodologies that underlie the techniques of *scientific computation*. More prosaically, consider the button on your calculator that computes the sine of the number in the display. Exactly how does the calculator know that correct value? When we speak of using the computer to solve a complicated mathematics or engineering problem, exactly what is involved in making that happen? Are computers "born" with the knowledge of how to solve complicated mathematical and engineering problems? No, they are not. Mostly they are *programmed* to do it, and the programs implement algorithms that are based on the kinds of things we will talk about in this text.

Textbooks and courses in this area generally follow one of two main themes: Those titled "Numerical methods" tend to emphasize the implementation of the algorithms, perhaps at the expense of the underlying mathematical theory that explains why the methods work; those titled "Numerical analysis" tend to emphasize this underlying mathematical theory, perhaps at the expense of some of the implementation issues. The best approach, of course, is to properly mix the study of the algorithms and their implementation ("methods") with the study of the mathematical theory ("analysis") that supports them. This is the goal of the present text.

Whenever someone speaks of using a computer to design an airplane or predict the weather, or otherwise solve a complex science or engineering problem, that person is talking about using numerical methods and analysis. The problems and areas of endeavor that