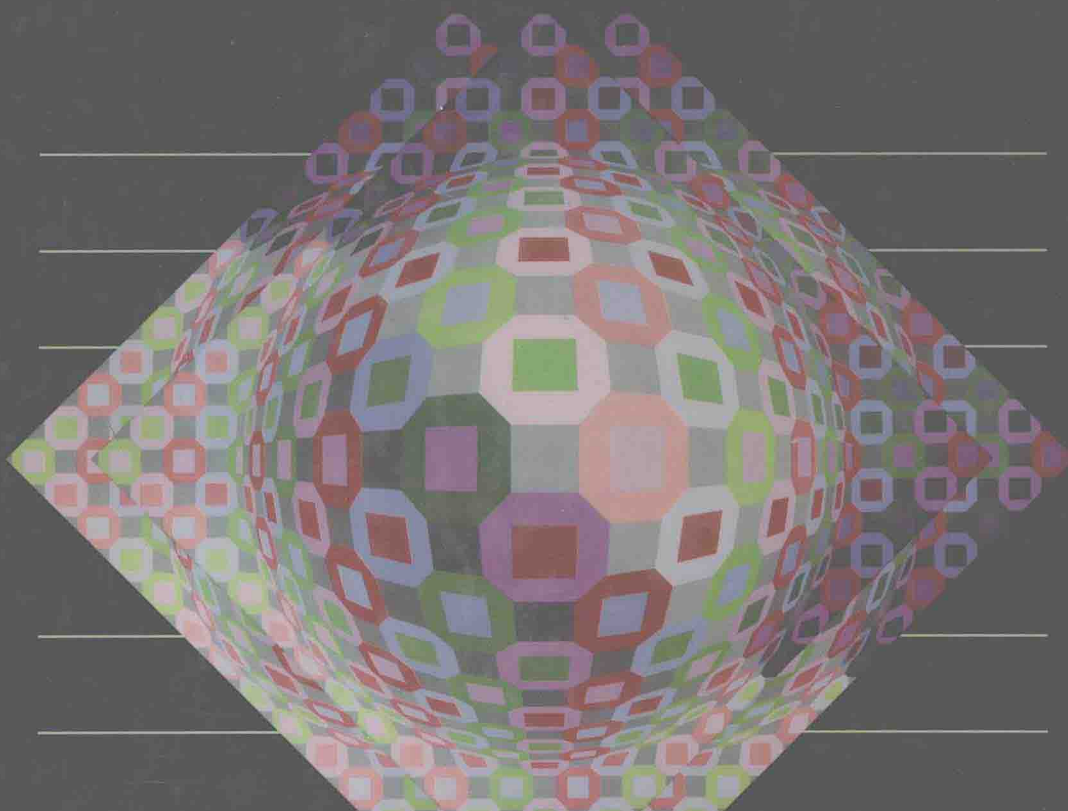
THIRD EDITION

DISCRETE MATHEMATICAL STRUCTURES



Kolman • Busby • Ross

THIRD EDITION

DISCRETE MATHEMATICAL STRUCTURES

Bernard Kolman
Drexel University

Robert C. Busby
Drexel University

Sharon Ross
DeKalb College

PRENTICE HALL, Upper Saddle River, New Jersey 07458

Library of Congress Cataloging-in-Publication Data

Kolman, Bernard.
Discrete mathematical structures / Bernard Kolman, Robert C. Busby, Sharon Ross.—[3rd ed.]
p. cm.
Previous eds. published under title: Discrete mathematical structures for computer science.
Includes index.
ISBN 0–13–320912–1 (alk. paper)
1. Computer science—Mathematics. I. Busby, Robert C. II. Ross, Sharon Cutler. III. Kolman, Bernard. Discrete mathematical structures for computer science. IV. Title.
QA76.9.M35K64 1996
511'.6—dc20 95–9049
CIP

Acquisition Editor: George Lobell
Director of Production and Manufacturing: David W. Riccardi
Editor-in-Chief: Jerome Grant
Production Editor: Elaine Wetterau
Creative Director: Paula Maylahn
Art Director: Amy Rosen
Art Production: Marita Froimson
Cover Design: Christine Gehring-Wolf
Marketing Manager: Frank Nicolazzo
Manufacturing Buyer: Alan Fischer
Cover Art: *Lator*, by Vasarely, Copyright © 1995; ARS, NY/ADAGP, Paris

Printed in the United States of America

10 9 8 7 6 5 4

ISBN 0-13-320912-1

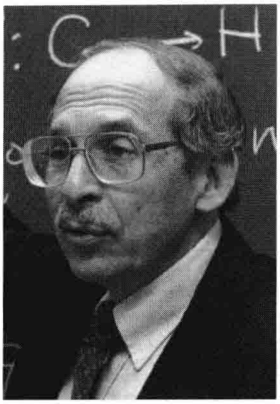
Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

To the memory of Lillie
B. K.

