

Interpreting the Quantum World

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This is a book about the interpretation of quantum mechanics, in particular about how to resolve the measurement problem introduced by the orthodox interpretation of the theory.

The heart of the book is a new result that shows how to construct all possible 'no collapse' interpretations, subject to certain natural constraints and the limitations imposed by the hidden variable theorems. From this perspective one sees precisely where things have gone awry and what the options are. Various interpretations, including Bohm's causal interpretation, Bohr's complementarity interpretation, and the modal interpretation are shown to be special cases of this result, for different choices of a 'preferred' observable. A feature of the book is a novel treatment of the main hidden variable theorems, and an extended critique of contemporary 'decoherence' theories of measurement. The discussion is self-contained and organized so that the technical portions may be skipped without losing the argument.

This book will be of interest to advanced undergraduates and graduate students in philosophy of science, physics, and mathematics with an interest in foundational problems in quantum physics. General readers with some technical sophistication will also find the book of value.

Interpreting the Quantum World

For David Bohm

Preface

It was Michael Whiteman,¹ applied mathematician and mystic at the University of Cape Town, who first introduced me to the Einstein–Podolsky–Rosen ‘paradox’ and other mysteries of quantum mechanics. That was in 1962, and I was hooked. The following year I went to London on a Jan Smuts Memorial Scholarship (a princely sum of £500 for two years, renewable for a third), with the idea of studying philosophy of physics under Popper at the London School of Economics. Popper was visiting the US at the time and, apart from attending Lakatos’ fascinating evening seminar on mathematical discovery,² I soon withdrew from the program. The sudden immersion in courses on the history of philosophy was too much of a culture shock after four years as a mathematics and physics undergraduate.

Whiteman had suggested that I look up his friend G.J. Whitrow³ if things didn’t work out at the LSE. Whitrow’s advice was clear: if I was interested in foundational problems of quantum mechanics, the choice was between Bohm in London or Rosenfeld in Copenhagen. Since a move to Copenhagen seemed too daunting a prospect at the time, I telephoned Bohm at Birkbeck College. He agreed to a meeting, with the understanding that he wasn’t accepting any new graduate students. Eventually, on the strength of a paper I had written on the Einstein–Podolsky–Rosen argument (with some embarrassingly critical comments on Bohm’s ‘hidden variable’ theory), I persuaded him to change his mind.

The theoretical physics group at Birkbeck consisted of Bohm, Hiley, and about half a dozen graduate students. Bohm was interested in understanding algebraic topology as a process-based rather than object-based formal language for physics, and we worked through the section on polyhedral complexes in Hodge’s (1952) book on harmonic integrals in his graduate seminar. I recall being mystified. On the days when he came into Birkbeck, Bohm would arrive in the morning, begin a discussion on a topic he’d been thinking about, and continue until late afternoon. We’d all troop down to the cafeteria for lunch, or sometimes (mercifully, since the food was pretty awful) to one of the restaurants on Charlotte Street nearby. There were lots of gems in those discussions

¹ Author of *Philosophy of Space and Time* (1967), and *The Mystical Life* (1961).

² The material was later published as a four part article in *The British Journal for the Philosophy of Science* **14**, 1963–4, and in an expanded version in Lakatos (1976).

³ Author of *The Natural Philosophy of Time* (1961).

but invariably, after we had brainstormed with Hiley over a particularly intriguing or puzzling idea, Bohm had thought of some entirely new way of looking at the matter by the next session.

After a while, it began to seem increasingly unlikely to me that I'd ever manage to work on anything long enough to launch a PhD dissertation. I started to spend more time away from Birkbeck doing my own reading, and I also began to visit the LSE regularly, where I sat in on Popper's seminar after he returned. It was during this period that I stumbled on a paper by Margenau (1963a) on the measurement problem. I read all the references in the bibliography I could lay my hands on, including London and Bauer's *La Théorie de l'Observation en Mécanique Quantique* (1939), which I managed to obtain from the publisher in Paris and laboriously translated into English.

