Basic Mathematics

What Every College Student Should Know

Shirley O. Hockett

BASIC MATHEMATICS:
WHAT EVERY COLLEGE STUDENT SHOULD KNOW

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To my beloved typist
(and husband)

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PREFACE

This book is intended to prepare the student for a college-level course in mathematics: advanced algebra, trigonometry, analytic geometry, precalculus. It was written to be read and used by the student. It covers the essentials of arithmetic and algebra, with an emphasis on word problems. The content of these subjects and the basic skills considered necessary for further work in mathematics are described, illustrated, and explained in detail. Each set of exercises is followed immediately by the answers, which are often accompanied by explanations or relevant equations. The review exercises at the end of each chapter, designed to reinforce the material of the individual sections, are also followed by all of the answers.

Each chapter begins with a pretest which should enable a student to determine how much of the chapter, if any, he needs to study. Since the questions are keyed to the sections they cover, a student can tell quickly which specific topics in the chapter need review. Each chapter ends with two tests which can be used to measure achievement.

A section of chapter 5 is devoted entirely to the metric system of measurement. Now that this system has been officially adopted by the United States, it will be the one in predominant use in the future throughout the world. The approach to the topic of measurement in this book encourages the student to think metric, to get a feel for the approximate sizes of ordinary objects in metric units; it minimizes conversions between systems.

Another feature is a set of eight supplements included for enrichment, on a wide range of topics, from bases other than ten to complex numbers.

The book can be used in the following ways:

(1) As a text for a course taught in the usual manner by an instructor;

(2) As a text for a class in which the students pace themselves, proceeding through the material at their own individual speeds, with an instructor or assistant available for additional guidance;

(3) By a student who is working independently to review familiar or to learn new mathematics.

The book is especially suitable for a course in community and junior colleges, in vocational schools, or in adult education classes. It can also be used effectively in a college or university that has an open-admissions policy. Students with little, or no, knowledge of mathematics can begin with the rudiments of arithmetic in chapters 1 through 5. Others may be able to skip some or all of these, starting instead with elementary algebra in chapter 6.

If you are working through the material in this text on your own, you will find it instructive to take each pretest as you come to it. If you miss only a couple of questions, you can study the relevant sections of the chapter, then try questions on these sections in the Review Exercises for the chapter. If you miss as many as a third of the pretest questions, you will probably benefit from study of the entire chapter. You can then determine how well you have grasped the material by trying Chapter Test A, for which answers are given at the back of the book. The answers to Test B, along with Test C and the answers thereto, are available in the Instructor's Manual.

I am happy to acknowledge the many helpful suggestions of Albert W. Liberi and Bruce W. King, who read and commented on drafts of the manuscript. My special thanks go also to Edward Lugenbeel and Joseph Murray, who are responsible for my doing this book for Prentice-Hall, and to Nancy Milnamow who carefully and patiently supervised its production.

SHIRLEY O. HOCKETT

Ithaca, New York

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CHAPTER

1

WHOLE NUMBERS; THE OPERATIONS OF ARITHMETIC

PRETEST

Directions. Cover the right half of the page until you have written down all your answers. Then check. The section number after a question denotes the section in Chapter 1 that covers the topic tested; refer to it if necessary. Additional review exercises for each section are given towards the end of this chapter, just before the chapter tests.

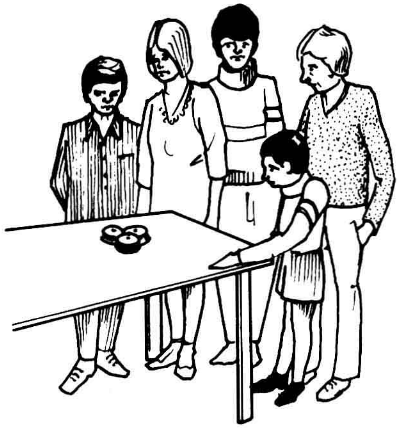
QUESTION	ANSWER
1. Which digit is in the hundreds place in the numeral 3094? (§ 1.1)	0
2. Fill in the blanks (§ 1.1): the numeral 82,409 = 8 × __ + 2 × __ + 4 × __ + 0 × __ + 9 × __.	8 × 10,000 + 2 × 1000 + 4 × 100 + 0 × 10 + 9 × 1

