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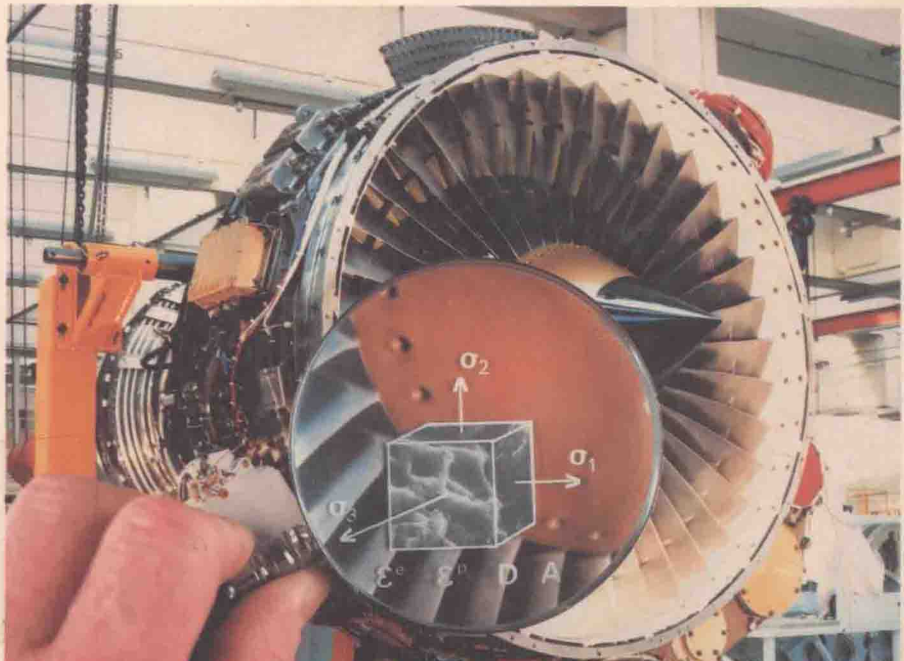

Mechanics of solid materials

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FOREWORD TO FRENCH EDITION

When my young colleagues, Jean Lemaitre and Jean-Louis Chaboche invited me to write a few introductory lines on the occasion of the publication of their work *Mécanique des Matériaux Solides* (Mechanics of solids materials), I very willingly acceded to their request as an expression of trust and friendship, even though on one hand, the aim of the present work is made abundantly clear by the title and the introduction, and on the other, the well-deserved fame of the authors is quite sufficient to attract and retain the attention of readers.

The originality, I would even say the identity, of this book becomes apparent only if we place it within the evolution of the scientific subjects during the last few decades. In fact, it is part of a triple current whose recent developments it assimilates and integrates. First of all, it borrows from continuum mechanics and thermodynamics the conceptual framework and methods which, starting with a few simple concepts, allow construction of a great variety of phenomenological models necessary for describing the extremely varied behaviour of solids. Secondly, it collects, rearranges, and above all, takes advantage of the observations, schematic representations, and empirical laws which generations of engineers have used with imagination and perspicacity in guiding and accomplishing their projects. Finally, it provides an inventory of physical phenomena, especially those observed at the microscopic, molecular or atomic scale, the scale at which events determining and explaining macroscopic behaviour occur. Even if these phenomena cannot be explained and expressed in terms of formulas, they are mentioned, whenever possible, in order to clarify the results and procedures.

To my mind, this triple heritage can never be overstated. This remarkable development of continuum thermodynamics would be no more than a pure theoretical elaboration if it did not include an effective and operational understanding of the many empirical laws, patiently deduced from experiments. The knowledge resulting from these latter sources can only contribute to the advancement of the mind in as much as it helps to explain, justify, and often inspire, the theoretical developments which receive their full recognition on the basis of experiments. Even as the physics of solids penetrates further and further into the elementary physical phenomena at the microscopic scale, it can acquire its full practical importance only if it is supplemented by macroscopic disciplines which support and perfect an ever bolder and an ever more efficient technique. These disciplines, as is shown by the present day research, will become more precise and refined in their methods so long as the disciplines close to them, physics and chemistry, continue to be perfected. It appears to me that the main characteristic of this work is that it lies at the crossroads of these trends of thoughts and that it brings out their mutual relationships which, in turn, reinforce their individual interests.