When I surfaced again at Birkbeck, Bohm asked me to give a seminar presentation to the group on what I had been doing. He seemed genuinely interested and suggested I take a look at papers by Wiener and Siegel (Wiener and Siegel, 1953, 1955; Siegel and Wiener, 1956) as a way of resolving the measurement problem. Bohm's thought was that one should be able to exploit the Wiener–Siegel 'differential space' approach to quantum mechanics to construct an explicit nonlinear dynamical 'collapse' theory for quantum measurement processes.⁴

After that, I knew I had my PhD topic. We eventually published the theory as 'A Proposed Solution of the Measurement Problem in Quantum Mechanics by a Hidden Variable Theory' (Bohm and Bub, 1966a), together with a critique of the Jauch and Piron 'no go' theorem for hidden variables (Bohm and Bub, 1966b). Both articles appeared in the same issue of *Reviews of Modern Physics* as Bell's seminal critique of 'no go' theorems (Bell, 1966). As I recall, the underlying ideas of the theory were Bohm's, but he left it to me to work out the details. Bohm was never very interested in mathematical rigour – he simply 'saw' that things would work out in a certain way, and his physical intuition was always on the right track (although it was sometimes a frustrating business to map out all the twists and turns in his thinking).

I was Bohm's graduate student from 1963 to 1965. After I left Birkbeck we corresponded regularly for a few years. Bohm continued to work on the 'collapse' theory during the late 1960s – I have a lengthy unpublished manuscript of his dating from that time – but eventually we both lost interest.⁵ I discovered quantum logic which, for a while, I thought was the answer to all the conceptual puzzles of quantum mechanics. Bohm found my fascination with quantum logic incomprehensible, and our correspondence languished.

⁴ Curiously, he did not bring up his own 1952 hidden variable theory in this connection, and I don't recall him discussing the theory while I was a student at Birkbeck.

⁵ The manuscript ('On the Role of Hidden Variables in the Fundamental Structure of Physics') has now been published in *Foundations of Physics* 26, 719–86 (1996). The Bohm–Bub theory has recently been resurrected by Ron Folman (1994, 1995), who has been looking for experimental confirmation in a possible deviation from the quantum mechanically predicted experimental distribution for the decay time of massive particles, specifically the tau lepton, in the very short decay time region. See also OPAL Collaboration (1996). For an account of some earlier experimental tests of the theory, see Belinfante (1973), chapter 4.

My first and most important intellectual debt is to Bohm, and this book is dedicated to his memory.

After I left Birkbeck, I had a one-year post-doctoral position in the Chemistry Department at the University of Minnesota. Alden Mead, a physical chemist in the department, visited Birkbeck on a sabbatical during my last year there and offered me a position as his assistant. I was supposed to work on fundamental length theories, but I don't think much came of that. Ford Hall, the home of the Minnesota Center for the Philosophy of Science, beckoned from just across the mall.

The Center was an exciting place. Herbert Feigl and Grover Maxwell were there permanently, and there were many visitors. Hilary Putnam passed through and gave a talk on quantum logic that profoundly influenced my thinking on the interpretation of quantum mechanics. I was captivated by von Neumann's (1939) notion of a non-Boolean logic for quantum systems, and the idea that what is conceptually puzzling about quantum mechanics relative to classical mechanics is that the properties of quantum systems 'fit together' in a non-Boolean way, and this is what we ought to try and understand.

At the Center I met Bill Demopoulos. We were both intrigued by quantum logic – after a brief joint flirtation with Whitehead's process philosophy it was like a breath of fresh air. Working through the Kochen and Specker papers (1965, 1967) together was the beginning of a long collaboration and friendship. The ideas on quantum logic in my book *The Interpretation of Quantum Mechanics* (1974) reflect this collaboration. I still think the essential difference between classical and quantum mechanics is captured by the insight that going from classical to quantum mechanics involves the transition from a Boolean to a non-Boolean possibility structure for the properties of a physical system (see Demopoulos, 1976).