	QUESTION	ANSWER
3.	What property or law is illustrated by this equation? (§ 1.4) $7 \times (9 + 3) = (7 \times 9) + (7 \times 3)$	The distributive law
4.	Add: 897 (§ 1.2) 1346 3584	5827
5.	Subtract 2098 from 8106 (§ 1.3).	6008
6.	Multiply 5038 by 916 (§ 1.4).	4,614,808
7.	Divide 67,233 by 219 (§ 1.5).	307
8.	Express with exponents, listing each factor only once (§ 1.6): $2 \times 3 \times 3 \times 3 \times 5 \times 7 \times 7$	$2^1 \cdot 3^3 \cdot 5^1 \cdot 7^2$
9.	Evaluate (§ 1.6): $2^3 \cdot 3^2 \cdot 10^2$	7200
10.	Evaluate (§ 1.4): $(4 + 5) \times (3 + 9)$	108
11.	If x and y represent numbers, what is meant by $(x^4)(y^2)$? (§1.6)	$x \cdot x \cdot x \cdot x \cdot y \cdot y$
12.	Round off 38,355 to the nearest hundred (§ 1.7).	38,400
13.	Express 2,090,000 as a product of a whole number with no final zeros and a power of 10 (§ 1.7).	209×10^4
14.	Express 96 as a product of prime factors (§ 1.8).	$2 \times 2 \times 2 \times 2 \times 2 \times 3$
15.	Which of the following are meaningless or undefined? (§§ 1.4, 1.5) $\frac{0}{0}$; 0×5; $\frac{0}{9}$; 1×0; $\frac{1}{0}$	$\frac{0}{0}$; $\frac{1}{0}$

1.1. NUMERALS AND COUNTING

At the top of page 3 is a picture of a small group of hungry people and some hamburgers. As soon as you look at this picture, it is immediately obvious that there are not enough hamburgers to go around. Your eye and brain tell you that instantly, without any fuss or bother.

But suppose that there are a lot of hungry people, perhaps in one room, and a big pile of hamburgers on a table in the next room. It is then more complicated to find out



whether there are enough hamburgers to go around. In fact, there are really just two ways to do it.

One way is to let the people file by the table one by one, each taking a single hamburger, until either all the people or all the hamburgers are gone. If the people give out first, you know there are more hamburgers than people; if the hamburgers give out first, then there are more people than hamburgers. If the last person in line takes the last hamburger, then there are, of course, exactly the same number of people and of hamburgers—and you can know this even if you don't know just what that number is. This technique can be called PAIRING OFF (or MATCHING). If it works out evenly, we say that the set of people and the set of hamburgers are in ONE-TO-ONE CORRESPONDENCE. If two sets are in one-to-one correspondence, then they have the same NUMBER of members.

The other way to solve the problem is by means of COUNTING, a method invented by our ancestors a very long time ago. To count the people in the first room, you can turn a finger up (or down) for each person, or put a check mark on a piece of paper, or push the button on an automatic counter. When you have finished doing this, the position your fingers are in, or the check marks on the paper, or the symbols showing on your counter, constitute a NUMERAL that represents the number of people in the room. You can then count the hamburgers, in the second room. If the resulting numeral is the same as that for the people, then the two sets have the same number.

A numeral is any symbol or name for a number. Suppose in our problem that the number of hamburgers is the same as the number of people. If you had used two different techniques to count the two sets, then the final numerals might have been physically different—say, nine fingers up for the people and nine tallies on paper for the hamburgers. But these numerals are just different names for the number nine.

Here are some numerals for the number nine:

nine   9   𝍸|||| 5 + 4   IX   九

To indicate that "9" and "5 + 4" are different names for the same number, we write

$$9 = 5 + 4$$

This centered statement is called an EQUATION.

Today everybody all over the world uses SPOKEN NUMERALS as the handiest kind for most everyday purposes—spoken words like our *one, two, three, four, five,* and so on, or the equivalents in some other language. That is probably how most of us would handle the problem of the people and the hamburgers. But other kinds of counting are probably older—especially counting on the fingers. We know this because in many languages, all over the world, the words for 'five' and 'hand' (or 'thumb') are the same or similar, or the words for 'ten' and 'person', or the like.