To write a good book, it is not enough to have a right conception, a clear objective, and an interesting aim. It is of no interest to mention the pitfalls and the dangers inherent in the task which our authors could have encountered, because to my mind they have succeeded in avoiding them. This book is not a '*summa*' where the reader could find a presentation of the three main branches of research that I have just mentioned. But it is essentially an account, a rather complete one but without a surfeit of details that obscure the ideas, of the constitutive laws of all the so-called solid materials, metals at room or elevated temperatures, polymers exhibiting very different behaviours from a vitrous state to a rubbery state, wood, concrete, and moreover, if I may say so, in every state of their existence: damaged, cracked, or aged. It is this primary concern which is the essence of this book.

The theoretical aspects of continuum mechanics are presented, without any superfluous development, with just what is required by the reader to establish a link, if necessary, with other works; the same is true as regards the finite element method of analysis without which the work would lose much of its interest. Similar observations hold true for the other disciplinary currents mentioned above: what is presented is quite sufficient to clarify the aim of this book in describing methods which can be used to test the proposed laws, and to explain them with numerical data useful in applications. This is the reason why I do not hesitate to recommend this book to a broad category of readers who, because of the connecting threads provided herein, will easily find valuable information for their work:

theoretical researchers, who will be happy to see that their past efforts have not been useless, will find here an inspiration for the future; mechanical and civil engineers will find here not only precious data but also an enlightening and stimulating framework for their thoughts; metallurgists, chemists and physicists will discover here the knowledge of macroscopic disciplines beyond their research. Of a reasonable size, written in a clear style, free from any over-specialized language, at the level of a master's degree which remains accessible to a vast scientific public, this book should enjoy a wide and well-deserved circulation.

Finally, I cannot help but note that this book provides a new testimony of the vitality of French mechanics of solids, of the unity and the interrelation between its views and its efforts, and of the interest of the results owed to it.

The French team of researchers have indeed contributed significantly to the theoretical developments on which Jean Lemaitre and Jean-Louis Chaboche have based their work: the method of virtual power, functional analysis and methods of finite element analysis, the formulation of constitutive laws by the method of local state in plasticity and especially in viscoplasticity, and application of these methods to damage and to the mechanics of cracked media. The merit of these authors is to have been able to assimilate the above developments, so as to go further and open new fields by combining all the needs arising from physical experiments and technical applications, and thus provide to French researchers a proof of the validity of the direction of their work and of the quality of the results arising from their coherent efforts. Both of these authors have already succeeded in enlivening this productive spirit and developing it by their own personal research at ONERA and at LMT at Cachan, by their teaching, and above all by their work in the '*Large Deformations and Damage Group*' (GRECO). The present work will spread the field of their influence even further, and as a consequence, the field of French mechanics as well. This will contribute to the betterment and greater vitality of our particular approach to mechanics.

In expressing publicly to Jean Lemaitre and to Jean-Louis Chaboche my gratitude and congratulations, I hope that this book will enjoy a wide circulation and that it will contribute to an awareness, beyond our borders, of the spirit and the quality of research in the mechanics of solids which the French teams have carried out in recent years.