Bohm's ideas on hidden variables, and von Neumann's concept of a quantum logic understood as a possibility structure for events, have always been the two main influences on my approach to the conceptual problems of quantum mechanics. In a sense, this book reconciles these two opposing themes, from the perspective of a 'modal' interpretation in the sense of van Fraassen (1973, 1974, 1981, 1991), although the implementation of this notion is very different from van Fraassen's. But my more immediate intellectual debt is to Rob Clifton, with whom I have enjoyed a lively and extremely productive email correspondence and collaboration for the past two years or so. Much of the book is an extended discussion of our joint paper on a uniqueness theorem for 'no collapse' interpretations of quantum mechanics (Bub and Clifton, 1996).

I had constructed a class of 'no collapse' interpretations (Bub, 1992a, b, 1993a, 1994b, 1996), which I presented variously as versions of the modal interpretation or as Bohmian interpretations (in the sense of Bohm's 1952 hidden variable theory). The basic idea came out of my analysis of Bohr's reply to the Einstein–Podolsky–Rosen argument (Bub, 1989, 1990). Clifton was working on modal interpretations that exploit the biorthogonal decomposition theorem and proved a result justifying the common

framework of the Kochen and Dieks formulations as unique, subject to certain constraints (Clifton, 1995b). After I received a draft of this theorem, I proved a uniqueness theorem for the class of ‘no collapse’ interpretations I had constructed (Bub, 1994a, 1995c). Later, Clifton saw the possibility of replacing the assumptions in my original uniqueness theorem with fewer and more natural assumptions, which eventually led to our joint theorem. My account of the motivation for the theorem, and the significance of the theorem for the modal interpretation, Bohmian mechanics, and Bohr’s complementarity interpretation draws on the analysis in our joint paper. Needless to say, I bear sole responsibility for any foolishness in this exposition.

Like many others working on problems in the foundations of quantum mechanics, I have found enlightenment and inspiration in the writings of John Bell and David Mermin, and over the years I have benefited from discussions on the interpretation problem and the measurement problem with Roger Cooke, Bas van Fraassen, R.I.G. Hughes, Allen Stairs, Itamar Pitowsky, Michael Redhead, Harvey Brown, Jeremy Butterfield and, more recently, David MacCallum, David Albert, Andrew Elby, Jeff Barrett, Ron Folman, Pekka Lahti, Bradley Monton, Michael Dickson, Guido Bacciagaluppi and Jitendra Subramanyam. Rob Clifton, Bradley Monton, David MacCallum, and Jo Clegg read the manuscript in various drafts, and I have incorporated many of their suggestions for improvements in both style and content.

Finally, I owe a special debt to my wife, Robin Shuster – muse extraordinaire and Socratic midwife to many of the ideas presented here. I doubt that I would have completed the book without her constant encouragement and support.

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Introduction

This is a book about the interpretation of quantum mechanics, and about the measurement problem. The conceptual entanglements of the measurement problem have their source in the orthodox interpretation of ‘entangled’ states that arise in quantum mechanical measurement processes. The heart of the book is a uniqueness theorem (Bub and Clifton, 1996; see chapter 4) that characterizes alternative ‘no collapse’ interpretations of the theory, in particular observer-free interpretations that don’t involve the measurement problem. From the perspective of the uniqueness theorem, one sees precisely where things have gone awry and what the options are.

One might wonder why, and in what sense, a fundamental theory of how physical systems move and change requires an interpretation. Quantum mechanics is an irreducibly statistical theory: there are no states of a quantum mechanical system in which all dynamical variables have determinate or ‘sharp’ values – no states that are ‘dispersion-free’ for all dynamical variables. Moreover, so-called ‘no go’ theorems exclude the possibility of defining new states in terms of ‘hidden variables,’ in which all dynamical variables – or even certain finite sets of dynamical variables – have determinate values, if we assume that the values assigned to functionally related dynamical variables by the new hidden variable states are subject to certain constraints, and we require that the quantum statistics can be recovered by averaging over these states. So it is standard practice to refer agnostically to ‘observables’ rather than dynamical variables (which suggest determinate values evolving in time), and to understand quantum mechanics as providing probabilities for the outcomes of measurements of observables under physically well-defined conditions.