To my wife, Patricia, and our sons, Robert and Scott
R. C. B.

To Bill and bill
S. C. R.

About the Authors



Bernard Kolman

Robert C. Busby

Sharon Cutler Ross

Bernard Kolman received his B.S. (summa cum laude with honors in mathematics and physics) from Brooklyn College in 1954, his Sc.M. from Brown University in 1956, and his Ph.D. from the University of Pennsylvania in 1965, all in mathematics. During the summers of 1955 and 1956 he worked as a mathematician for the U.S. Navy, and IBM, respectively, in areas of numerical analysis and simulation. From 1957–1964, he was employed as a mathematician by the UNIVAC Division of Sperry Rand Corporation, working in the areas of operations research, numerical analysis, and discrete mathematics. He also had extensive experience as a consultant to industry in operations research. Since 1964, he has been a member of the Mathematics Department at Drexel University, where he also served as Acting Head of this department. Since 1964, his research activities have been in the areas of Lie algebras and operations research.

Professor Kolman is the author of numerous papers, primarily in Lie algebras, and has organized several conferences on Lie algebras. He is also well known as the author of many mathematics textbooks that are used worldwide and have been translated into several other languages. He belongs to a number of professional associations and is a member of Phi Beta Kappa, Pi Mu Epsilon, and Sigma Xi.

Robert C. Busby received his B.S. in Physics from Drexel University in 1963 and his A.M. in 1964 and Ph.D. in 1966, both in mathematics from the University of Pennsylvania. From September 1967 to May 1969 he was a member of the mathematics department at Oakland University in Rochester, Michigan. Since 1969 he has been a faculty member at Drexel University, in what is now the Department of Mathematics and Computer Science. He has consulted in applied mathematics in industry and government. This includes a period of three years as a consultant to the Office of Emergency Preparedness, Executive Office of the President, specializing in applications of mathematics to economic problems. He has had extensive experience developing computer implementations of a variety of mathematical applications.

Professor Busby has written two books and has numerous research papers in operator algebras, group representations, operator continued fractions, and the applications of probability and statistics to mathematical demography.

Sharon Cutler Ross received an S.B. in mathematics from the Massachusetts Institute of Technology (1965), an M.A.T. in secondary mathematics from Harvard University (1966), and a Ph.D. also in mathematics from Emory University (1976). In addition, she is a graduate of the Institute for Retraining in Computer Science (1984). She has taught junior high, high school, and college mathematics. She has also taught computer science at the collegiate level. Since 1974, she has been a member of the Department of Mathematics at DeKalb College. Her current professional interests are in the areas of undergraduate mathematics education reform and alternative forms of assessment.

Professor Ross is the co-author of two other mathematics textbooks. She is well known for her activities with the Mathematical Association of America, the American Mathematical Association of Two-Year Colleges, and UME Trends. In addition, she is a full member of Sigma Xi and of numerous other professional associations.

PREFACE

Discrete mathematics for computer science is a difficult course to teach and to study at the freshman and sophomore level for several reasons. It is a hybrid course. Its content is mathematics, but many of its applications, and more than half of its students, are from computer science. Thus careful motivation of topics and previews of applications are important and necessary strategies. Moreover, the number of substantive and diverse topics covered in the course is high, so the student must absorb these rather quickly.

Approach

First, we have limited both the areas covered and the depth of coverage to what we deemed prudent in a *first* course taught at the freshman and sophomore level. We have identified a set of topics that we feel are of genuine use in computer science and that can be presented in a logically coherent fashion. We have presented an introduction to these topics along with an indication of how they can be pursued in greater depth.

For example, we cover the simpler finite-state machines, not Turing machines. We have limited the coverage of abstract algebra to a discussion of semigroups and groups and have given applications of these to the important topics of finite-state machines and error-detecting and error-correcting codes. Error-correcting codes, in turn, have been primarily restricted to simple linear codes.

Second, the material has been organized and interrelated to minimize the mass of definitions and the abstraction of some of the theory. Relations and digraphs are treated as two aspects of the same fundamental mathematical idea, with a directed graph being a pictorial representation of a relation. This fundamental idea is then used as the basis of virtually all the concepts introduced in the book, including functions, partial orders, graphs, and algebraic structures. Whenever possible, each new idea introduced in the text uses previously encountered material and, in turn, is developed in such a way that it simplifies the more complex ideas that follow. Thus partial orders, lattices, and Boolean algebras develop from general relations. This material in turn leads naturally to other algebraic structures.