Paul Germain
Professor at the École Polytechnique
Secrétaire perpétuel de l'Académie des Sciences
Paris, December 1984

FOREWORD TO THE ENGLISH EDITION

It is a special pleasure for me to introduce the English translation of the book by Jean Lemaitre and Jean-Louis Chaboche. Readers will find this an ambitious book written in a bold and adventurous style. I had the good fortune to spend four months as a visitor at the Laboratoire de Mécanique et Technologie at Cachan in 1983. It is evident that the book reflects the dynamic and refreshing style of research at this laboratory. The aim of the book is to answer the important question of how the mechanical properties of materials are affected by complex loading histories and how this behaviour in turn affects the performance of engineering components. The approach is global. Theoretical formulation is combined with discriminating experiments and the computational power of the computer to define and calculate the factors which affect the life and performance of engineering components subjected to severe loading conditions. A great merit of the approach is its ability to be constantly improved. Incremental improvements in constitutive laws, for example, are easily introduced as are efforts which help to bridge the physical processes within the material and the macroscopic behaviour observed in experiment. By this means research and development effort can be well coordinated and gaps of knowledge become more clearly identified. While the properties of metals attract most attention the importance of the general approach is illustrated with reference to other materials such as concrete and polymers. A particularly attractive feature of the book is the special emphasis given to damage mechanics and in this respect it probably represents the first attempt to present a unified treatment of this rather new topic. Damage mechanics plays a crucial role when estimating the life of engineering components. The resulting procedures are of particular importance at the initial design stages or when attempts are made to predict the remaining life of existing components.

This is a book which should appeal to both academic and practising engineers who are concerned with the means of predicting the life and performance of advanced engineering components. It should also appeal to those who are involved in the demanding business of attempting to span the scales of the processes occurring at the microstructural level and the engineering properties appropriate to design.

Fred Leckie
Professor and Head of the Department of
Theoretical and Applied Mechanics
University of Illinois, Urbana-Champaign, USA
March 1987

INTRODUCTION

Well done reader, you have been bold enough to open this book! and now you will be rewarded with a few explanations.

First on its title: the book deals with *mechanics*, that is the study of equilibrium and motion by considering relations between forces (or stresses) and displacements (or strains), time, and possibly temperature. It is applied to the structural *materials* used in mechanical and civil engineering: metals and alloys, natural organic materials (wood) or synthetics (plastics), and concrete. Thus, it is concerned with the study of the properties of *resistance* to *deformation* and *fracture*. These properties are intrinsic to the material, and are usually defined with reference to a volume element, independent of the geometry of the body under study.

On its spirit: the writing of the book has been organized to facilitate *transfer* of knowledge to engineering science: fundamental knowledge oriented towards practical applications, knowledge of macroscopic properties for formulating macroscopic laws of material behaviour, and a synthesis of the knowledge of theoretical and practical aspects. The recent progress in the mechanics of solid materials has resulted from simultaneous and decisive developments in all these fields:

> At the fundamental level, it is the synthesis of continuum mechanics with the method of local state in thermodynamics.
> At the level of applications, it is a recognition of the need to ensure higher safety and economy in the building of more sophisticated structures. It also recognizes the possibility of using faster computers to solve nonlinear problems numerically.
> At the level of microscopic properties, it is the theory of dislocations and the invention of electron microscopes.

At the level of macroscopic phenomena, it is the technique of identification of mathematical models from experimental results. At the theoretical level, it is the development of functional analysis and variational formulations.

Finally, at the experimental level, it is, of course, the introduction of electronics and microcomputers in testing machines and measurement procedures.

On its content: in order to emphasize this idea of transfer of knowledge, and for the education of the young and the continued training of the 'not so young', the book includes two chapters (Chapters 1 and 2) that recall the basic knowledge necessary to understand the rest of the book, one chapter (Chapter 3) that synthesizes different material behaviours, and five chapters (Chapters 4–8) that represent original contributions to each of the five broad classes of behaviour.

The first chapter is devoted to *physical mechanisms* of deformation and fracture of metals and alloys, polymers, concrete and wood. The chapter provides a brief summary of the principal mechanisms and is written with a view to justifying the physical hypotheses used in macroscopic modelling.

The second chapter reviews the elements of *continuum mechanics* and *thermodynamics of irreversible processes* which constitute the theoretical tools used in other chapters.

The third chapter presents a schematic *classification* of the behaviour of solids based on experimentation and identification, the main methods of which are described.