This neutrality only goes so far. All standard treatments of quantum mechanics take an observable as having a determinate value if the quantum state is an eigenstate of that observable.⁶ If the state is not an eigenstate of the observable, no determinate value is attributed to the observable. This principle – sometimes called the ‘eigenvalue–eigenstate link’⁷ – is explicitly endorsed by Dirac (1958, pp. 46–7) and von Neumann (1955, p. 253), and clearly identified as the ‘usual’ view by Einstein, Podolsky, and

⁶ For an account of quantum states and their representation in Hilbert space see the appendix.

⁷ The term is due to Arthur Fine (1973, p. 20).

Rosen (1935) in their classic argument for the incompleteness of quantum mechanics (see chapter 2). Since the dynamics of quantum mechanics described by Schrödinger's time-dependent equation of motion is linear, it follows immediately from this orthodox interpretation principle that, after an interaction between two quantum mechanical systems that can be interpreted as a measurement by one system on the other, the state of the composite system is not an eigenstate of the observable measured in the interaction, and not an eigenstate of the indicator observable functioning as a 'pointer.' So, on the orthodox interpretation, neither the measured observable nor the pointer reading have determinate values, after a suitable interaction that correlates pointer readings with values of the measured observable. This is the measurement problem of quantum mechanics.

There are three possible ways of resolving the measurement problem: We adopt what Bell (1990) has termed a 'FAPP' ('for all practical purposes') solution, or we change the linear dynamics of the theory (which, as I see it, means changing the theory), or we change the orthodox Dirac–von Neumann interpretation principle.

FAPP solutions range from the Daneri–Loinger–Prosperi (1962, 1966) quantum ergodic theory of macrosystems⁸ to the currently fashionable 'decoherence' theories. Essentially, the idea here is to exploit the fact that a macroscopic measuring instrument is an open system in virtually continuous interaction with its environment. Because of the typical sorts of interactions that take place in our world between such systems and their environments, it turns out that almost instantaneously after a measurement interaction, the 'reduced state' of the measured system and measuring instrument as a composite subsystem of the universe is, for all practical purposes, indistinguishable from a state that supposedly can be interpreted as representing a classical probability distribution over determinate but unknown values of the pointer observable. The information required to exhibit characteristic quantum interference effects between different pointer-reading states is almost immediately irretrievably lost in the many degrees of freedom of the environment. Since there are well-known difficulties with such an 'ignorance interpretation,' there is usually a further move involving an appeal to Everett's (1957, 1973) 'relative state' or 'many worlds' interpretation of quantum mechanics, where determinateness is only claimed in some relative sense. I discuss versions of this approach in chapter 8, where I argue that the measurement problem is not resolved by this manoeuvre.

The Bohm–Bub 'hidden variable' theory (1966a) modifies the linear dynamics by adding a nonlinear term to the Schrödinger equation that effectively 'collapses' or projects the state onto an eigenstate of the pointer reading and measured observable in a measurement process (the resulting eigenstate depending on the hidden variable). Currently the Ghirardi–Rimini–Weber theory (1986), with later contributions by Pearle (Ghirardi, Grassi, and Pearle, 1990, 1991; Pearle, 1989, 1990), is a much more

⁸ For a critique, see Bub (1968).

sophisticated stochastic dynamical ‘collapse’ theory, formulated as a continuous spontaneous localization theory.