What Is New in the Third Edition

We have been very pleased by the warm reception given to the first two editions of this book. We have repeatedly been told that the book works well in the classroom because of the unifying role played by two key concepts: relations and digraphs. Thus we have not drastically interfered with the organization or flow of the material. We have added some more flexibility and modularity while continuing the centrality of relations and digraphs. In preparing this edition, we have incorporated many faculty and student suggestions. Although many changes have been made in this edition, our goal continues to be that of *maximizing the clarity of presentation.* To achieve this goal, the following features have been developed in this edition:

New Sections Have Been Added on

- Mathematical Structures (showing similarities and differences in the structure of sets and set operations, integers and integer arithmetic, and matrices and matrix operations).
- The predicate calculus.
- Recurrence relations.
- Functions for computer science.
- Growth of functions.
- Minimal spanning trees.

- A new chapter has been added on Graph Theory.
- Appendix B, Experiments in Discrete Mathematics, has been added.
- Coding exercises have been included in each chapter.
- More material on recursion has been included.
- More material on logic and methods of proof has been presented.

- The presentation on permutations and combinations has been expanded.
- More figures and illustrative examples have been prepared.
- The Exercise Sets have been revised. Many of the routine exercises have been kept, others of this type have been created, and more emphasis has been placed on exercises asking the student to explain and describe.

Exercises

The exercises form an integral part of the book. Many are computational in nature, whereas others are of a theoretical type. Many of the latter and the experiments, to be further described below, require verbal solutions. Answers to all odd-numbered exercises appear in the back of the book. Solutions to all exercises appear in the **Instructor's Manual**, which is available (to instructors only) gratis from the publisher. The Instructor's Manual also includes notes on the pedagogical ideas underlying each chapter, goals and grading guidelines for the experiments further described below, and a test bank.

Experiments

Appendix B contains a number of assignments that we call experiments. These provide an opportunity for discovery and exploration, or a more-in-depth look at various topics discussed in the text. These are suitable for group work. Content prerequisites for each experiment are given in the Instructor's Manual.

End of Chapter Material

Every chapter contains a summary of Key Ideas for Review and a set of Coding Exercises.

Content

Chapter 1 contains a miscellany of basic material required in the course. This includes sets, subsets, and their operations; sequences; division in the integers; and matrices. New to this edition is a section on Mathematical Structures, showing the similarities and differences among some of the concepts discussed earlier in the chapter. Chapter 2 covers logic and related material, including methods of proof and mathematical induction. It includes two sections that are new to this edition: Conditional Statements and Methods of Proof. Chapter 3, on counting, deals with permutations, combinations, the pigeonhole principle, elements of probability, and a new section on Recurrence Relations.

Chapter 4 presents basic types and properties of relations, along with their representation as directed graphs. Connections with matrices and other data structures are also explored in this chapter. Chapter 5 deals with the notion of a function and gives several important examples of functions, including permutations. New to this edition are sections on Functions for Computer Science and Growth of Functions. Chapter 6, new to this edition, provides an elementary introduction to some of the ideas and applications of graph theory. It gives additional flexibility and modularity to the text.

Chapter 7 covers partially ordered sets, including lattices and Boolean algebras. Chapter 8 introduces directed and undirected trees. New to this edition is a section on Minimal Spanning Trees. In Chapter 9 we give the basic theory of semigroups and groups. These ideas are applied in Chapters 10 and 11. Chapter 10 is devoted to finite-state machines. It complements and makes effective use of ideas developed in previous chapters. Chapter 11 treats the subject of binary coding.

Appendix A discusses Algorithms and Pseudocode. The simplified pseudocode presented here is used in some text examples and exercises; these may be omitted without loss of continuity. Appendix B gives a collection of experiments dealing with extensions or previews of topics in various parts of the course.

Use of This Text

This text can be used by students in mathematics as an introduction to the fundamental ideas of discrete mathematics, and as a foundation for the development of more advanced mathematical concepts. If used in this way, the topics dealing with specific computer science applications can be ignored or selected independently as important examples. The text can also be used in a computer science or computer engineering curriculum to present the foundations of many basic computer-related concepts, and provide a coherent development and common theme for these ideas. The instructor can easily develop a suitable course by referring to the chapter prerequisites, which identify material needed by that chapter.

Acknowledgments

We are pleased to express our thanks to the following reviewers of the first two editions: Harold Fredricksen, Naval Postgraduate School; Thomas E. Gerasch, George Mason University; Samuel J. Wiley, La Salle College; Kenneth B. Reid, Louisiana State University; Ron Sandstrom, Fort Hays State University; Richard H. Austing, University of Maryland; Nina Edelman, Temple University; Paul Gormley, Villanova University; Herman Gollwitzer and Loren N. Argabright, both at Drexel University; and Bill Sands, University of Calgary, who brought to our attention a number of errors in the second edition; and of the third edition: Moshe Dror, University of Arizona, Tucson; Lloyd Gavin, California State University at Sacramento; Robert H. Gilman, Stevens Institute of Technology; Earl E. Kymala, California State University at Sacramento; and Art Lew, University of Hawaii, Honolulu. The suggestions, comments and criticisms of these people greatly improved the manuscript.