The fourth chapter marks the begining of the modelling of different material behaviours. All the subsequent chapters follow the same outline, namely: domain of validity defining more or less precisely the conditions for using the models, phenomenological aspects derived experimentally, general formulation based on thermodynamics, determination and identification of particular models with examples for common materials, and a concise presentation of the associated structural analyses. Within this framework, the fourth chapter presents *linear elasticity*, *thermoelasticity*, and *viscoelasticity*.

The fifth chapter is devoted to *plasticity*. Classical isotropic plasticity is formulated starting with the dissipation potential associated with the flow criterion. Above all, we insist on plasticity with anisotropic hardening which can be used to consider the

cyclic behaviour so important for the prediction of fatigue failure.
The sixth chapter is concerned with the same questions but this time applied to metals and alloys under loads at such intermediate or high temperatures which give rise to the phenomena of viscosity: *viscoplasticity*.
The seventh chapter approaches the fracture of a volume element through the *continuum damage mechanics*. Different models have been worked out for considering and predicting the phenomena of ductile fracture, brittle fracture and fatigue fracture.
The eighth chapter (take heart, this is the last one!) deals with crack mechanics of solids. The *fracture mechanics of crack growth* is approached by energy methods which logically introduce the concept of the energy release rate. This variable, associated with stress intensity factors, is used to formulate models of fracture by instability, of ductile fracture, and of fatigue crack growth.

The book therefore covers the whole field of the mechanics of materials, but in a highly condensed fashion. For a more detailed study, it is advisable to consult the important works listed in the bibliography at the end of each chapter. This book is therefore intended for the reader, who has a good knowledge of the basic elements of continuum mechanics or of the strength of materials, but who wishes to introduce more physics into the design and manufacture of products, with or without the help of computers, or in the safety analysis of structures.

The subject matter of the book occupies a central position between what is taught in the final undergraduate years at engineering schools, at the master's degree and postgraduate levels at universities, and what is expected of professional engineers engaged in research, project engineers, and engineers engaged in the testing of materials. In order to facilitate the use of the book as a manual, each chapter has been written in a way that it can be read independently. Consequently, there is unavoidable repetition of some material. Although, most of the analysis has been carried out in intrinsic notation, the main results are expressed using index notation.

The wise reader can now appreciate the fact that the scope of this book is very wide, and in order to be able to accomplish this, the authors took full advantage of their particularly favourable human environment at the Division Résistance-Fatigue de la Direction des Structures at ONERA, created at the initiative of R. Mazet of the Laboratoire de Mécanique et Technologie at Cachan (ENS de Cachan/Université Paris 6/CNRS). Much of the material of this book is based on the notes of courses given at

Université Paris 6 (3e Cycle de Mécanique Appliquée à la Construction), at ENS de Cachan, at École Polytechnique Féminine, at École Centrale, and at various training sessions of continuing education, summer schools, and foreign universities. The authors here would like to thank publicly all those who participated in numerous and engaging discussions.

The endorsement of the GRECO CNRS '*Grandes Déformations et Endommagement*' was obtained after a critical reading of each chapter by specialists: A. Pineau, F. Sidoroff, A. Zaoui, M. Predeleanu, G. Duvaut, D. Marquis, G. Touzot, C. Oytana, K. Dang Van, D. François, H.D. Bui, R. Labourdette, and a reading in its entirety by P. Muller.

The authors wish to express their firm belief in this method. Thanks to you all for your advice. Thanks are also due to G. Combourieux who prepared the figures, and to Marie-Christine Senechal for her patience in the difficult task of reading the manuscript to ensure a neat and clear typescript.