The remaining possibility is to adopt an alternative principle for selecting the set of observables that have determinate values in a given quantum state. This was Bohm’s approach, and also – very differently – Bohr’s. Bohm’s 1952 hidden variable theory or ‘causal’ interpretation (Bohm, 1952a; Bohm and Hiley, 1993) takes the position of a system in configuration space⁹ as determinate in every quantum state. Certain other observables can be taken as determinate at a given time together with this ‘preferred’ always-determinate observable, depending on the state at that time. Alternative formulations of Bohm’s theory present different accounts of ‘nonpreferred’ observables such as spin. On the formulation proposed here, the theory is a ‘modal’ interpretation of quantum mechanics, in the broad sense of van Fraassen’s notion (see chapter 6). For Bohr, an observable has a determinate value only in the context of a specific, classically describable experimental arrangement suitable for measuring the observable. Since the experimental arrangements suitable for locating a quantum system in space and time, and for the determination of momentum–energy values, turn out to be mutually exclusive, there is no unique description of the system in terms of the determinate properties associated with the determinate values of a fixed preferred observable. So which observables have determinate values is settled pragmatically by what we choose to observe, via the classically described measuring instruments we employ, and is not defined for the system alone. Bohr terms the relation between space–time and momentum–energy concepts ‘complementary,’ since both sets of concepts are required to be mutually applicable for the specification of the classical state of a system.

What is generally regarded as the ‘Copenhagen interpretation’ is some fairly loose synthesis of Bohr’s complementarity interpretation and Heisenberg’s ideas on the significance of the uncertainty principle. It is usual to pay lip service to the Copenhagen interpretation as the ‘orthodox’ interpretation of quantum mechanics, but the interpretative principle behind complementarity is very different from the Dirac–von Neumann principle. (I discuss the relationship in detail in sections 7.1 and 7.2). Unlike Dirac and von Neumann, Bohr never treats a measurement as an interaction between two quantum systems, and hence has no need for a special ‘projection postulate’ to replace the linear Schrödinger evolution of the quantum state during a measurement process. Both Dirac and von Neumann introduce such a postulate to describe the stochastic projection or ‘collapse’ of the state onto an eigenstate of the pointer reading and measured observable – a state in which these observables are determinate on their interpretation. (See Dirac, 1958, p. 36, and von Neumann, 1955, p. 351 and pp. 417–18.) The complementarity interpretation avoids the measurement problem by selecting as determinate an observable associated with an individual quantum ‘phenomenon’ manifested in a measurement interaction involving a specific classically describable experimental arrangement. Certain other observables, regarded as measured in the

⁹ For an N -particle system, the configuration space of the system is a $3N$ -dimensional space, coordinatized by the $3N$ position coordinates of the particles.

interaction, can be taken as determinate together with this observable and the quantum state.

Einstein viewed the Copenhagen interpretation as ‘a gentle pillow for the true believer.’¹⁰ For Einstein, a physical system has a ‘being-thus,’ a ‘real state’ that is independent of other systems or the means of observation (see the quotations in section 1.1 and section 6.1). He argued that realism about physical systems in this sense is incompatible with the assumption that the state descriptions of quantum mechanics are complete. What Einstein had in mind by a ‘completion’ of quantum mechanics is not entirely clear, but on one natural way of understanding this notion (as an observer-free ‘no collapse’ interpretation subject to certain physically plausible constraints), the possible completions of quantum mechanics are fully characterized by the uniqueness theorem in chapter 4.¹¹

This book begins with a survey of the problem of interpretation, as it arises in the debate between Einstein and Bohr. Einstein’s discomfort with quantum mechanics cannot be attributed to an aversion to indeterminism. He did not argue that quantum mechanics must be incomplete *because* ‘God does not play dice with the universe.’ Rather, as Pauli put it, Einstein’s ‘philosophical prejudice’ was realism, not determinism (section 1.1). It is not that all indeterministic or stochastic theories were problematic for Einstein. What Einstein objected to were stochastic theories that violate certain realist principles; or rather, he objected to taking such theories as anything more than predictive instruments that would ultimately be replaced by a complete explanatory theory.