Although, strictly speaking, the words "numeral" and "number" do not have the same meaning, it is not always convenient to keep them apart and in this book we will not always try to. Especially, we will often refer to numerals as numbers. This is frequently done, even by mathematicians, and should cause no confusion.

The Hindu-Arabic Numeral System

People have invented many different systems of written numerals. The system we use today was invented by the Hindus about 500 A.D. and then carried to Europe by the Arabs. It is therefore called the Hindu-Arabic system. In this system, we can represent any number by a numeral that uses only the following ten symbols, called DIGITS:

0 1 2 3 4 5 6 7 8 9

(It is interesting to note that "digit" also means 'finger' or 'toe'.)

The Hindu-Arabic numeral system is much better than any other because it is based on PLACE VALUE. This means that the value of a digit in a numeral depends on its place or position. The rightmost digit tells us how many ones or units there are; then, in order, from right to left, the digits indicate the number of tens, hundreds, and so on, as shown in this diagram:

. . .	TEN-MILLIONS	MILLIONS	HUNDRED-THOUSANDS	TEN-THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES
	10,000,000's	1,000,000's	100,000's	10,000's	1000's	100's	10's	1's

In the numeral "586", for example, the digit "5" in its position indicates five HUNDREDS, the digit "8" indicates eight TENS, and the digit "6" indicates six ONES or UNITS. When we add, we get

$$5 \text{ hundreds} + 8 \text{ tens} + 6 \text{ units}$$

or

$$5 \times 100 + 8 \times 10 + 6 \times 1$$

also written

$$5(100) + 8(10) + 6(1)$$

yielding

$$500 + 80 + 6$$

or 586. We note that $5(100) = 5 \times 100$ and that $8(10) = 8 \times 10$.

To appreciate the significance of place value, consider now the numeral 856:

$$\begin{aligned} 856 &= 8 \text{ hundreds} + 5 \text{ tens} + 6 \text{ units} \\ &= 8 \times 100 + 5 \times 10 + 6 \times 1 \\ \text{or } &\ 8(100) + 5(10) + 6(1) \\ &= 800 + 50 + 6 \end{aligned}$$

Of course, the numeral 856 has exactly the same three digits as 586. But would you settle for $586 from someone who owed you $856?

Note, also, in our place-value system, that

Each place has value ten times the place just to its right.

The Hindu-Arabic system is also called the DECIMAL SYSTEM, because it is based on ten (and the Latin word for 'ten' is *decem*).*

Now we compare two more numerals, 37 and 307:

$$\begin{aligned} 37 &= 3 \text{ tens} + 7 \text{ units} \\ &= 3 \times 10 + 7 \times 1 \\ &= 30 + 7 \end{aligned}$$

* In Supplement 1 numeral systems to bases other than ten are discussed.

but

$$\begin{aligned} 307 &= 3 \text{ hundreds} + 0 \text{ tens} + 7 \text{ units} \\ &= 3 \times 100 + 0 \times 10 + 7 \times 1 \\ &= 300 + 0 + 7 \end{aligned}$$

The only difference between these numerals is the digit "0" (zero) which serves as a PLACE-HOLDER in "307", telling us that there are no tens.

In "4097", the zero holds the hundreds place, indicating that there are no hundreds. In the numeral "50", the zero signals the absence of units.

In our decimal system, place value and place-holding (using zero) make it possible to write a numeral for any number easily and efficiently.

Expanded Form

Numerals written out in a form that exposes the values of the places (as in the last examples above) are said to be EXPANDED. The expanded form may involve either the times sign ("×") or parentheses, as illustrated below.

Examples

1. $914 = 9 \times 100 + 1 \times 10 + 4 \times 1.$

2. $2538 = 2 \times 1000 + 5 \times 100 + 3 \times 10 + 8 \times 1.$

3. $65 = 6 \times 10 + 5 \times 1.$

4. $871 = 8 \times 100 + 7 \times 10 + 1 \times 1.$

5. $817 = 8 \times 100 + 1 \times 10 + 7 \times 1.$

6. $1509 = 1 \times 1000 + 5 \times 100 + 0 \times 10 + 9 \times 1$
or $1(1000) + 5(100) + 0(10) + 9(1).$

7. $1095 = 1 \times 1000 + 0 \times 100 + 9 \times 10 + 5 \times 1$
or $1(1000) + 0(100) + 9(10) + 5(1).$

8. $7002 = 7 \times 1000 + 0 \times 100 + 0 \times 10 + 2 \times 1$
or $7(1000) + 0(100) + 0(10) + 2(1).$

Note how zero serves as a place-holder in examples 6, 7, and 8.