J. Lemaitre – J. L. Chaboche
Spring, 1979–Autumn 1984

NOTATION

Operators

Operator	Meaning
X	Scalar
$\vec{x}$	Vector with components x_i
$\mathbf{X}$	Second order tensor with components X_{ij}
$\hat{X}$	Virtual quantity
$[X]$	Matrix
$\{X\}$	Column
$\dot{X}$	Time derivative of X $(= \mathrm{d}X/\mathrm{d}t)$
$\delta X/\delta N$	Pseudo derivative (as a function of the cycle number)
$\mathbf{X}^{\mathrm{T}}$	Transpose of $\mathbf{X}$
X^{+}	Laplace–Carson transform
$X \otimes Y$	Convolution product of X with Y
$\mathrm{Re}\, X$	Real part of X
$\mathrm{Im}\, X$	Imaginary part of X
f, g	Unidentified functions
$\mathcal{F}$	Unidentified functional
$Sgn(x)$	$+$ or $-$ sign of scalar x
$\langle x \rangle$	$\langle x \rangle = x$ if $x > 0$, $\langle x \rangle = 0$ if $x < 0$
$\mathbf{X}:\mathbf{Y}$	Double contracted product of $\mathbf{X}$ with $\mathbf{Y}$
X_{I}	First invariant of $\mathbf{X}$: $X_{\mathrm{I}} = Tr(\mathbf{X})$
X_{II}	Second invariant of $\mathbf{X}$: $X_{\mathrm{II}} = \frac{1}{2} Tr(\mathbf{X}^2)$
X_{III}	Third invariant of $\mathbf{X}$: $X_{\mathrm{III}} = \frac{1}{3} Tr(\mathbf{X}^3)$
$\mathbf{X}'$	Deviator of $\mathbf{X}$: $\mathbf{X}' = \mathbf{X} - \frac{1}{3} X_{\mathrm{I}} \mathbf{1}$
J_1	$J_1(\mathbf{X}) = X_{\mathrm{I}}$
J_2	$J_2(\mathbf{X}) = (3X'_{\mathrm{II}})^{1/2}$
J_3	$J_3(\mathbf{X}) = (\frac{27}{2} X'_{\mathrm{III}})^{1/3}$
$H(x)$	$H(x) = 1$ if $x \geqslant 0$, $H(x) = 0$ if $x < 0$
δ	Kronecker delta $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$
X_{Max}	Maximum value of X
X_m	Minimum value of X
$\bar{X}$	Mean value of X
ΔX	Peak to peak amplitude of X (range of X)

List of symbols used as coefficients defined every time they are used.

a	c	e	k	M	Q	S	α	η	ν
A	C	F	K	n	r	v	β	θ	τ
b	d	G	L	N	R	V	γ	λ	
B	D	H	m	P	s	w	δ	μ	

Modelling is the corruption of notation. The authors, unfortunately, have not been able to avoid using the same letter several times for denoting different quantities.

Symbols

Symbol	Meaning
a	Crack length
$\mathbf{a}$	Tensor of elastic moduli
A_{II}	Octahedral shear amplitude
A_k	Thermodynamically associated variable
$\mathbf{A}$	Tensor of elastic compliances
$\vec{b}$	Burgers' vector
c	Specific heat
C^*	Contour integral
D	Damage
D_c	Critical damage
D_u	Ductility
e	Specific internal energy
E	Internal energy
E	Young's modulus of elasticity
$\mathbf{E}$	Finite elastic transformation tensor
f	Plasticity criterion function
$\vec{f}$	Force density
f	Coefficient of Coulomb friction
$\vec{F}$	Force vector
g	Hardening function
$\vec{g}$	Vector $\vec{g} = \overrightarrow{\text{grad}}\ T$
G	Elastic shear modulus
G	Elastic energy release rate
G_s	Threshold of elastic energy release rate
G_c	Critical elastic energy release rate
G_n	Reduced elastic energy release rate
h	Hardening modulus
ΔH	Activation energy
I	Bui integral
J	Viscoelastic creep function
J	Rice integral