Chapter 1 continues with a discussion of the transition from classical to quantum mechanics, and a formulation of the measurement problem as a problem generated by the orthodox (Dirac–von Neumann) interpretation of the theory. My main aim here is to bring out the different ways in which dynamical variables and properties are represented in the two theories. In classical mechanics, the dynamical variables of a system are represented as real-valued functions on the phase space of the system and form a commutative algebra. The subalgebra of idempotent dynamical variables (the characteristic functions) represent the properties of the system and form a Boolean algebra, isomorphic to the Boolean algebra of (Borel) subsets of the phase space of the system. In quantum mechanics, the dynamical variables or ‘observables’ of a system are represented by a noncommutative algebra of operators on a Hilbert space, a linear vector space over the complex numbers, and the subalgebra of idempotent operators (the projection operators) representing the properties of the system is a non-Boolean algebra isomorphic to the lattice of subspaces of the Hilbert space. So the transition from classical to quantum mechanics involves the transition from a Boolean to a non-Boolean structure for the properties of a system.

There are restrictions on what sets of observables can be taken as simultaneously determinate without contradiction, if the attribution of determinate values to observ-

¹⁰ In a letter to Schrödinger, dated May, 1928. Reprinted in Przibram (1967, p. 31).

¹¹ See Fine (1986), especially chapter 4, for a different interpretation of Einstein’s view.

ables is required to satisfy certain constraints. The ‘no go’ theorems for hidden variables underlying the quantum statistics provide a series of such results that severely limit the options for a ‘no collapse’ interpretation of the theory.

In chapter 2, I present the Einstein–Podolsky–Rosen (1935) incompleteness argument, and several versions of Bell’s extension of the argument to a ‘no go’ theorem demonstrating the inconsistency of stochastic and deterministic hidden variables, satisfying certain locality and separability constraints, with the quantum statistics.

Chapter 3 deals with the Kochen and Specker (1967) ‘no go’ theorem, showing the impossibility of assigning determinate values to certain finite sets of observables if the value assignments are required to preserve the functional relations holding among the observables. I present a new proof of the theorem for a set of 33 observables (1-dimensional projectors), based on a classical tautology that is quantum mechanically false proposed by the logician Kurt Schütte in an early (1965) unpublished letter to Specker.

Chapter 4 introduces the problem of interpretation, and contains the proof of the uniqueness theorem demonstrating that, subject to certain natural constraints, all ‘no collapse’ interpretations of quantum mechanics can be uniquely characterized and reduced to the choice of a particular preferred observable as determinate. The preferred observable and the quantum state at time t define a (non-Boolean) ‘determinate’ sublattice in the lattice \mathcal{L} of all subspaces of Hilbert space – the sublattice of propositions that can be true or false at time t . The actual properties of the system at time t are selected by a 2-valued homomorphism (a yes–no map) on the determinate sublattice at time t , so the range of possibilities for the system at time t is defined by the set of 2-valued homomorphisms on the determinate sublattice. From this ‘modal’ perspective, the possibility structure of a quantum world is represented by a *dynamically evolving* (non-Boolean) sublattice in \mathcal{L} , while the possibility structure of a classical world is fixed for all time as the Boolean algebra \mathcal{B} of subsets of a phase space. The dynamical evolution of the quantum state tracks the evolution of possibilities (and probabilities defined over these possibilities) through the evolution of the determinate sublattice, rather than actualities, while the dynamically evolving classical state defines the actual properties in a classical world as a 2-valued homomorphism on \mathcal{B} and directly tracks the evolution of actual properties. In a quantum world, the dynamical state is distinct from the ‘property state’ (defined by a 2-valued homomorphism on the determinate sublattice), while the classical state doubles as a dynamical state and a property state.

Different choices for the preferred determinate observable correspond to different ‘no collapse’ interpretations of quantum mechanics. In chapter 5, I show how the orthodox (Dirac–von Neumann) interpretation without the projection postulate can be recovered from the theorem, and how the measurement problem is avoided in ‘no collapse’ interpretations by an appropriate choice of the preferred determinate observable. Property states must evolve in time so as to reproduce the quantum statistics over the determinate sublattices defined by the dynamical evolution of