We wish to express our thanks to Stephen Kolman, who swiftly and skillfully prepared the index; to Emily Whaley, DeKalb College, who helped field-test the experiments; and to Lilian Brady, who critically read the page proofs.

Finally, a sincere expression of thanks goes to Elaine Wetterau, Production Editor, who patiently steered this book through rough seas; to George Lobell, Executive Editor; and to the entire staff of Prentice Hall for their support, encouragement, enthusiasm, interest, and unfailing cooperation during the conception, design, production, and marketing phases of this edition.

B. K.
R. C. B.
S. C. R.

Discrete Mathematical Structures

CONTENTS

CHAPTER 1

FUNDAMENTALS

Prerequisites

There are no formal prerequisites for this chapter; the reader is encouraged to read carefully and work through all examples.

In this chapter we introduce some of the basic tools of discrete mathematics. We begin with sets, subsets, and their operations, notions with which you may already be familiar. Next we deal with sequences, using both explicit and recursive patterns. Then we review some of the basic divisibility properties of the integers. Finally, we introduce matrices and their operations. This gives us the background needed to begin our exploration of mathematical structures.

1.1. Sets and Subsets

Sets

A **set** is any well-defined collection of objects, called the **elements** or **members** of the set. For example, the collection of all wooden chairs, the collection of all onelegged black birds, or the collection of real numbers between zero and one is each a set. Well defined just means it is possible to decide if a given object belongs to the collection or not. Almost all mathematical objects are first of all sets, regardless of any additional properties that they may possess. Thus, set theory is, in a sense, the foundation on which virtually all of mathematics is constructed. In spite of this, set theory (at least the informal brand we need) is easy to learn and use.

One way of describing a set that has a finite number of elements is by listing the elements of the set between braces. Thus the set of all positive integers that are less than 4 can be written as

$$\{1, 2, 3\}. \tag{1}$$

The order in which the elements of a set are listed is not important. Thus $\{1, 3, 2\}$, $\{3, 2, 1\}$, $\{3, 1, 2\}$, $\{2, 1, 3\}$, and $\{2, 3, 1\}$ are all representations of the set given in (1). Moreover, repeated elements in the *listing* of the elements of a set can be ignored. Thus $\{1, 3, 2, 3, 1\}$ is another representation of the set given in (1).

We use uppercase letters such as A, B, C to denote sets and lowercase letters such as a, b, c, x, y, z, t to denote the members (or elements) of sets.

We indicate the fact that x is an element of the set A by writing $x \in A$. We also indicate the fact that x is not an element of A by writing $x \notin A$.

Example 1. Let $A = \{1, 3, 5, 7\}$. Then $1 \in A, 3 \in A$, but $2 \notin A$. ◆

Sometimes it is inconvenient or impossible to describe a set by listing all its elements. Another useful way to define a set is by specifying a property that the elements of the set have in common. We use the notation $P(x)$ to denote a sentence or statement P concerning the variable object x. The set defined by $P(x)$, written $\{x \mid P(x)\}$ is just the collection of all objects x for which P is sensible and true. For example, $\{x \mid x \text{ is a positive integer less than } 4\}$ is the set $\{1, 2, 3\}$ described in (1) by listing its elements.

Example 2. The set consisting of all the letters in the word "byte" can be denoted by {b, y, t, e} or by $\{x \mid x \text{ is a letter in the word "byte"}\}$. ◆

Example 3. We introduce here several sets and their notations that will be used throughout this book.

(a) $Z^+ = \{x \mid x \text{ is a positive integer}\}$.
Thus Z^+ consists of the numbers used for counting: $1, 2, 3, \ldots$.

(b) $N = \{x \mid x \text{ is a positive integer or zero}\}$.
Thus N consists of the positive integers and zero: $0, 1, 2, \ldots$.

(c) $Z = \{x \mid x \text{ is an integer}\}$.
Thus Z consists of all the integers: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$.

(d) $\mathbb{R} = \{x \mid x \text{ is a real number}\}$.

(e) The set that has no elements in it is denoted either by $\{\ \}$ or the symbol $\varnothing$ and is called the empty set. ◆

Example 4. Since the square of a real number is always nonnegative, $\{x \mid x \text{ is a real number and } x^2 = -1\} = \varnothing$. ◆