K	Kinetic energy
K	Elastic bulk modulus in compression
K, K_I	Stress intensity factors
K_{IC}	Toughness
L	Reference length
L_p	Useful length of a specimen
$\bar{m}$	Coefficient of friction of boundary layer
M	Material point
$\vec{n}$	Vector in the normal direction
N	Current number of cycles
N_R	Number of cycles to failure
N_F	Number of cycles to failure due to pure fatigue
p	Accumulated plastic strain
p_R	Accumulated plastic strain at fracture
P	Power
$\mathbf{P}$	Finite inelastic transformation tensor
$\{q\}$	Column of degrees of freedom
$\vec{q}$	Heat flux vector
Q	Rate of heat input
$\{Q\}$	Column of nodal forces
$\mathbf{Q}$	Tensor of reduced damage
q, Q	Hardening memory variables
r	Heat production per unit volume
r_Y	Dimension of plastic zone
R	Stiffness
R	Relaxation function in viscoelasticity
R	Resistance to ductile tearing
R	Isotropic hardening variable
s	Specific entropy
$\vec{s}$	Unit vector
S	Entropy
S	Specimen cross-section
$\mathbf{S}$	First Piola–Kirchhoff stress tensor
$\mathbf{S}^*$	Second Piola–Kirchhoff stress tensor
t	Time
t_R	Time to fracture
$\vec{t}$	Unit vector
T	Absolute temperature
T_m	Melting point
$\vec{T}$	Stress vector
u	One-dimensional displacement
u_e	Elastic displacement
u_p	Plastic displacement
$\vec{u}$	Displacement vector
$\vec{v}$	Velocity vector
V	Volume
V_k	Internal variable

$\mathscr{V}$	Potential energy
w	Strain energy density
w_e	Elastic strain energy density
w_p	Plastic strain energy density
W	Strain energy
W^*	Complementary energy
$\mathbf{X}$	Tensor variable of kinematic hardening
Y	Elastic energy density release rate
z	Complex variable
Z	Airy stress function
α	Phase of a metal
α	Coefficient of dilatation
$\boldsymbol{\alpha}$	Kinematic hardening variable
γ	Phase of a metal
γ	Density of decohesion energy
$\vec{\gamma}$	Acceleration vector
Γ	Crack front
$\boldsymbol{\Gamma}$	Couple stress tensor
δ	Crack opening displacement (COD)
$\boldsymbol{\Delta}$	Green–Lagrange finite strain tensor
ε	Uniaxial strain
ε_v	True strain
ε_e	Elastic strain
ε_p	Plastic strain
ε_R	Fracture strain
ε_D	Damage threshold strain
ε_H	Hydrostatic strain
ε_{eq}	Von Mises equivalent strain
$\boldsymbol{\varepsilon}$	Strain tensor
$\boldsymbol{\varepsilon}'$	Deviator of strain tensor
$\boldsymbol{\varepsilon}^e$	Elastic strain tensor
$\boldsymbol{\varepsilon}^p$	Plastic strain tensor
η	Exponent in the Paris crack growth law.
θ	Temperature
λ	Lame's constant of elasticity
λ	Multiplying factor
μ	Lame's constant of elasticity in shear
ν^*	Contraction coefficient
ν	Poisson's ratio
ξ	Hardening memory variable
ρ	Mass density
ρ_D	Density of dislocations
ρ	Length of plastic zone
ρ	Electric resistivity

σ	Uniaxial stress
σ_v	True stress
σ_s	Plastic threshold
σ_Y	Yield stress
σ_u	Ultimate stress at fracture
σ_H	Hydrostatic stress
σ_{eq}	Von Mises equivalent stress
$\tilde{\sigma}$	Effective stress
σ^*	Equivalent damage stress
$\boldsymbol{\sigma}$	Stress tensor
$\boldsymbol{\sigma}'$	Deviatoric stress tensor
σ_{ij}	Components of $\boldsymbol{\sigma}$
σ_i	Principal components of $\boldsymbol{\sigma}$
τ	Time
τ	Shear stress
τ_Y	Shear stress at elastic limit
τ_R	Shear stress at fracture limit
φ	Dissipation potential
φ^*	Dual potential
Φ	Specific power of dissipation
χ	Equivalent stress in creep fracture
ψ	Specific free energy
Ω	Potential of viscoplastic dissipation
$\boldsymbol{\Omega}$	Rotation rate tensor
$\boldsymbol{\Omega}$	Area reduction tensor due to damage