Exercises

Write each numeral in expanded form:

1. 29 **2.** 630 **3.** 7805 **4.** 23,092 **5.** 4001

Answers

1. $29 = 2 \times 10 + 9 \times 1$ or $2(10)+9(1)$ (can be written either way)
2. $630 = 6(100)+3(10)+0(1)$
3. $7805 = 7(1000)+8(100)+0(10)+5(1)$
4. $23{,}092 = 2 \times 10{,}000 + 3 \times 1000 + 0 \times 100 + 9 \times 10 + 2 \times 1$
5. $4001 = 4 \times 1000 + 0 \times 100 + 0 \times 10 + 1 \times 1$

The numbers we've used so far are called WHOLE NUMBERS. They are the ones in the set 0, 1, 2, 3, 4, ... (Here the three dots indicate that the whole numbers continue without end, that the set of whole numbers is INFINITE.) The whole numbers other than 0 are also called the NATURAL NUMBERS or the COUNTING NUMBERS or the POSITIVE INTEGERS.

1.2. ADDITION OF WHOLE NUMBERS

If we add \$2 and \$3, we get \$5. If we add \$3 and \$2, we also get \$5. This illustrates the COMMUTATIVE PROPERTY of addition:

> When numbers are added, the ORDER in which they are taken does not affect the sum.

If you find the sum of a column of figures by adding from top to bottom, you can use the commutative property to check the answer by adding from bottom to top.

Numbers to be added are called ADDENDS.

Given the three addends 35, 8, and 2, one may proceed either as shown on the left or as shown on the right:

$(35 + 8) + 2$	$35 + (8 + 2)$
$= 43 + 2$	$= 35 + 10$
$= 45$	$= 45$

The parentheses above tell us which two of the addends to add first. Obviously, it doesn't matter, since the answer is the same. This is an example of the ASSOCIATIVE PROPERTY of addition:

> The sum of three or more numbers does not depend on the GROUPING.

If you are fast at mental arithmetic then you probably exploit both the commutative and the associative properties of addition. To add 54 + 16 + 29 + 71, you might think "54 plus 6 equals 60, plus 10 equals 70; 29 plus 1 equals 30, plus 70 equals 100; 70 plus 100 equals 170." Can you tell how and where the commutative and associative properties of addition were used in this example?

To see, in greater detail, why we add two three-digit numbers as we do, let's find the sum of 538 and 296. First note that

$$538 = 5 \text{ hundreds} + 3 \text{ tens} + 8 \text{ ones}$$

$$296 = 2 \text{ hundreds} + 9 \text{ tens} + 6 \text{ ones}$$

Now write these numbers so that the ones digits are in the same column, and add the columns, starting at the right.

HUNDREDS	TENS	ONES
5	3	8
2	9	6
		14

In the ones column we get 8 + 6 or 14 ones. But since 14 ones equal 1 ten and four ones, we "carry" the 1 ten—

	1	
5	3	8
2	9	6
	13	4

—to the tens column and then add the tens column: 1 + 3 + 9 = 13. Now we note that 13 tens equal 1 hundred and 3 tens. We carry the 1 hundred—

1	1	
5	3	8
2	9	6
8	3	4

—to the hundreds column, and then add the hundreds column: 1 + 5 + 2 = 8. The answer is 8 hundreds + 3 tens + 4 ones, or just 834.

Usually, of course, we do much of this in our heads.* For instance, we might perform the addition from right to left as shown at the bottom above by saying (to ourselves) something like: "8 plus 6 is 14; write 4 and carry 1; 1 (the carry) plus 3 plus 9 equals 13; write 3 and carry 1; 1 (the carry) plus 5 plus 2 equals 8; write 8."

* Clearly, the ability to add numbers of this sort quickly and correctly depends very much on familiarity with the sum of ANY two one-digit numbers. If you cannot instantly reel off the sum of 9 and 7, or of 6 and 8, or of any other such pair, then memorize them NOW before proceeding.